MODERN PHYSICS

THE QUANTUM PHYSICS OF ATOMS, SOLIDS, AND NUCLEI

THIRD EDITION

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PREFACE

An analytical study of the quantum physics of atoms, molecules, solids, and nuclei is presented in this textbook. This area of physics, usually but not altogether appropriately called "modern physics," is frequently taught in a largely descriptive fashion in an introductory college course and in a highly mathematical fashion in a graduate course. Our aim in this book has been to cultivate a middle ground, in which a quantitative understanding of atomic, molecular, solid-state, and nuclear physics is attained through the application of elementary quantum mechanics. Quantum mechanics, once thought to be too mysterious for undergraduates, is presented here at an introductory level with emphasis on its concepts and methods, which become familiar to the student through analysis of one-dimensional problems. Application of these concepts and methods permits answering the most intriguing questions of modern physics—questions like: What holds matter together? What holds matter apart? How does the variety of chemical properties of different elements arise? How do electrons move through solids? Why do the nuclei occurring in nature have only certain combinations of protons and neutrons?

This third edition is intended as a text for a course following an introductory general physics course and is addressed to both science and engineering majors. Science students and engineers will probably build on the subject matter of this book in different ways. For example, physics or chemistry majors may use the knowledge gained here in their intermediate theory and laboratory courses and later return to quantum mechanics in graduate-level courses before doing original research; electrical engineers may combine the knowledge gained here with work in circuit theory to prepare themselves for conducting original work in solid-state devices or nuclear particle detectors.

Although students' paths will diverge after studying this book, it seems to us that the aims of science and engineering majors in studying modern physics at the level presented here should be basically the same, namely, to understand as fully and as quantitatively as possible the fundamental processes of quantum physics.

In keeping with the analytical character of this book, we have placed considerable emphasis on the solution of problems by the student. It is not much of an exaggeration to say that no topic is introduced in this text unless it is possible for the student to solve meaningful problems after studying that topic. The number of problems has been increased in the new edition, primarily to provide illustrations, applications, and extensions of new material in the text. The order of presenting problems at the end of each chapter parallels the order of presentation of the relevant topics in the text. The relative difficulty of problems varies widely; some merely demonstrate orders of magnitude or simple applications, but many constitute substantial extensions of the text, frequently guided by appropriate hints. Answers to problems marked by asterisks are given in Appendix H, primarily as a help to readers who are studying this book without the benefit of an instructor.

This third edition has added considerable depth, while still being accessible to students who begin with no knowledge of quantum physics or of advanced mathematics. The physics and mathematics of the harmonic oscillator and the hydrogen atom are here carried out completely, rather than merely sketched as they were in earlier editions. The new Chapter 4 provides new depth in operators, eigenfunctions, wave packets, and perturbation theory. Nevertheless, as explained in the introduction to that chapter, students need not be frightened by these more abstract sections; they can pick up the thread of the previous discussion later in that chapter even if they skip those sections but, of course, they will then miss some of the marvelous richness and subtlety of the quantum world.

The additional depth permits a more thorough and revealing discussion later in the book of electrons in solids, chemical binding, superconductivity, lasers, the theory of the deuteron, and other topics.

The third edition has been also brought up to date on the rapidly moving applications of quantum physics. The most striking example is the physics of semiconductors, and that chapter has been completely rewritten. But major changes and additions have also been made in particle physics, lasers, magnetism, superconductivity, surface physics, and nuclear reactors. To keep the length manageable, most of the second edition's chapter on physical electronics, some introductory material, and some specialized discussions have been deleted.

The order in which modern physics is discussed here remains the same as in the second edition. The first chapter presents elementary descriptions of the particles composing atoms and nuclei, the ingredients of modern physics, and the demonstration that atomic physics can be split into the study of the extranuclear structure of the atom and the study of the nucleus. Chapter 2 develops the concept of a distribution function, needed later; it also adduces evidence on the size of atoms and shows that any system in thermal equilibrium, such as an atom or a solid, will be in nearly the lowest possible energy state for that system. These two chapters may be review for many students, since the material contained in them (and even much in Chapter 3) is now commonly included in a first course in physics.

The heart of modern physics is quantum mechanics, and the central part of this book is devoted to it. Chapter 3 presents the experiments that show the *necessity* for quantum theory, that produce much interesting information about atoms, and that provide hints of the form quantum theory must take. Chapter 4 describes quantum mechanics and some simple, artificial examples that illustrate the *method* of using it. The *application* of quantum mechanics begins in Chapter 5 and continues throughout the remainder of the book.

Attention could be turned again to nuclear physics after Chapter 5, but the development of molecular and solid-state physics follows more naturally (Chapters 6 through 10). Then the subject of nuclear physics is taken up again at the point where it was left at the end of Chapter 1. The discussion of nuclear physics in Chapters 11 and 12 can thus use not only quantum theory but also several topics from Chapters 9 and 10 that help to explain the instrumentation of nuclear experiments. The teacher who objects to the dispersal of nuclear physics may decide to interpose Chapters 11 and 12 between Chapters 5 and 6.

A detailed picture of the structure of this book can be obtained by reading the "Introduction" sections of every chapter. The student would be well advised to read all these sections before embarking upon the study of this book and to read all of them again after completing it.

Relatively little attention is paid in this book to the history of the development of twentieth century physics, for three principal reasons. First, each year quantum physics becomes less of a novel curiosity and more of a vital part of the core of physics; and it seems to us that its parentage should naturally begin to recede into the background, just as the development of Newtonian mechanics now claims little space in a classical mechanics textbook. Furthermore, lectures (rather than a textbook) are probably a more suitable medium for presenting the fascinating history of the ideas of quantum physics. And finally, there are already excellent books available that feature the historical approach.

We are grateful to the many students and instructors who have given us the benefit of their experience with the first and second editions. We remain deeply indebted to those who provided indispensable help with the earlier editions: Doctors D. S. Billington, P. J. Leurgans, A. R. Moore, E. M. Pell, and R. L. Pritchard; Professors D. R. Corson, J. W. DeWire, H. F. Newhall, Jay Orear, H. S. Sack, L. P. Smith, and above all J. A. Krumhansl and C. S. Smith. In preparing the third edition we have been greatly helped by Doctors R. E. Ansorge, D. D. Clarke, D. H. White, R. L. White, and T. O. White. We are particularly indebted to Professors R. J. Friauf and J. D. Sullivan for their careful, critical, but constructive reading of the manuscript.

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1

ATOMS, NUCLEI, AND PARTICLES

1-1 INTRODUCTION

An analytical introduction to much of the basic physics developed in the twentieth century is presented in this book. The new physics has been of great intrinsic interest, almost a new science in itself, and in addition has provided applications in engineering that are already considerable and are expanding rapidly. The study of modern physics has led to new devices and energy sources, to more convenient and accurate instruments, to new materials of construction, and to a clearer understanding of existing materials.

This book is primarily concerned with physical laws and processes, but applications in other sciences and in engineering will be described frequently. Television camera tubes, transistors, nuclear reactors, and other devices will be analyzed as part of the application of the basic physics; but most of the applications will be found in other science and engineering courses and in engineering practice.

The atom can be said to mark the boundary between nineteenth and twentieth century physics. Although the idea that matter is composed of atoms was popular in the nineteenth century, it was only at the end of the century that consistent measurements of atomic size became available, and the atomic theory placed on a sound quantitative footing.

Since 1900 physics has been increasingly concerned with the internal structure of atoms, and such studies have provided an important element in the development of twentieth century physics. In this first chapter we parallel the historical development by describing a number of experiments that have been used to investigate atomic structure in more and more detail, and in so doing we introduce many important concepts and theories, useful later in the book. At various places in this chapter, as indeed in the first few chapters, it will frequently be necessary to assert some properties of particles and to assume the existence of sources and detectors. These assertions and assumptions will be justified only much later in the book. However, in the later chapters we attempt a uniformly analytical approach; that is, we make assertions or conclusions only by logical arguments based on experiments or on theories well tested by experiment.

This chapter opens with an account of a modern experiment that effectively allows one to see atoms and gives estimates of atomic sizes and binding energies. The properties of two subatomic particles, the electron and the proton, are described in Sec. 1-3, and this is followed by an account of the first investigation of the structure of the atom. The conclusion of this investigation is that the atom consists of a tiny, positively charged core, the nucleus, surrounded by one or more electrons. Although most of our discussion of the nucleus is left to Chapters 11 and 12, Sec. 1-6 gives some idea of the sizes and binding energies of nuclei. However, to provide a background to nuclear physics, the variation of mass with velocity and the famous Einstein $E = Mc^2$ relation are presented in Sec. 1-5. Finally, a section is devoted to particle physics, where we try to give an idea of the properties of highly energetic particles.

As the chapter progresses, we look at the atom (or nucleus) on a finer and finer spatial scale and in general at effects that occur at higher and higher energies. These spatial and energy scales bring us to the most important conclusion of this chapter, that the problem of the interactions among particles can be divided into two separate problems: (1) The binding together of protons and neutrons into the nucleus at the center of the atom; (2) the motions of the electrons around the nucleus. This division can be achieved because (1) the energies of interaction between protons and neutrons in a nucleus are very much greater, by a factor of between 10^2 and 10^6, than the energies of interaction between electrons and nuclei; (2) a nucleus is much smaller than an atom. Therefore the size and energy scales of atomic experiments are such that the nuclei remain unchanged; we can consider the nucleus as a heavy particle and can ignore its size and internal structure. Nuclear experiments, on the other hand, involve such large energies and small distances that the presence of electrons surrounding the nucleus are of little consequence. Therefore we can deal separately with atomic physics (Chapters 3 to 6) and nuclear physics (Chapters 11 and 12). Furthermore, we shall see in Sec. 1-7 that the whole field of particle physics can be considered almost independently of the question of the structure of the nucleus.

1-2 THE ATOM

The idea that matter is composed of atoms is so basic to modern physics that it is difficult to realize that it was only at the turn of the century that scientists were finally convinced of the truth of the atomic theory. Philosophically, the idea is of long standing, but scientists were persuaded of the truth of the theory only when consistent measurements of atomic size became available. Perhaps the most important of these measurements was provided by X-ray diffraction experiments; these will be described in Sec. 3-5. A more recent technique, known as field-ion microscopy, is described in this section and comes as close as we can get to an actual picture of an atom.

In the field ion microscope a very fine needle of tungsten, niobium, or some other refractory metal is placed at the center of an evacuated chamber (as shown in Fig. 1-1*a*). A large potential difference is established between the needle, which is positive, and a negative or ground electrode close to the outer wall. The glass surface of the chamber is coated with a phosphor so that a flash of light is produced when it is struck by energetic particles (just as a television tube produces a flash of light when the screen is struck by electrons). Typically, the radius of curvature of the needle is approximately 100 nm, so that a voltage of 5000 V applied between needle and screen gives an electric field at the tip of the needle of 5×10^{10} V m^{-1} (Problem 1-1). When a small amount of helium gas is let into the chamber, a pattern of spots appears on the phosphor, as shown in Fig. 1-2.

Even without any detailed explanation, this picture gives strong evidence that matter is not continuous but discrete on a scale of approximately 10^{-10} m. This scale can be evaluated by combining a measurement of the spot separation on the screen with the magnification of about 10^7, the ratio of the effective radii of the tip and the chamber.

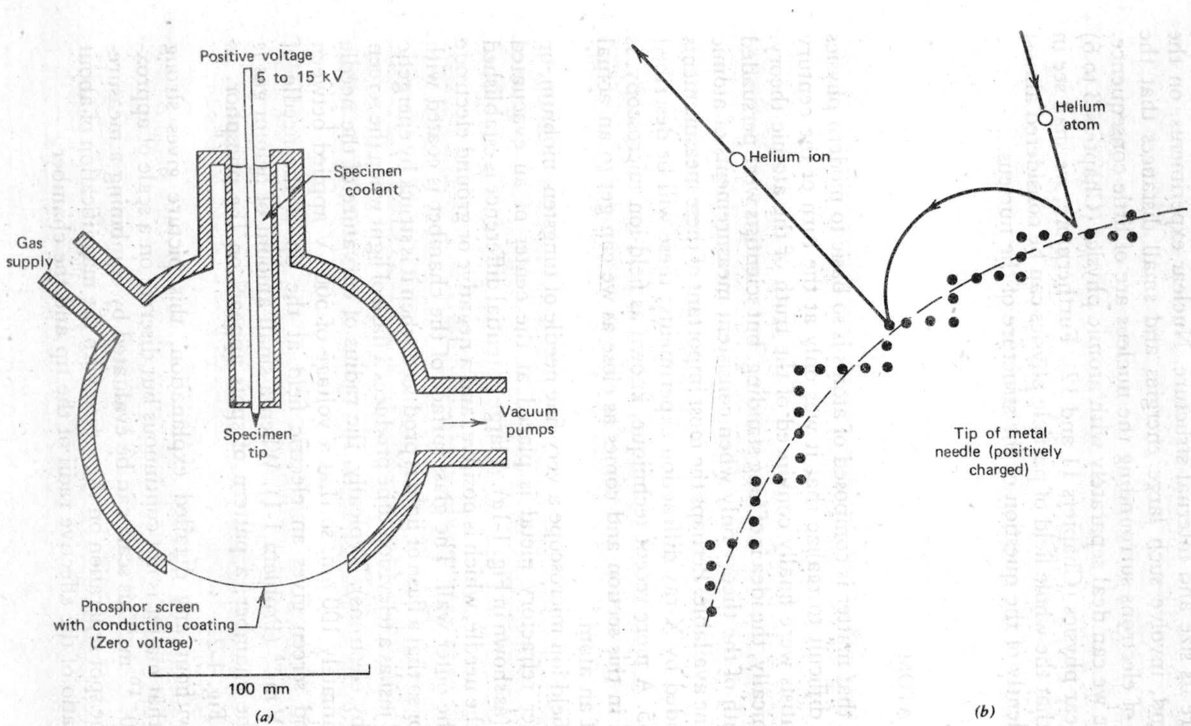

FIGURE 1-1
The field-ion microscope, showing (*a*) a schematic of the experimental arrangement and (*b*) ionization at regions of enhanced electric field.

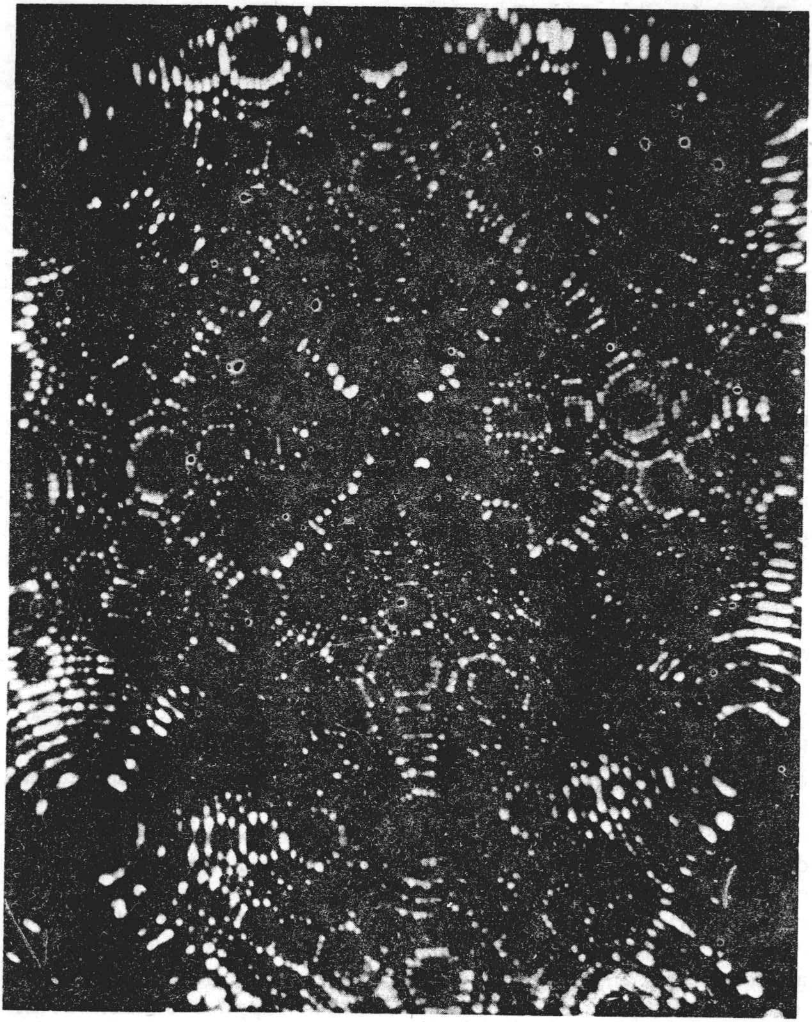

FIGURE 1-2

Field ion microscope picture of gold, showing clearly the positions of individual atoms. The tip has a radius of approximately 50 nm and this part of the pattern occupies a region of approximately 10 × 20 mm on the screen. (Photograph courtesy of M. J. Southon and E. D. Boyes, Cambridge University.)

The explanation of the effect makes use of the well-known concentration of electric field on the more pointed parts of a conductor (Problem 1-1). The average field at the tip of the needle is large, because of the small radius of curvature, but on an atomic scale (Fig. 1-1*b*) there are field variations between one atom and the next, and there is field enhancement at the bumps. A helium atom striking the tips may lose an electron through ionization (described in Sec. 5-4), which occurs preferentially at points where the field is largest. The resulting positive ion is strongly repelled from the surface and travels almost exactly along a radius, starting from the point at which it lost an electron. The pattern of spots on the phosphor produced by the helium ions therefore provides an enormously magnified image of the surface irregularities. As can be seen in Fig. 1-2, individual atoms can be resolved.

X-ray diffraction experiments are visually less dramatic but give more precise measurements of the distances between atoms in solids. If it is assumed that the atoms are packed so closely together that they touch, these experiments give a measurement of the size of atoms. Other estimates of atomic size are provided by studies of collisions between atoms in a gas (Sec. 2-5). All these experiments give the same result: atoms have average diameters of just over 10^{-10} m.*

1-3 THE ELECTRON AND THE PROTON

It is a little ironic that just as precise measurements of atomic size were becoming available, evidence was also accumulating that atoms had an internal structure. The electron was the first subatomic particle to be identified from studies of the internal glow observed when an electrical discharge is struck in a gas at low pressure. We now know that such a discharge consists of a mixture of negatively charged electrons, positively charged ions, and neutral gas atoms. The charged particles are accelerated by the electric field and by colliding with neutral atoms can produce more electrons and ions so that the discharge is self-sustaining. Electrons are attracted to the anode, and by making a hole in the anode we can obtain a beam of electrons. The term *cathode ray* was first applied to the electrons, as it was thought that they originated at the cathode.

* We shall work with distances of this order of magnitude so frequently that it is convenient to introduce here the unit of length called the *angstrom*. One angstrom (1 Å) equals 10^{-10} m.

The identification of the cathode rays as electrons was made by J. J. Thomson in 1897 through a measurement of the ratio e/m, where e is the charge on the electron and m is the mass. This ratio, as we shall see, labels a particle almost unambiguously, although, as described in Sec. 1-5, at velocities close to the velocity c of light the mass of a particle depends on its velocity. The value of e/m that is quoted for a particular particle is, strictly speaking, the ratio e/m_0 appropriate to zero velocity. Since throughout this book we deal with velocities very much less than c, we usually omit the subscript. Where both m and m_0 appear in an equation m will be the actual mass and m_0 will be the mass of the electron at rest. A similar convention will be followed where necessary for other particles.

The original apparatus used by Thomson is shown in Fig. 1-3. Although his result for e/m is now known to be rather inaccurate, we describe his method, since the principles involved are essentially the same as those used in more recent experiments. Electrons from the discharge (on the left) pass through the pair of slits which define their direction, and then between a pair of aluminum deflection plates. The end of the tube is coated with a phosphor, a material that emits light when struck by electrons and so allows them to be seen. The physics of this process is described in Sec. 9-6. A potential difference can be applied between these

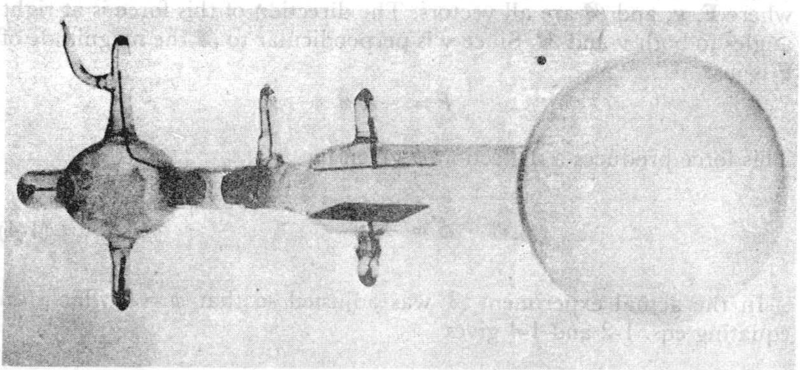

FIGURE 1-3
The original tube used by Thomson to measure e/m for the electron.