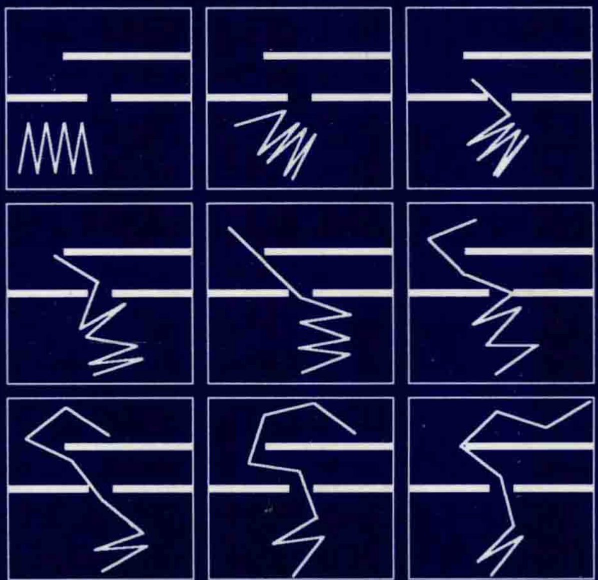
ROBOT MOTION PLANNING



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To Claudine, Laurence, and Emmanuel

Preface

One of the ultimate goals in Robotics is to create *autonomous robots.* Such robots will accept high-level descriptions of tasks and will execute them without further human intervention. The input descriptions will specify *what* the user wants done rather than *how* to do it. The robots will be any kind of versatile mechanical device equipped with actuators and sensors under the control of a computing system.

Making progress toward autonomous robots is of major practical interest in a wide variety of application domains including manufacturing, construction, waste management, space exploration, undersea work, assistance for the disabled, and medical surgery. It is also of great technical interest, especially for Computer Science, because it raises challenging and rich computational issues from which new concepts of broad usefulness are likely to emerge.

Developing the technologies necessary for autonomous robots is a formidable undertaking with deep interweaved ramifications in automated reasoning, perception and control. It raises many important problems. One of them — *motion planning* — is the central theme of this book. It can be loosely stated as follows: How can a robot decide what motions to perform in order to achieve goal arrangements of physical objects? This capability is eminently necessary since, by definition, a robot accomplishes tasks by moving in the real world. The minimum one would expect from an autonomous robot is the ability to plan its own motions.

At first glance motion planning looks relatively simple, since humans deal with it with no apparent difficulty in their everyday lives. In fact, as is also the case with perception, the elementary operative intelligence that people use unconsciously to interact with their environment — e.g. preparing and serving coffee, assembling a device, moving in a building — turns out to be extremely difficult to duplicate using a computer-controlled robot. It is true that some naive methods can produce apparently impressive results, but the limitations of these methods quickly become obvious. The unaware reader will be surprised by the amount of nontrivial mathematical and algorithmic techniques that are necessary to build a reasonably general and reliable motion planner.

The research in robot motion planning can be traced back to the late 60's, during the early stages of the development of computer-controlled robots. Nevertheless, most of the effort is more recent and has been conducted during the 80's. Over the last few years, the theoretical and practical understanding of some of the issues has increased rapidly thanks to the combined work of researchers in Artificial Intelligence, theoretical Computer Science, Mathematics, and Mechanical Engineering. In addition to producing effective planning methods, this work has contributed to advancing our knowledge of the mathematical structure of the problems and to pinpointing their inherent computational complexity. This book is an attempt to give a comprehensive account of recent results in motion planning and to organize them in a unified framework.

An incorrect, but still widespread view is that motion planning essentially consists of doing some sort of collision checking or collision avoidance. In fact, motion planning is much more than that. It involves such diverse aspects as computing collision-free paths among possibly moving obstacles, coordinating the motions of several robots, planning sliding and pushing motions to achieve precise relations among objects, reasoning about uncertainty to build reliable sensory-based motion strategies, dealing with models of physical properties such as mass, gravity and friction, and planning stable grasps of objects. Therefore, motion planning requires the robot to consider geometrical constraints, as well as physical and temporal constraints. In addition, uncertainty may require that it plan not only motion commands, but also their interaction with sensing. When knowledge at planning time is too incomplete, it may become necessary to interweave planning and execution in order to collect appropriate information through sensing.

The concept of *configuration space* is used throughout this book in order to organize the various facets of motion planning in a coherent framework. This concept, which has been popularized in Robotics by Lozano-Pérez, is essentially a representational tool. It consists of treating the robot as a point in an appropriate space, the configuration space of the robot. The geometry of the task can be mapped in this configuration space. Physical concepts, such as force and friction, can also be represented in this space as additional geometrical constructs. Various tools from Geometry, Topology and Algebra apply nicely to this representation and provide the theoretical basis of motion planning.

Configuration space is sometimes regarded in the literature as a motion planning approach. This is an incorrect view. Configuration space is only a tool for formulating motion planning problems precisely. Planning methods that have been viewed as alternatives to the "configuration space approach", such as the "potential field" methods, can be consistently and usefully described using this tool. Configuration space has also been criticized for its inefficiency. As a representational tool, it cannot be efficient or inefficient. This unfounded criticism is probably a reflection of the complexity of motion planning in general. Hopefully, these facts will rapidly become obvious to the reader.

The concept of configuration space will make it possible to study the various aspects of motion planning in a unified fashion, hence facilitating the presentation and comparison of various planning approaches which are often considered separately in the literature. In addition, the description of the methods will proceed incrementally by considering a *basic motion planning problem* and extensions of this problem. Taking the basic problem as the first target, the major concepts and computational approaches to motion planning will be exposed in detail (the first half of this book). Then, several extensions of the basic problems will be studied; the methods presented previously will be extended and new ones will be described (the second half of the book).

The book consists of 11 chapters:

- Chapter 1 is an introduction to motion planning. It presents the basic motion planning problem, which is to plan a collision-free path for a rigid object translating and rotating among static obstacles. It introduces the concept of configuration space and the main computational approaches to motion planning. It discusses various extensions of the basic problem and gives an account of the important results relative to the computational complexity of motion planning. It reviews some key publications which have contributed to the development of the motion planning field.

- Chapters 2 and 3 develop the notion of configuration space for a rigid object that can translate and rotate freely among obstacles in a two- or three-dimensional workspace, i.e. the type of robot considered in the basic motion planning problem. Chapter 2 focuses on the differential and topological structure of this space. It introduces most of the mathematical concepts that will be used in the rest of the book. Chapter 3 describes how the objects contained in the robot's workspace (the "obstacles") map into configuration space when all the objects (including the robot) are described as semi-algebraic sets. The particular case where the objects are modeled as polygons and polyhedra are analyzed in detail.

- Chapters 4 through 7 describe four computational approaches for solving the basic motion planning problem. Chapter 4 presents the roadmap approach, which consists of capturing the global topology of the set of collision-free configurations of the robot in the form of a network of one-dimensional curves. Chapters 5 and 6 describe the exact and approximate cell decomposition approaches, which represent the set of collision-free configurations as a collection of cells and search the graph representing the adjacency relation among these cells. Chapter 7 describes the potential field approach, which regards the robot as a particle moving under the action of forces generated by an artificial potential field attracting the robot toward the goal while pushing it away from the obstacles.

- Chapters 8 through 11 present various extensions of the basic motion planning problem, and describe both modifications of the previous methods and new planning methods applying to these extensions. Chapter 8 considers the case where there are moving obstacles, multiple robots, and/or articulated robots. Chapter 9 analyzes the impact of nonholonomic kinematic constraints (the kind of constraints which apply to an automobile car) on motion planning. Chapter 10 investigates motion planning in the presence of uncertainty, in particular in robot control and in sensing. Chapter 11 considers the case where the robot can grasp/ungrasp movable objects in order to accomplish sophisticated manipulation tasks.

All the chapters are deliberately oriented toward presenting the fundamentals of motion planning. System engineering and implementation choices, which often depend closely on the domain of application and the features of the hardware systems, are considered only in a few places, and to the extent that they help understand the basic themes. The material presented, once understood, should be highly useful for designing practical systems.

Taken together, the methods presented in this book go beyond Robotics, and can be regarded as a contribution to the mechanization of *spatial reasoning*, i.e. the capacity to represent and reason about the geometry, spatial arrangement, and physics of the objects in the real world. Although the mainstream of Artificial Intelligence has been quite successful in developing various reasoning paradigms leading notably to efficient expert systems, it has done relatively little to automate spatial reasoning in a realistic fashion. Nonetheless, automatic spatial reasoning capabilities will be critical for building new, more advanced computer systems in a variety of domains related to design, manufacturing and engineering. A comprehensive computational theory of spatial reasoning will obviously increase our understanding of the interaction between design and manufacturing, eventually opening a new era of better designed and better manufactured products. The ideas and methods developed for robot motion planning can be a source of knowledge and inspiration for future developments in these domains.

This book is aimed at graduate students intending to study further in Robotics and Spatial Reasoning, and at engineers having to develop advanced computer-based systems in design, manufacturing, construction and automation. It assumes good "undergraduate-level" knowledge in Mathematics (Geometry, Topology, Algebra) and in Algorithms. A glossary of mathematical definitions is provided in Appendix A for the reader's convenience. Some background in Algorithms is also given in Appendices B (computational complexity), C (graph searching), and D (line sweeping). The book assumes elementary knowledge in Robotics and Mechanics.

A collection of exercises is included at the end of each chapter. Three types of exercises are typically proposed. Some exercises are quite formal and call for precise answers. Other exercises consist of writing programs and experimenting with them. They are intended to familiarize the reader with some implementation issues, and may be a good preparation before starting to work on a larger implementation project. They can be performed using almost any kind of personal computer or workstation with such programming languages as C, Pascal, and Lisp. Graphical inputs/outputs are suitable for several programming exercises. Finally, other exercises suggest topics for informal reflection, and are often used to introduce ideas not presented in the text.

I would like to acknowledge the help of many people who contributed their time reading and commenting on early drafts of this book. Many thanks to José Bañon, Jérôme Barraquand, Philippe Caloud, Wonyun Choi, Bill Dixon, Joseph Friedman, Maria Gini, François Ingrand, Ramesh Jain, Krasimir Kolarov, Oussama Khatib, Jean-Paul Laumond, Anthony Lazanas, Claude Le Pape, Jocelyne Pertin-Troccaz, Sean Quinlan, Jean-François Rit, Bernie Roth, Shashank Shekhar, Yoav Shoham, Dominique Snyers, Tom Strat, Yong Se Kim, Mark Yim, Randall Wilson, and David Zhu.

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Jean-Claude Latombe

Notations

The following conventions and notations are used throughout the book. Most of them are introduced in Chapter 1.

1. The robot is called $\mathcal{A}$. If there are several robots, they are called $\mathcal{A}_i$ $(i = 1, 2, ...)$.

2. The robots' workspace is denoted by $\mathcal{W}$ and is modeled as the Euclidean space $\mathbf{R}^N$, with $N = 2$ or 3. ($\mathbf{R}$ is the set of the real numbers.)

3. A Cartesian frame $\mathcal{F}_{\mathcal{A}}$ is attached to $\mathcal{A}$. Another Cartesian frame $\mathcal{F}_{\mathcal{W}}$ is attached to $\mathcal{W}$. The origin of $\mathcal{F}_{\mathcal{A}}$ (resp. $\mathcal{F}_{\mathcal{W}}$) is denoted by $O_{\mathcal{A}}$ (resp. $O_{\mathcal{W}}$).

4. The obstacles in $\mathcal{W}$ are denoted by $\mathcal{B}_i$ $(i = 1, 2, ...)$. The symbol $\mathcal{B}$ sometimes denotes a particular obstacle. More often, it denotes the union of all the obstacles (called the obstacle region).

5. The configuration space of a robot $\mathcal{A}$ is denoted by $\mathcal{C}$. It is a manifold of dimension m. An element of $\mathcal{C}$ (i.e. a configuration) is denoted by $\mathbf{q}$. The region of the workspace occupied by the robot $\mathcal{A}$ at configuration $\mathbf{q}$ is denoted by $\mathcal{A}(\mathbf{q})$.

6. An obstacle $\mathcal{B}$ in the workspace maps in configuration space to a region called C-obstacle and denoted by $\mathcal{CB}$.

7. The free space in configuration space is denoted by $\mathcal{C}_{free}$; the contact space by $\mathcal{C}_{contact}$. $\mathcal{C}_{valid} = \mathcal{C}_{free} \cup \mathcal{C}_{contact}$ is the valid space.

8. A path is denoted by τ. It is usually a function of a parameter denoted by s that takes its values in the interval $[0, 1]$. When τ is expressed as a function of time, it is called a trajectory.

9. A vector is denoted by a symbol with an arrow on top of it, e.g. $\vec{\nu}$. Its modulus is written $\|\vec{\nu}\|$. The inner product of two vectors $\vec{\nu}_1$ and $\vec{\nu}_2$ is denoted by $< \vec{\nu}_1, \vec{\nu}_2 >$ or $\vec{\nu}_1 \cdot \vec{\nu}_2$. The outer product of $\vec{\nu}_1$ and $\vec{\nu}_2$ is denoted by $\vec{\nu}_1 \wedge \vec{\nu}_2$. The angle between $\vec{\nu}_1$ and $\vec{\nu}_2$ is written $angle(\vec{\nu}_1, \vec{\nu}_2)$.

10. The distance between two points x and y of a Euclidean space is denoted by $\|x - y\|$.

11. An interval in $\mathbf{R}$ bounded by a and b is denoted by $[a, b]$ if it is closed at both ends and by (a, b) if it is open at both ends. It is denoted by $[a, b)$ or $(a, b]$ if it is closed at one end and open at the other.

12. Let a be a real number. $sign(a)$ denotes the "sign" of a; it is $+1$ if $a > 0$, -1 if $a < 0$, and 0 otherwise.

13. The symbols $\oplus$ and $\ominus$ are the Minkowski operators for affine set addition and subtraction, respectively.

14. Let E be a topological space and F be a subset of it. $cl(F)$, $int(F)$, and $\partial(F)$ denote the closure, the interior, and the boundary of F, respectively. $E \backslash F$ denotes the complement of F in E.

15. Most of the time, calligraphic letters, e.g. $\mathcal{S}$, denote sets.

16. Symbols in typewriter type style are used to denote boolean expressions or predicates, e.g. `CB`, `Achieve`. The symbols `true` and `false` are the logical values "true" and "false".

Contents

Chapter 1

Introduction and Overview

A robot is a versatile mechanical device — for example, a manipulator arm, a multi-joint multi-fingered hand, a wheeled or legged vehicle, a free-flying platform, or a combination of these — equipped with actuators and sensors under the control of a computing system. It operates in a workspace within the real world. This workspace is populated by physical objects and is subject to the laws of nature. The robot performs tasks by executing motions in the workspace.

In this book we are interested in giving the robot the capability of planning its own motions, i.e. deciding automatically what motions to execute in order to achieve a task specified by initial and goal spatial arrangements of physical objects. Creating autonomous robots is a major undertaking in Robotics. It definitively requires that the ability to plan motions automatically be developed. Indeed, except in limited and carefully engineered environments, it is not realistic to anticipate and explicitly describe to the robot all the possible motions that it may have to execute in order to accomplish requested tasks. Even in those cases where such a description is feasible, it would certainly be useful to incorporate automatic motion planning tools in off-line robot programming