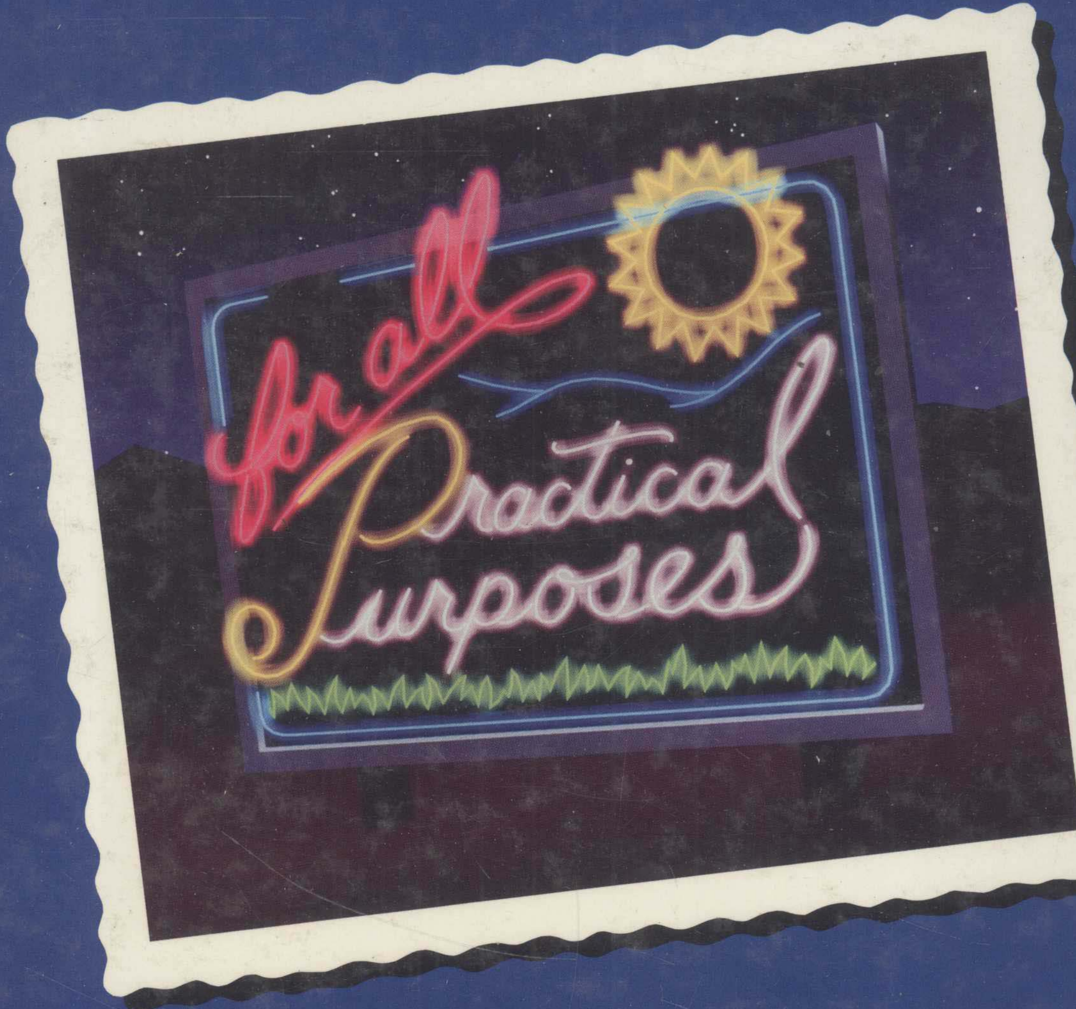

INTRODUCTION TO

Contemporary Mathematics

FOR ALL PRACTICAL PURPOSES

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Contemporary Mathematics

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Preface

Every mathematician at some time has been called upon to answer the innocent question, "Just what is mathematics used for?" With understandable frequency, usually at social gatherings, the question is raised in similar ways: "What do mathematicians do, practice, or believe in?" At a time when success in our society depends heavily on satisfying the need for developing quantitative skills and reasoning ability, the mystique surrounding mathematics persists. *For All Practical Purposes: Introduction to Contemporary Mathematics* is our response to these questions and our attempt to fill this need.

For All Practical Purposes represents our effort to bring the excitement of contemporary mathematical thinking to the nonspecialist, as well as help him or her develop the capacity to engage in logical thinking and to read critically the technical information with which our contemporary society abounds. We attempt to implement for the study of mathematics Thomas Jefferson's notion of an "enlightened citizenry," in which individuals having acquired a broad knowledge of topics exercise sound judgment in making personal and political decisions. Environmental and economic issues dominate modern life, and behind these issues are complex matters of science, technology, and mathematics that call for an awareness of fundamental principles.

To encourage achievement of these goals, *For All Practical Purposes* stresses the connections between contemporary mathematics and modern society. Since the technological explosion that followed World War II, mathematics has become a cluster of mathematical sciences encompassing statistics, computer science, operations research, and decision science, as well as the more traditional areas. In science and industry mathematical models are the tools par excellence for solving complex problems. In this book our goal is to convey the power of mathematics as illustrated by the great variety of problems that can be modeled and solved by quantitative means.

This book is designed for use in a one-term course in liberal arts mathematics or in courses that survey mathematical ideas. The assumed background of our audience is varied. We expect some ability in arithmetic, geometry, and elementary algebra. Even though a few of the topics included here are traditionally thought of as part of advanced mathematics, we aim to develop in the reader strong conceptual understanding and appreciation, not computational expertise. Our stated bias throughout the book is on presenting the subject through its contemporary applications.

In determining the selection of topics for this book, the authors posed a question to leading mathematicians and educators nationwide: "What would you teach students if they took only one semester of math during their entire college career?" Their answers are best demonstrated by the main topic selections for the text: management science, statistics, social choice, the geometry of size and shape, and mathematics for computer science. These topics were chosen both for their basic mathematical importance and for the critical role their applications play in a person's economic, political, and personal life.

Because readers will approach this book with a great range of expertise in using mathematical symbols, the material has been designed in layers to accommodate different objectives and diverse backgrounds. *For All Practical Purposes* concentrates on discussions about mathematics—about its nature, its content, its applications. At various places, in separate boxes or optional sections, certain mathematical issues are pursued with explicit use of equations and appropriate mathematical symbols. Problems are also divided by type, with some emphasizing descriptive matters and others providing practice in calculation and symbolic manipulation.

Additionally, *For All Practical Purposes* includes interviews with practitioners—the people who put mathematics to work. Each major topic section is complete and self-contained so that instructors can adjust the topic ordering to suit their particular needs. A large number of photographs and 69 full-color illustrations (in 24 pages of inserts) have been incorporated to emphasize the fact that contemporary mathematics is visually alive. Many of the color images are computer-generated and represent the accomplishments at the forefront of some mathematical research.

Perhaps the most distinctive new feature of this text is its relation to the 26 half-hour television programs that constitute the series "For All Practical Purposes." In 1983, the Consortium for Mathematics and Its Applications (COMAP) received a grant from The Annenberg/CPB Project, with co-funding from the Carnegie Corporation of New York, to produce a telecourse in mathematics for public broadcast on PBS. Development of the textbook was funded by the Alfred P. Sloan Foundation. An impressive array of people committed to educational excellence in mathematics as well as experienced television producers and computer graphics specialists were gathered to work on this project from the original outlines to final shows and text. Though this book stands alone from the video series, the authors worked simultaneously on developing television scripts and book chapters.

Some instructors will use the text and television shows in concert. Some will use the text alone in a traditional lecture course. Still others will make the videos available to students on tape for enrichment or special credit. The choice of how best to use this mix of resources will depend on each individual's particular preference and course requirements. An Instructor's Manual that relates the video programs with the text is available. Information about television-course licensing, off-air taping rights, and prerecorded video cassettes can be obtained by calling (202) 955-5251 (collect), or by writing The Annenberg/CPB Project at 1111 Sixteenth Street NW, Washington, DC 20036.

Students enrolled in the telecourse will view the videos and read the text on their own. To aid them a Course Guide has been developed by the text authors that provides an overview of each program, skill objectives, and a sample short-answer examination. We hope that these materials will provide a rich and exciting environment in which to learn more about the power and centrality of mathematics in our world.

Acknowledgments

First, it is a pleasure to acknowledge the support of the Alfred P. Sloan Foundation, which provided a generous grant to help fund the production of this book. Over the almost four years of this undertaking, a remarkable number of people have made significant contributions. It is difficult to find words expressing our gratitude and appreciation. For the authors, this was no ordinary writing task. The revision process was enriched but certainly made a great deal more complicated by the relationship with the television series. With so many contributors, coordination was of necessity a key factor. The cluster leaders—Donald Albers, Zaven Karian, William F. Lucas, David S. Moore, and Joseph Malkevitch—did yeoman duty.

Professor Malkevitch truly deserves a very special recognition. To a great extent the underlying philosophy of this book is a reflection of his ideas, beliefs, and dedication.

We also owe a huge debt to Stephanie Stewart, John Rubin, and David Gifford, the writers, producers, and directors of the Chedd-Angier Production Company, who created "For All Practical Purposes" as a television series. The selection of locations, the interviews, and the scripts were all their domain. In particular, we acknowledge the tireless efforts of Joseph Blatt, the series' producer. His ingenuity and creativity, along with those of producer Olga Rakich, permeate this book. Their words and their spirit are found throughout the text.

We are indebted to the many instructors at colleges and universities around the country who offered us their critical comments of the manuscript during the development and production of *For All Practical Purposes*. Their efforts helped improve the conceptual and pedagogical quality of the text.

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Many of the outstanding full color images were obtained from research scientists. We thank them for allowing us to reproduce some of their important work: H.E. Benzinger, S.A. Burns, and J. Palmore, University of Illinois (chaotic basins of attraction); Douglas Dunham, University of Minnesota (hyperbolic patterns); David Hoffman, University of Massachusetts (minimal surfaces); Benoit Mandelbrot, IBM (fractal images); Heinz-Otto Peitgen, University of Santa Cruz (Mandelbrot sets); Roger Penrose, Oxford University (Penrose tilings).

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The writing of this book by our author team took place simultaneously with the production of the television series, and the complications and deadlines were numerous. The manuscript and complex illustrations would not have come together without the expert guidance of the W. H. Freeman and Company staff. In particular we wish to thank Susan Moran, project editor, Susan Stetzer, production coordinator, Mike Suh, art director, and Bill Page, art coordinator. Lloyd Black, senior development editor, arrived on the scene just in time to pull the words and figures together and guide the book through the production process. We send a special note of thanks to Jeremiah Lyons, senior mathematics editor. His faith in the project and efforts over two plus years have brought order out of chaos.

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Solomon Garfunkel
COMAP

Lynn A. Steen
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FOR ALL PRACTICAL PURPOSES

Contents

PART I

MANAGEMENT SCIENCE

The first steps on the moon represented a great leap for management science. Here, Buzz Aldrin poses for Neil Armstrong. [NASA.]

Those who were watching live television one night in July 1969 will never forget the spectacle of seeing the first man walk on the moon. The element of danger and uncertainty, heightened by a history of trial and error, added to the suspense of the lunar landing. Before the 1960s, no one really knew if rockets would ever be able to carry humans into space.

Neil Armstrong's first step onto the moon's surface was a triumph for American science and technology and the culmination of a national quest that had begun in the office of President John F. Kennedy. It was Kennedy's goal to put a man on the moon before the decade was out, a goal realized in the Nixon administration.

In the eight years from 1961 to 1969 we moved from a president's vision to the reality of a lunar landing. Most of us think of this achievement in terms of the tremendous scientific advances it represented — in physics, engineering, chemistry, and associated technologies. But there was another side to this far-reaching project. Someone had to set the objectives, commission the work, suffer the setbacks, overcome unforeseen obstacles, and tie together a project with thousands of disparate components. The kind of science responsible for such details is a branch of mathematics called **management science.**

NASA administrators faced many new problems in putting a man on the moon: they had to choose the best design for the spacecraft, design realistic ground simulations, and weigh the priorities of conducting experiments with immediate returns against carrying out tests that would serve long-term goals. When NASA commissioned the Apollo module, it was asking several hundred companies to design, build, test, and deliver components and systems that had never been built before.

Supporting these space age goals, however, were the nuts and bolts issues that make up the major concerns of management science, namely, finding ways to make the operations as productive and economical as possible (see the accompanying box on the Apollo 11 launch). Issues of efficiency are important in all organizations. In business, industry, or government, operating efficiently is hardly a novel idea. Directors of large corporations and those in the high ranks of the military have always pursued efficiency. What is new about management science is that it distinguishes between trial-and-error — "seat of the pants" — approaches and new ideas guided by systematic mathematical analysis.

The need to establish scientific principles for operations management arose in World War II. The progenitors of management science were mathematicians and industrial technicians associated with the armed services who worked together to improve military operations. In applying quantitative techniques to project planning, these pioneers founded a new science.

Management science, or **operations research,** as it sometimes is called, turned out to be a powerful notion with wide-ranging applications. It enabled mathematicians to bring a long history of pure research to bear on practical problems. We will explore some of these problems and solutions in the sections ahead.

Apollo 11 Launch Owes Success to Management Science

Today, Captain Robert F. Freitag is director of Policy and Plans for NASA's space station. Back in 1969, however, Freitag headed the team responsible for landing the Apollo 11 safely on the moon. The success of the lunar mission can be traced to management-science techniques that ensured that thousands of small tasks would come together to meet a single giant objective. Freitag shares his observations about that historic event:

> I think the feeling most of us in NASA shared was, "My gosh, now we really have to do it." When you think that the enterprise we were about to undertake was ten times larger than any that had ever been undertaken, including the Manhattan Project, it was a pretty awesome event. But we knew it was the kind of thing that could be broken down into manageable pieces and that if we could get the right people and the right arrangement of these people, it would be possible.

Captain Robert F. Freitag, NASA.

In the case of the Apollo program, it was very important that we take a comprehensive system engineering approach. We had to analyze in a very strict sense exactly what the mission was going to be, what each piece of equipment needed was and how it would perform, and all the elements of the system from the concept on through to the execution of the mission, to its recovery back on earth.

We started out, in a very logical way, by having a space station in earth orbit. We would then take the lunar space craft and build it in orbit, and then send it off to the moon and bring it back. It turned out that this approach was probably a little more risky and took a lot longer, so with the analyses we made, we shifted our whole operation to building a rocket that would go all the way to the moon after it took off from Cape Canaveral. It would then go into orbit around the moon rather than landing on the moon, and from orbit around the moon would descend to the surface of the moon and perform its exploration. Then it would return to its orbit around the moon and come back home.

Well, that was a very comprehensive analysis job. It was probably more deep-seated than the kind of job one would do for building an airplane or a dam because there were so many variables involved. What you do is break it down into pieces: the launch site, the launch vehicles, the space craft, the lunar module, and world-wide tracking networks, for example. Then, once these pieces are broken down, you assign them to one organization or another. They, in turn, take those small pieces, like the rocket, and break it down into engines or structures or guidance equipment. And this breakdown, or "tree," is the really tough part about managing.

In the Apollo program, it was decided that three NASA centers would do the work. One was Huntsville, where Dr. Von Braun and his team built the rocket. The other was Houston, where Dr. Gerous and his team built the space craft and controlled the flight operations. The third was Cape Canaveral, where Dr. Debries and his team did the launching and the preparation of the rocket.

Those three centers were pieces, and they could break their pieces down into about 10 or 20 major industrial contractors who would build pieces of the rocket. And then each of those industrial contractors would break them down into maybe 20 to 30 or 50 subcontractors—and they, in turn, would break them down into perhaps 300,000 or 400,000 pieces, each of which would end up being the job of one person. But you need to be sure that the pieces come together at the right time, and that they work when put together. Management science helps with that. The total number of people who worked on the Apollo was about 400,000 to 500,000, all working toward a single objective. But that objective was clear when President Kennedy said, "I want to land a man on the moon and have him safely returned to the earth, and to do so within the decade." Of course, Congress set aside $20 billion. So you had cost, performance, and schedule, and you knew what the job was in one simple sentence. It took a lot of effort to make that happen.