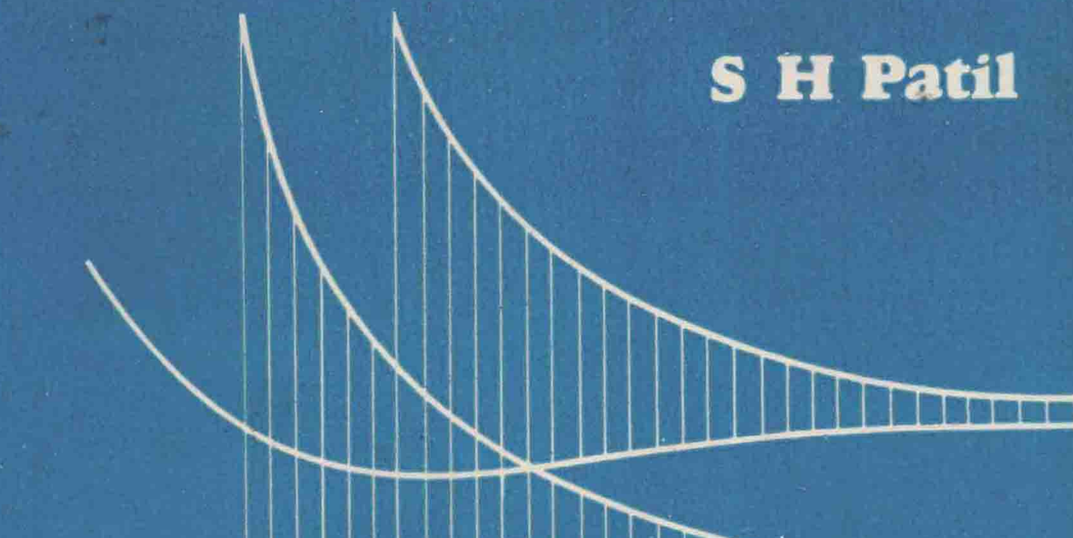
Tata Mcgraw-Hill Series in Physics

Elements of MODERN PHYSICS



S H Patil

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ELEMENTS OF MODERN PHYSICS

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To
My Parents

Preface

This book attempts to provide a perspective of the important concepts and applications in contemporary physics.

With modern physics developing so rapidly, there is a constant need to revise and update the presentation. The present book tries to do this. Starting with a discussion of special theory of relativity and quantum theory, it describes their applications to atoms, molecules, solids and nuclei. There are two special chapters on the modern description of elementary particles and on general theory of relativity and cosmology. The emphasis is on a logical development of ideas, and historical aspects are referred to mainly as an aid to this. An effort has been made to maintain rigour analytical discussions and precision in descriptions. It is hoped that the book will be useful to an advanced undergraduate student, and as a review to a graduate student.

I am grateful to my colleagues, Dr. S.M. Bharati, Dr. S.M. Chitre, Dr. P.P. Divakaran, Dr. Y.K. Gambhir, Dr. G.V. Dass, Dr. Dipan K. Ghosh, Dr. K.S. Kulkarni, Dr. R.C. Mehrotra, Dr. C.H. Mehta, Dr. G. Mukhopadhyay, Dr. R.S. Patil and Dr. G. Thyagarajan who ungrudgingly gave me their valuable time in reading parts of the manuscript and made valuable suggestions. I also thank Mr. Sunil Somalwar for going through a part of the manuscript.

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S H PATIL

Fundamental Constants

$c = 2.997925 \times 10^{8}$ m/s
$h = 6.6256 \times 10^{-34}$ J s
$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
$e = 1.60206 \times 10^{-19}$ C
$k = 1.38044 \times 10^{-23}$ J/K
$m_p = 938.211$ MeV/c^2
$m_n = 939.505$ MeV/c^2
$\epsilon_0 = 8.85434 \times 10^{-12}$ F/m or C^2/Nm^2
$\mu_0 = 4\pi \times 10^{-7}$ H/m or N/A^2
$N = 6.022 \times 10^{26}$/kmol, number of atoms in 12 kg of ^{12}C

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Introduction

The development of classical physics reached its climax during the latter half of the nineteenth century. The laws of mechanics as stated by Newton, and their formal development due to Lagrange, Hamilton, Poisson and Jacobi, exhibiting the inherent power and beauty of these laws, seemed complete and universal. Maxwell's equations gave a satisfactory description of electromagnetic fields, while kinetic theory provided a microscopic basis for the thermodynamic properties of matter. The success of these laws in explaining the physical observations was so impressive that the nineteenth century scientists implicitly believed that they were potentially capable of describing all physical phenomena. Scarcely could they have foreseen the revolution that was about to take place in our understanding of the physical universe and its laws.

The laws of classical physics are quite unsuccessful in explaining the phenomena involving high velocities and the physical properties of small bodies such as atoms at short distances, which were observed around the turn of the century. Here, high velocities are velocities comparable to the velocity of light, i.e. $v \sim c = 3 \times 10^8$ m/s, and short distances are distances comparable to atomic distances, i.e. $d \sim 10^{-10}$ m. The attempts to explain these observations led to a drastic reformulation of the ideas of the physical nature, resulting in the special theory of relativity and the quantum theory of matter. These two theories and their applications, together with the generalizations to the general theory of relativity and the quantum field theory, form the subject of what is popularly known as modern physics or the twentieth century physics.

Ideas of the special theory of relativity are considered first. This is logically appropriate since the theory defines the basic framework of space, time, and observation. It allows us to choose a convenient frame of coordinates in which the physical laws take on a simple form. What is more, the physical relations exhibit meaningful symmetry when expressed in the language of the special theory of relativity, e.g. energy and momentum can be expressed as the components of a 4-component vector. This helps in developing the ideas of quantum mechanics and in introducing the electromagnetic interaction of matter and radiation.

In the second part the concepts of quantum theory, which have far-reaching implications on the meaning of measurement and observation, are discussed. The calculational methods of quantum mechanics are developed and applied to describe the properties of atoms and molecules, which provide definite, quantitative support to the validity of quantum mechanics. The methods are then extended to apply to many-particle systems, in particular, to the solid state. This leads to a qualitative understanding of the properties of solids, which can be exploited for technological applications.

In the third part are described some important properties of nuclei and elementary particles and their interactions. Their analysis requires a deeper probe, and hence larger energies, with the consequence that relativistic effects become significant in this domain. The study of elementary particles is oriented towards a search for the building blocks of matter and their basic interactions. Considerable progress has been made in this direction though it cannot be said that the end is as yet in sight. Finally, we end with a brief description of the applications of modern physics to astrophysics, in particular the implications of the general theory of relativity for the origin and evolution of the universe. This is quite appropriate for the reason that the general theory of relativity stands out as one of the most elegant and profound theories in modern physics.

The mks system of units (metre, kilogram and second have been used in most places, though occasionally there are digressions into the cgs system. There are also examples at the end of each chapter, to illustrate and elaborate the material in the chapter.

1

Special Theory of Relativity

1.1 Inertial frames of reference
1.2 Galilean transformations
1.3 Velocity of light
1.4 Postulates of special relativity
1.5 Lorentz transformations
1.6 Simultaneity and time dilation
1.7 Length contraction
1.8 Transformation of velocities
1.9 Lorentz four-vector
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1.11 Electromagnetic interaction
1.12 Zero-mass particles and doppler shift
1.13 Examples
Problems

We begin our discussion of modern physics with the theory of relativity which aims at relating the observations made by observers in relative motion with respect to each other. Here *only* the restrictive case of the special theory of relativity is analysed, in which the observers are moving with constant velocity with respect to each other. This will help in choosing appropriate frames of reference and in presenting the later topics in a unified manner. After a brief consideration of the drawbacks of the classical theory, the main results of the special theory of relativity are obtained, and applied to describe some specific physical situations.

1.1 Inertial Frames of Reference

Most physical observations describe the behaviour of certain objects in space as a function of time. Since the position of a body can be stated only relative to some other bodies, the description of these observations requires a *frame of reference* which is a technical term for the combination of a set of spatial coordinate axes and a time variable.

It was realised by Galileo and others, that the form of the laws of nature depends on the choice of the frame of reference. Among all the possible frames of reference, there exists a class called the *inertial* frames of reference, in which these laws take a simple form. Inertial frames of reference are those in which a body that is not acted upon by external forces, moves with constant velocity. It is implicit here that if two reference frames move with constant velocity with respect to each other, and one of them is inertial, the other also is an inertial frame. It was found that the laws of mechanics take on the same form in all inertial frames of reference.

1.2 Galilean Transformations

Consider two inertial frames of reference F and F', such that their coordinate axes coincide at $t = 0$, and F' moves with velocity $\mathbf{v}$ along the x-axis with respect to F. Then, it may be expected that the coordinates in the two frames are related by the equations

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \tag{1.1}$$

called *Galilean transformations*. In writing these relations, it is assumed that (i) it is possible to define a time t which is the same for all inertial frames of reference, and (ii) the distance between two points is independent of the frames of reference.

For Galilean transformations, it is easy to show that the velocities and accelerations in the two frames are related by

$$\mathbf{u}' = \mathbf{u} - \mathbf{v} \tag{1.2}$$

where $\mathbf{v}$ is along the x-direction, and

$$\mathbf{a}' = \mathbf{a} \tag{1.3}$$

respectively. Then, if the interaction potential V is a function of only the distances between particles, Newton's equations in the two frames are

$$\begin{aligned} m_i\mathbf{a}_i &= -\ \nabla_i V \\ m_i\mathbf{a}_i' &= -\ \nabla_i' V \end{aligned} \tag{1.4}$$

where the subscript i is the particle index. These equations are related by the transformations (1.1) and are of the same form. However, it was observed that the Galilean transformations are not consistent with the dynamical theory of electromagnetic fields as formulated by Maxwell (1865).

1.3 Velocity of Light

It follows from Maxwell's equations for electromagnetic fields that electromagnetic waves travel in vacuum with a speed equal to the ratio of the electromagnetic unit to the electrostatic unit of charge. This ratio is essentially equal to the speed of light so that light itself is taken as a form of electromagnetic radiation.

Now, how does the velocity of light transform from one inertial frame to another ? According to Galilean transformations, the velocities are different in different frames and are related by Eq. (1.2). However, Maxwell's equations have no reference to the velocity of the inertial frame and hence imply that the speed of light is independent of the velocity of the inertial frame. Observationally also, the Michelson-Morley experiment (1887) analysed below suggests that the speed of light is independent of the velocity of the inertial frame.

Suppose, the earth is moving with velocity $\mathbf{v}$ in the x-direction with respect to the 'standard' frame in which the velocity of light is c in all directions. Then according to Eq. (1.2), the velocity of light with respect to an observer on Earth is $\mathbf{c} - \mathbf{v}$. The time taken for light to travel along the limb AB of the interferometer (Fig. 1.1), from A to B and back is

$$t_1 = \frac{l_1}{c - v} + \frac{l_1}{c + v} \tag{1.5}$$

While travelling from A to C and back, the velocity $\mathbf{c} - \mathbf{v}$ is parallel to AC and hence perpendicular to $\mathbf{v}$. Therefore

$$\mathbf{c} \cdot \mathbf{v} = v^2 \tag{1.6}$$

and the magnitude of $\mathbf{c} - \mathbf{v}$ is $(c^2 - v^2)^{1/2}$, so that the time taken for light to travel from A to C and back, is given by

$$t_2 = \frac{2l_2}{(c^2 - v^2)^{1/2}} \tag{1.7}$$

Thus the difference in the two times is

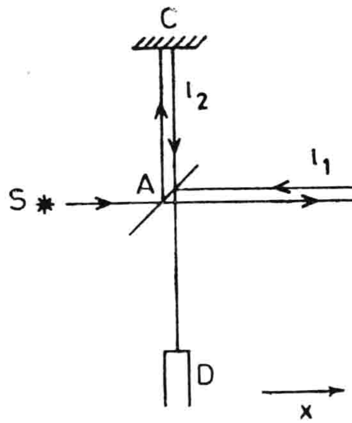

Fig. 1.1 Schematic diagram of the Michelson-Morley experiment.

$$\Delta = t_1 - t_2 = \frac{2l_1 c}{c^2 - v^2} - \frac{2l_2}{(c^2 - v^2)^{1/2}} \tag{1.8}$$

If the apparatus is turned through 90°, the roles of l_1 and l_2 are interchanged, and the difference in the times becomes

$$\Delta' = t_1' - t_2' = \frac{2l_1}{(c^2 - v^2)^{1/2}} - \frac{2l_2 c}{c^2 - v^2} \tag{1.9}$$

The expected shift in the interference fringe at D, is

$$\begin{aligned} \delta &= \frac{c(\Delta' - \Delta)}{\lambda} \\ &= \frac{2(l_1 + l_2)}{\lambda}\left[\frac{1}{(1 - v^2/c^2)^{1/2}} - \frac{1}{1 - v^2/c^2}\right] \\ &\approx -\frac{(l_1 + l_2)}{\lambda}\left(\frac{v^2}{c^2}\right) \text{ for } v \ll c \end{aligned} \tag{1.10}$$

In the experiment of Michelson and Morley, $l_1 + l_2$ was 22 m, and $\lambda = 5.9 \times 10^{-7}$ m. The value of v is at least of the order $v \approx 30$ km/s corresponding to the velocity of the earth's motion around the sun, even if the motion of the solar system around the galactic centre is ignored. For these values

$$\delta \approx 0.37 \tag{1.11}$$

No such shift was observed in the experiment.

The above result is based on Eq. (1.2) for the transformation of the velocity of light, which was derived from the Galilean transformations (1.1). An attempt was made to salvage the Galilean transformations by postulating that a hypothetical medium called *ether*, responsible for the propagation of light, is dragged along by the earth as it moves in space. Then the speed of light with respect to an observer on the earth would remain unaffected by the motion of the earth, analogous to the speed of sound in which case the air is dragged along. This would explain the null result of the Michelson-Morley experiment, but this is in conflict with the observed aberration of starlight received on the earth. It is found that in order to observe a star, the telescope should be tilted in the direction of the earth's velocity (Fig. 1.2). This tilt would not be needed if the light-propagating ether was dragged along by the earth. Actually, what is observed is not the absolute tilt but the variation of the tilt as the earth changes its velocity along its orbit around the sun. The ether-based explanation became even less tenable after Lodge (1892) showed that the velocity of light is unaffected in the vicinity of rapidly rotating bodies.

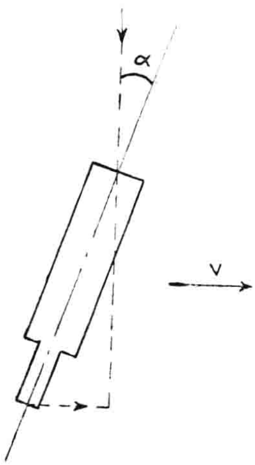

Fig. 1.2 In order that starlight passes along a telescope moving with velocity v, the telescope should be tilted at an angle of $\alpha \approx v/c$.

1.4 Postulates of Special Relativity

Einstein (1905) proposed a radically different but, in retrospect, a simple approach to the problem posed by the Michelson-Morley experiment. He started with the principle of relativity but also postulated that the speed of light is the same in all inertial frames of reference, thus giving it the status of a physical law. This immediately explains the Michelson-Morley result but requires that the Galilean transformations (1.1) be discarded. He found that space and time are related in an intimate manner and should be treated on an equal basis. Their relation has a far-reaching influence on the laws of physics. We begin the discussion of Einstein's results with a formal statement of the postulates of the special theory of relativity.

1. The laws of nature are of the same form in all inertial frames of reference.

2. The speed of light is the same in all inertial frames of reference, and is independent of the motion of the source.

It is implicit in the first postulate that, since the coordinates of the different inertial frames are related, the laws of nature written in the various inertial frames can be deduced from one another. It also follows that the Galilean transformations (1.1) relating the coordinates of the inertial frames, cannot be right since they would imply that the speed of light is different in different inertial frames, in contradiction to the second postulate. Hence, a more general relation between the coordinates must be obtained, which incorporates the information that the speed of light is the same in all inertial frames.

1.5 Lorentz Transformations

In deriving the transformation equations consistent with the postulates of the special theory of relativity, it was assumed that space is homogeneous, i.e. that all points in space and time are equivalent. This means that the separation between space-time points should remain invariant under translations which implies that the relations between the coordinates of different inertial frames should be linear.

Let us consider again the inertial frames F and F' mentioned in Sec. 1.2, whose axes coincide at time $t = t' = 0$, and F' moves with velocity $\mathbf{v}$ along the x-axis with respect to F. It is assumed that the y-axis is perpendicular to the x'-axis since otherwise the inclinations of the positive and negative y-axis with respect to the x-axis would be different, violating the rotational (or alternatively, left-right) symmetry about the direction of relative velocity. It is also assumed that the y- and z-axes are orthogonal to each other in either of the frames of reference. Finally, since the lengths of two rods, which are at rest in frames F and F' respectively, and which are perpendicular to the x-axis, can be compared while they are passing each other,

$$\begin{aligned} y' &= y \\ z' &= z \end{aligned} \tag{1.12}$$

in order that the relations between F and F' be reciprocal. For the transformation of the x-coordinate, it is noted that the origin of F' travels with velocity $\mathbf{v}$ with respect to frame F, which implies that

$$x' = \alpha\,(x - vt) \tag{1.13}$$

Taking into account the possibility that time may not be a universal variable,

$$t' = \gamma\,(t - \beta x) \tag{1.14}$$

is for the transformation of the time coordinate.

Let an electromagnetic signal be emitted at $t = 0$, from the origin of F, which also coincides with the origin of F' at that time. Since the speed of light is the same in all inertial frames, the wavefront is described in the two frames by the equations