Reliability-Based Design in Civil Engineering

MILTON E. HARR

Reliability-Based Design in Civil Engineering

Milton E. Harr
Professor of Civil Engineering
Purdue University

McGraw-Hill Book Company
New York St. Louis San Francisco Auckland Bogotá
Hamburg Johannesburg London Madrid Milan
Mexico Montreal New Delhi Panama
Paris São Paulo Singapore
Sydney Tokyo Toronto

Library of Congress Cataloging-in-Publication Data

Harr, Milton Edward, 1925-
Reliability-based design in civil engineering.

Bibliography: p.
Includes index.
1. Reliability (Engineering) 2. Engineering design.
3. Civil engineering. I. Title.
TA169.H37 1987 624 86-21479
ISBN 0-07-026697-2

1234567890 DOC/DOC 8932109876

ISBN 0-07-026697-2

The editors for this book were Joan Zseleczky and Ingeborg M. Stochmal, the designer was Naomi Auerbach, and the production supervisor was Teresa F. Leaden. It was set in Century Schoolbook by Denver Data Center.

Printed and bound by R. R. Donnelley & Sons, Inc.

To Faith, Harvey, Heather, Nicholas,
Karen, Gerald, Jamie, Seth,
Robert . . .

Preface

This book was written with the following main objectives.

1. To provide practicing civil engineers and civil engineering students with concepts and techniques for evaluating the reliability of engineering systems.
2. To provide civil engineers the means of assessing and improving upon the reliability of their designs.
3. To present this information as an organized, logical, and systematic body of knowledge.
4. To provide the civil engineer with an understanding of the important idea of probability theory as the encoding of information and to illustrate its relevance with application to practical problem situations.
5. To provide this knowledge so as to enable the civil engineer to understand and assimilate the literature being developed.

In no field of engineering are practitioners faced with a more complex set of operating conditions than those concerned with the action of civil engineering systems subject to natural or induced loadings. Unlike their colleagues in other engineering disciplines, who often have the advantage of observing the performance of many prototypes under anticipated loadings, civil engineers are generally faced with systems that are custom-built and tailored to special demands and specific locations. To be sure, similarities do exist and information can be and is transferred from previous successes and failures. However, to paraphrase Tolstoy: All successes are similar, but each failure is unique!

This book has evolved from lecture notes and from the author's involvement as a consulting engineer in a number of civil engineering projects. Most of the material has been taught in courses in civil engineering at Purdue University. In addition, the author has presented much of this material in a number of one-week short courses to many groups of practicing engineers and at universities throughout the world.

The form of this book was designed to meet the needs of the practitioner who must be self-taught, as well as the student who will be guided through the material. Consequently, special efforts were made to make the text self-contained and to provide many examples to illustrate usage. The text presupposes a working knowledge of mathematics and an engineering background at the level of civil engineering juniors; however, no previous formal training in probability or statistics is required.

A number of computer program listings are included as figures in the last sections to the chapters. They were written so as to be user-friendly (interactive) in portable Fortran 77. All programs have been used successfully by the author and his students at Purdue University. Even so, there is no guarantee that they are free of bugs. Although the text presupposes that the reader has the capability of running the computer programs, many graphs and charts are presented that permit direct determination of results.

Since much of the material and many of the results in this book are new (or are not readily obtainable), it is important that the reader have available exercises and examples that promote assimilation and application. Consequently, approximately 250 problems are given at the end of the chapters (most with answers), as well as over 140 worked examples.

The author would like to acknowledge his indebtedness to the many students, associates, and colleagues who attended his lectures and who through their participation and comments led to the development of this book. To Catherine Ralston go special thanks for her splendid assistance in the preparation of the manuscript. Finally, the author would like to express his appreciation to Joan Zseleczky for her encouragement and to Ingeborg Stochmal for her editorial acumen.

Milton E. Harr

Contents

Chapter

1

Elements of Probability

1.1 Introduction

The trend in civil engineering today, more than ever before, is toward providing economical designs at specified levels of safety. Often these objectives necessitate a prediction of the performance of a system for which there exists little or no previous experience. Current design procedures, which generally have been learned only after many trial and error iterations, lacking precedence, often fall short of expectations in new or alien situations. In addition, there is an increasing awareness that the raw data, on which problem solutions are based, themselves exhibit significant variability. It is the aim of this book to demonstrate how concepts of probability theory may be used to supplement the civil engineer's judgment in such matters.

The primary function of engineering is to accommodate the transfer of energy through a system. The kind of energy generally specifies the type of engineer: aeronautical, agricultural, biomedical, chemical, electrical, industrial, mechanical, nuclear, and that of the civilian population. In this sense the system is a filter that is required to transform the induced applied (external) energy to tolerable levels so as to accomplish an intended purpose. It is within this framework that the civil engineers plan, design, fabricate, construct, and maintain buildings, bridges, dams and levees, foundations, power plants, offshore structures, tunnels, aqueducts, highways, airports, harbors and ports, canals, pipelines, sanitary facilities, pavements, earth embankments, excavations, and so on.

Analysis is the central theme of civil engineering design. It is the idealization of a system, which admits to simple but logical mathematical solution and still contains the essential elements of the actual system. Traditionally, induced loadings are modeled by completely defined, simple geometric or analytical representations. Material characterization is assumed to be complete, and inherent properties are taken to be stable and uniquely defined. Until very recently it was the objective of developed analytical procedures to provide the civil engineer with methods that could be used in making value judgments concerning likely scenarios while working at a desk with a piece of paper, a pencil, and a slide rule.

Induced loadings to civil engineering systems are never completely known. Among these are uncertainties with respect to the frequency and intensity of earthquakes; the flow of surficial water and groundwater and of toxic and hazardous materials; the action and variability of wind and waves, heat and cold, freezing and thawing, chemical and environmental factors; vibrations and shock; vehicular and pedestrian traffic, its distribution and weights; construction equipment, sequencing, and processes. Almost all induced loadings are random, and all systems may often be subjected to overloads.

All engineering materials contain microcrystalline imperfections or faults called *dislocations* (Radavich, 1980), which initiate cracks or permit their propagation. No two manufactured objects can be exactly the same. To date no general theory exists that relates the strength to the deformation of a body. No single framework accounts for such common phenomena as plastic flow and brittle fracture of metals, fatigue and creep, elastic and inelastic response. Defects exist from at least the atomic packing level up through the largest of manufactured objects. Bourgault (1980) quotes S. A. Wenk, who gave the following definitions:

Material—collection of defects

Acceptable material—fortuitous or organized collection of defects

Unacceptable material—unfortunate collection of defects

All civil engineering systems are founded on, or in, the soil, which, in turn, is composed of complex aggregations of discrete particles, in arrays of varying shapes, sizes, and orientations. Voids between the particles are of various orientations and sizes, and may serve to transport several fluids.

Figure 1.1.1 compares the elements of the civil engineering design process to the links of a chain. One first obtains samples from which parameters (properties) are extracted using established testing procedures,

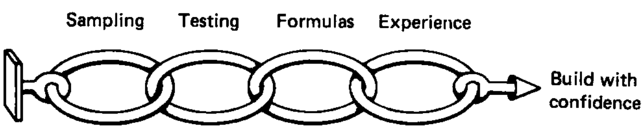

Figure 1.1.1 Model of the civil engineering design process.

which are then substituted into appropriate formulas. Other variations and other linkages are possible, and not all links apply to all civil engineering systems. It is the last link of the metaphorical chain that has special importance in the present context. The fourth link, stated as experience, is generally specified in civil engineering as a *factor of safety* and is very much problem-oriented. For example, the stability of the upstream slope of an earth dam requires minimum (tolerable) factors of safety ranging from 1.05 for rapid drawdown to 10.0 or more for an adequate impervious upstream blanket, with a factor of 1.3 for long-term stability. The factors are measures that were discovered with great difficulty and only after considerable experience with many similar structures. Clearly, lacking prior experience, conventional design procedures may be sadly deficient in new situations. However, within the established design technology of today, these factors present considerable information as to the adequacy of a system and represent important indexes of performance.

The ways in which civil engineering systems fail, the occurrence and frequency of failure, its economic and social consequences, demonstrate considerable differences between hypothetical and actual systems. Induced loadings, site characterization, properties of materials, developed formulations and procedures, and the adequacy of predicted sizes and shapes of the system and its elements are far from certain. All are subject to complex interrelationships, material defects, structural deficiencies, human errors, and hence to varying degrees of randomness. A guiding motto proposed by Alfred North Whitehead (1920) is: "Seek simplicity and distrust it."

What is failure? A bridge or building collapses: it is failure. A dam is breached: it is failure. Is it failure if a section of a heavily traveled highway is "jammed" during rush hours, but operates adequately at other times? Add to this that the money that could have been used to improve this section of road was used to make another part of the system safer.

The failure of a system is assessed by its inability to perform its intended function adequately on demand for a period of time and under specified conditions. Its antithesis, the measure of success, is called *reliability*. Failure is highly qualitative and subjective; reliability, on the other hand, can be defined, quantified, tested, and confirmed.

The customary engineering definition of reliability is as follows: *Reliability is the probability of an object (item or system) performing its required function adequately for a specified period of time under stated conditions*. This definition contains four essential elements:

1. Reliability is expressed as a probability.
2. A quality of performance is expected.

3. It is expected for a period of time.
4. It is expected to perform under specified conditions.

That any meaningful scale of success or failure must be addressed in probabilistic terms is evident from the variability and lack of determinacy of civil engineering systems. Probabilities are objective measures of the likelihood of occurrence of random events and as such provide quantitative assessments of system adequacy. It is the purpose of reliability-based design to produce an engineered system whose failure would be an event of very low probability. Probabilities of failure are the most significant indexes of reliability. Being objective, they admit directly to comparisons of the risk of failure of different systems, or of the components of a single system, and under varying operating conditions. This capability for both traditional and untried scenarios is the very fabric of civil engineering design.

1.2 Axioms

Within the context of engineering usage there are two primary definitions of the concept of probability: *relative frequency* and *subjective* interpretation.

Historically, the measure first offered for the probability of an outcome was its relative frequency: if an outcome A occurs T times in N equally likely trials, the probability of the outcome A is

$$P[A] = \frac{T}{N} \tag{1.2.1a}$$

Implied in Eq. (1.2.1*a*) is that the probability of an outcome A equals the number of outcomes favorable to A (within the meaning of the experiment) divided by the total number of possible outcomes, or

$$P[A] = \frac{\text{favorable outcomes}}{\text{total possible outcomes}} \tag{1.2.1b}$$

This definition was first formulated by Laplace in 1812.

Example 1.2.1 Find the probability of drawing a red card from an ordinary well-shuffled deck of 52 cards.

Solution Of the 52 equally likely outcomes, there are 26 favorable outcomes (red cards). Hence,

$$P[\text{drawing red card}] = \frac{26}{52} = \frac{1}{2}$$

Understood in the example is that if one were to repeat the process a very large number of times, a red card would appear in one-half of the trials. This is an example of the relative frequency interpretation. Now, what meaning can be associated with the statement: the probability of failure of a proposed structure is 1% (P[failure] = 0.01)? The concept of repeated trials is meaningless; the structure will be built only once, and it will either fail or be successful during its design lifetime. It cannot do both. Here we have an example of the subjective interpretation of probability. It is a measure of information as to the *likelihood* of the occurrence of an outcome.

In engineering applications, subjective probability is generally more useful than the relative frequency concept. However, the basic rules governing both are identical. As an example, we note that both concepts specify the probability of an outcome to range between, and to include numerical values of, zero and one. The lower limit indicates that there is no likelihood of occurrence; the upper limit corresponds to a certain outcome, that is, the probability of an outcome A ranges between zero and unity,

Axiom I $$0 \leq P[A] \leq 1 \tag{1.2.2a}$$

The certainty of an outcome C is a probability of unity,

Axiom II $$P[C] = 1 \tag{1.2.2b}$$

Equations (1.2.2*a*) and (1.2.2*b*) provide two of the three axioms of the theory of probability. The third axiom requires the concept of *mutually exclusive* outcomes. Two outcomes are mutually exclusive if they cannot occur simultaneously. The third axiom states that the probability of the occurrence of the sum of a number of mutually exclusive outcomes A_1, $A_2, \ldots, A_N$ is the sum of their individual probabilities (*addition rule*), or

Axiom III $$P[A_1 + A_2 + \cdots + A_N] = P[A_1] + P[A_2] + \cdots + P[A_N] \tag{1.2.2c}$$

As a very important application of these axioms consider a proposed design for a structure. After construction, only one of two outcomes can occur in the absolute structural sense: either it is successful or it fails. These are mutually exclusive outcomes. They are also *exhaustive* in that, within the sense of the example, no other outcomes are possible. Hence, the second axiom, Eq. (1.2.2*b*), requires

$$P[\text{success} + \text{failure}] = 1$$

Since they are mutually exclusive, the third axiom specifies that

$$P[\text{success}] + P[\text{failure}] = 1$$

The probability of the success of a structure is its reliability R. Designating the probability of failure as $p(f)$, we have the important expression

$$R + p(f) = 1 \tag{1.2.3}$$

Example 1.2.2 Only three contractors, A, B, and C, are bidding for a contract. A has half the chance that B has; B is two-thirds as likely as C to be awarded the contract. What is the probability for each to get the job if only one of them will be successful?

Solution Since there are only three contractors and one must be successful,

$$P[A + B + C] = 1$$

As only one will be successful, they are mutually exclusive,

$$P[A] + P[B] + P[C] = 1$$

Given $P[A] = P[B]/2$ and $P[B] = 2\,P[C]/3$, we have

$$\frac{P[B]}{2} + P[B] + \frac{3}{2}P[B] = 1$$

whence, $P[A] = 1/6$, $P[B] = 1/3$, and $P[C] = 1/2$.

1.3 Conditional Probability

A useful graphic representation of outcomes is the *Venn diagram.* Outcomes are usually shown as simple geometric shapes. Examples are given in Fig. 1.3.1. The large rectangle represents the universal set, or sample space, S. The rectangles labeled P_1 and P_2 and the circle M designate subsets of outcomes. For example, the sample space may represent all the concrete delivered to a highway pavement construction site. P_1 and P_2 may correspond to the concrete produced at two plants.

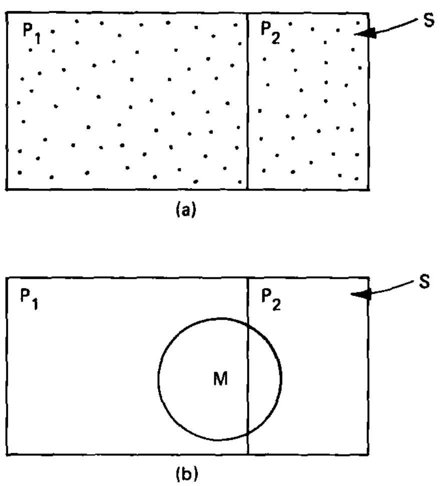

Figure 1.3.1 Venn diagram. (*a*) Sample space for two outcomes. (*b*) Sample space for three outcomes.

If the rectangular region S represents the total collection of pertinent probabilities, $P[S] = 1$. The probability associated with a particular outcome is the sum of the elementary outcomes that are contained within the respective regions in the Venn diagram. For example, suppose the dots shown in Fig. 1.3.1*a* represent the percentage of concrete delivered at a highway pavement construction site during the entire job. The order of delivery is not significant. Then the percentage of the concrete provided by a particular plant (P_1 or P_2) would be the sum of the percentages contained within its designated region. Two plants P_1 and P_2 are assumed here to provide all the concrete.

Suppose test results on concrete cylinders judge the concrete to meet specified standards if the 28-day unconfined compressive strength is not less than 4500 lb/in^2. Continuing, suppose that plant P_1 produces 66% and plant P_2 34% of the concrete used, and that 90% of the concrete produced from plant P_1 met the standard, whereas 95% did so from plant P_2. These suppositions indicate that, on the average, approximately 92 tests per 100 will prove adequate [$0.66(90) + 0.34(95) \approx 92$]. Stated another way, the probability that a sample of concrete chosen at random will meet the specification is approximately 0.92. On the other hand, if all the concrete was obtained from plant P_1, the probability of getting substandard concrete would be 0.10. It is apparent that information as to where the concrete was produced will affect the probability of a sample meeting standards. Such probabilities are said to be *conditional,* that is, the occurrence of one outcome (information as to which plant produced the concrete) will modify the chance of the occurrence of another outcome (the probability that a number of test samples of the concrete will test above standard). Before specifying the origin of the concrete, the *unconditional* probability was 0.92 that a test sample would meet the specification.

The conditional probability of an outcome A, given that an outcome B has occurred, denoted by $P[A|B]$, is defined as

$$P[A|B] = \frac{P[AB]}{P[B]} \tag{1.3.1}$$

where $P[AB]$ denotes the probability that both outcomes A and B will occur and $P[B]$ is the probability of the occurrence of outcome B. If $P[B] = 0$, the conditional probability is not defined. For the foregoing example, if the region M in Fig. 1.3.1*b* is designated as the region of meeting standards, the probability of doing so for plant P_1, $P[M|P_1]$, is the number of elementary events common to both M and P_1, $P[MP_1]$, divided by the sum of the elementary events in P_1, or

$$P[M|P_1] = \frac{P[MP_1]}{P[P_1]}$$

The subset M here denotes all elementary events (tests) that will meet standards from both plants P_1 and P_2. It is seen that for the conditional probability with regard to plant P_1, the sample space reduces to that of P_1, and $P[MP_1]$ is the probability of the joint occurrence, or *intersection*, of M and P_1.

Example 1.3.1 Find the probability of drawing the king of hearts (a) from an ordinary well-shuffled deck of cards and (b) given that a heart was drawn.

Solution (a) Here we have the unconditional probability $P[K_h] = 1/52$. (b) The sample space has been reduced from 52 elementary events (cards) to 13 (only hearts). Hence, $P[K_h|h] = 1/13$.

Equation (1.3.1) also suggests

$$P[AB] = P[B]P[A|B]$$

or $$P[AB] = P[A]P[B|A] \tag{1.3.2}$$

Two outcomes are said to be *independent* if the occurrence or nonoccurrence of one has no effect on the probability of occurrence of the other. Independence between A and B requires

$$P[A|B] = P[A]$$

and $$P[B|A] = P[B] \tag{1.3.3}$$

Hence, if A and B are independent, Eqs. (1.3.2) become

$$P[AB] = P[A]P[B] \tag{1.3.4}$$

Example 1.3.2 What is the probability of drawing a king in each of two draws from a deck, (a) without replacement (the card drawn is not returned to the deck before the next draw) and (b) with replacement?

Solution (a) Here the second draw depends upon the results of the first draw. Thus,

$$P[K_1K_2] = P[K_1]P[K_2|K_1] = \frac{4}{52}\left(\frac{3}{51}\right) = \frac{1}{221}$$

(b) Replacing the card, we obtain independence and

$$P[K_1K_2] = P[K_1]P[K_2] = \frac{4}{52}\left(\frac{4}{52}\right) = \frac{1}{169}$$

For a number of independent events $A_1, A_2, \ldots, A_N$, Eq. (1.3.4) generalizes to (*multiplication rule*)

$$P[A_1A_2A_3 \cdots A_N] = P[A_1]P[A_2] \cdots P[A_N] \tag{1.3.5}$$

Example 1.3.3 A fair coin (P[head] = P[tail] = ½) is tossed three times; what is the probability of getting three heads?

Solution Assuming each toss is independent of the others, Eq. (1.3.5) produces

$$P[\text{three heads}] = 1/2\,(1/2)\,(1/2) = (1/2)^3 = 1/8$$

Figure 1.3.2 shows the eight outcomes of the three-step process of tossing a fair coin three times. Each outcome is an elementary event with a probability of occurrence of $1/8$. Hence, the probability of obtaining two heads in three tosses $P[HHT + HTH + THH] = P[HHT] + P[HTH] + P[THH] = 1/8 + 1/8 + 1/8 = 3/8$. It should be evident why the representation in Fig. 1.3.2 is called a *tree diagram*. The total number of outcomes is the same as the number of paths, which is $2 \times 2 \times 2 = 8$. A tree diagram such as shown in Fig. 1.3.2 is, in a sense, a graphic multiplier.

Example 1.3.4 How many possible outcomes are there for drawing five cards from a 52-card deck?

Solution There are 52 possible outcomes for the first card, 51 for the second, 50 for the third, and so on. Hence, there are

$$52 \times 51 \times 50 \times 49 \times 48 = 3.12 \times 10^8 \quad \text{possible hands}$$

Granted replacement and hence independence, we would have

$$52^5 = 3.80 \times 10^8 \quad \text{possible hands}$$

Equations (1.3.2) give the probability of the joint occurrence of two dependent outcomes A and B. These can be generalized. For three dependent outcomes A, B, and C we would have

$$P[ABC] = P[A]P[B|A]P[C|AB] \tag{1.3.6}$$

where $P[C|AB]$ is the probability that outcome C occurs given that the joint outcomes of A and B have already occurred.

Example 1.3.5 Find the probability of drawing hearts on three consecutive draws, without replacement, from a standard deck of cards.

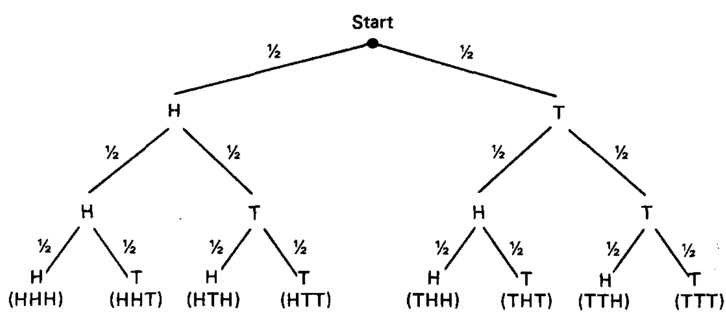

Figure 1.3.2 Tree diagram of three-step tossing of a fair coin.