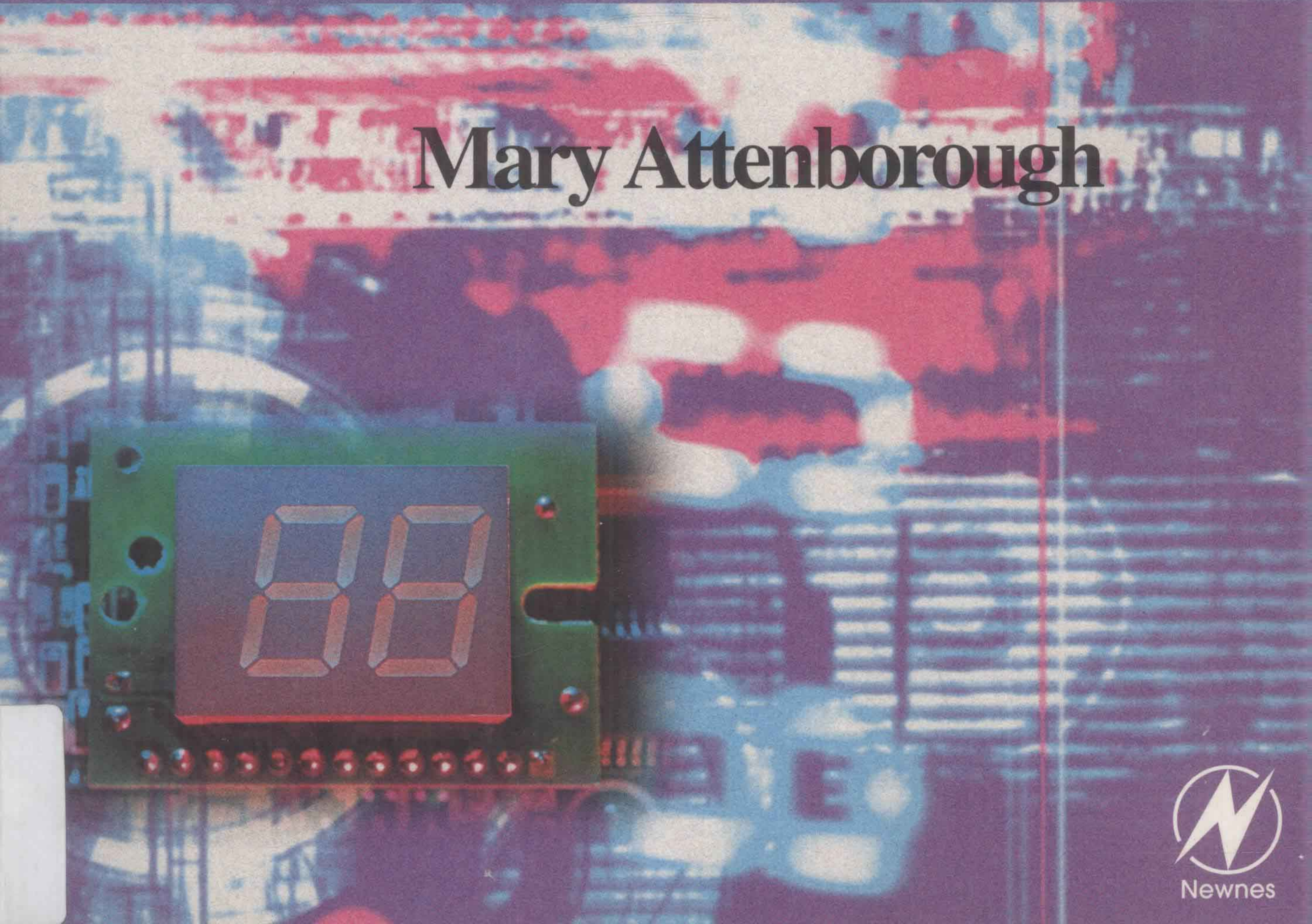
Mathematics for Electrical Engineering and Computing

Mary Attenborough



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Preface

This book is based on my notes from lectures to students of electrical, electronic, and computer engineering at South Bank University. It presents a first year degree/diploma course in engineering mathematics with an emphasis on important concepts, such as algebraic structure, symmetries, linearity, and inverse problems, clearly presented in an accessible style. It encompasses the requirements, not only of students with a good maths grounding, but also of those who, with enthusiasm and motivation, can make up the necessary knowledge. Engineering applications are integrated at each opportunity. Situations where a computer should be used to perform calculations are indicated and 'hand' calculations are encouraged only in order to illustrate methods and important special cases. Algorithmic procedures are discussed with reference to their efficiency and convergence, with a presentation appropriate to someone new to computational methods.

Developments in the fields of engineering, particularly the extensive use of computers and microprocessors, have changed the necessary subject emphasis within mathematics. This has meant incorporating areas such as Boolean algebra, graph and language theory, and logic into the content. A particular area of interest is digital signal processing, with applications as diverse as medical, control and structural engineering, non-destructive testing, and geophysics. An important consideration when writing this book was to give more prominence to the treatment of discrete functions (sequences), solutions of difference equations and z transforms, and also to contextualize the mathematics within a systems approach to engineering problems.

Acknowledgements

I should like to thank my former colleagues in the School of Electrical, Electronic and Computer Engineering at South Bank University who supported and encouraged me with my attempts to re-think approaches to the teaching of engineering mathematics.

I should like to thank all the reviewers for their comments and the editorial and production staff at Elsevier Science.

Many friends have helped out along the way, by discussing ideas or reading chapters. Above all Gabrielle Sinnadurai who checked the original manuscript of *Engineering Mathematics Exposed*, wrote the major part of the solutions manual and came to the rescue again by reading some of the new material in this publication. My partner Michael has given unstinting support throughout and without him I would never have found the energy.

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Part 3 Functions of more than one variable

Part 4 Graph and language theory

Part 5 Probability and statistics

Part 1 Sets, functions, and calculus

1 Sets and functions

1.1 Introduction

Finding relationships between quantities is of central importance in engineering. For instance, we know that given a simple circuit with a 1000 Ω resistance then the relationship between current and voltage is given by Ohm's law, $I = V/1000$. For any value of the voltage V we can give an associated value of I. This relationship means that I is a function of V. From this simple idea there are many other questions that need clarifying, some of which are:

1. Are all values of V permitted? For instance, a very high value of the voltage could change the nature of the material in the resistor and the expression would no longer hold.
2. Supposing the voltage V is the equivalent voltage found from considering a larger network. Then V is itself a function of other voltage values in the network (see Figure 1.1). How can we combine the functions to get the relationship between this current we are interested in and the actual voltages in the network?
3. Supposing we know the voltage in the circuit and would like to know the associated current. Given the function that defines how current depends on the voltage can we find a function that defines how the voltage depends on the current? In the case where $I = V/1000$, it is clear that $V = 1000I$. This is called the inverse function.

Another reason exists for better understanding of the nature of functions. In Chapters 5 and 6, we shall study differentiation and integration. This looks at the way that functions change. A good understanding of functions and how to combine them will help considerably in those chapters.

The values that are permitted as inputs to a function are grouped together. A collection of objects is called a set. The idea of a set is very simple, but studying sets can help not only in understanding functions but also help to understand the properties of logic circuits, as discussed in Chapter 10.

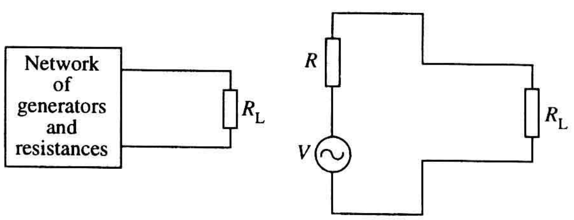

Figure 1.1 *The voltage V is an equivalent voltage found by considering the combined effect of circuit elements in the rest of the network.*

1.2 Sets

A *set* is a collection of objects, called *elements*, in which the order is not important and an object cannot appear twice in the same set.

Example 1.1 Explicit definitions of sets, that is, where each element is listed, are:

A = {a, b, c}

B = {3, 4, 6, 7, 8, 9}

C = {Linda, Raka, Sue, Joe, Nigel, Mary}

a ∈ A means 'a is an element of A' or 'a belongs to A'; therefore in the above examples:

3 ∈ B

Linda ∈ C

The *universal set* is the set of all objects we are interested in and will depend on the problem under consideration. It is represented by $\mathcal{E}$.

The *empty set* (or *null set*) is the set with no elements. It is represented by ∅ or { }.

Sets can be represented diagrammatically – generally as circular shapes. The universal set is represented as a rectangle. These are called *Venn diagrams*.

Example 1.2

$\mathcal{E}$ = {a, b, c, d, e, f, g}, A = {a, b, c}, B = {d, e}

This can be shown as in Figure 1.2.

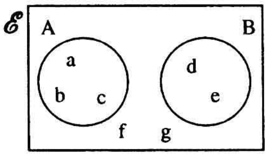

Figure 1.2 *A Venn diagram of the sets* $\mathcal{E}$ = {a, b, c, d, e, f, g}, A = {a, b, c}, *and* B = {d, e}.

We shall mainly be concerned with sets of numbers as these are more often used as inputs to functions.

Some important sets of numbers are (where '...' means continue in the same manner):

The set of *natural numbers* $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$
The set of *integers* $\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$
The set of *rationals* (which includes fractional numbers) $\mathbb{Q}$
The set of *reals* (all the numbers necessary to represent points on a line) $\mathbb{R}$

Sets can also be defined using some rule, instead of explicitly.

Example 1.3 Define the set A explicitly where $\mathcal{E} = \mathbb{N}$ and $A = \{x \mid x < 3\}$.

Solution The $A = \{x \mid x < 3\}$ is read as 'A is the set of elements x, such that x is less than 3'. Therefore, as the universal set is the set of natural numbers, $A = \{1, 2\}$

Example 1.4 $\mathcal{E}$ = days of the week and A = {x | x is after Thursday and before Sunday}. Then A = {Friday, Saturday}.

Subsets

We may wish to refer to only a part of some set. This is said to be a subset of the original set.

A ⊆ B is read as 'A is a subset of B' and it means that every element of A is an element of B.

Example 1.5

$\mathscr{E} = \mathbb{N}$

$A = \{1, 2, 3\}, \quad B = \{1, 2, 3, 4, 5\}$

Then $A \subseteq B$

Note the following points:

- All sets must be subsets of the universal set, that is, $A \subseteq \mathscr{E}$ and $B \subseteq \mathscr{E}$
- A set is a subset of itself, that is, $A \subseteq A$
- If $A \subseteq B$ and $B \subseteq A$, then $A = B$

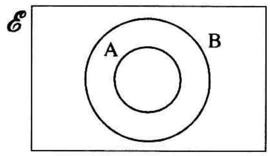

Figure 1.3 *A Venn diagram of a proper subset of* B*:* A ⊂ B.

Proper subsets

A ⊂ B is read as 'A is a proper subset of B' and means that A is a subset of B but A is not equal to B. Hence, A ⊂ B and simultaneously B ⊂ A are impossible.

A proper subset can be shown on a Venn diagram as in Figure 1.3.

1.3 Operations on sets

In Chapter 1 of the background Mathematics notes available on the companion website for this book, we study the rules obeyed by numbers when using operations like negation, multiplication, and addition. Sets can be combined in various ways using set operations. Sets and their operations form a Boolean Algebra which we look at in greater detail in Chapter 4, particularly its application to digital design. The most important set operations are as given in this section.

Complement

$\bar{A}$ or A′ represents the complement of the set A. The complement of A is the set of everything in the universal set which is not in A, this is pictured in Figure 1.4.

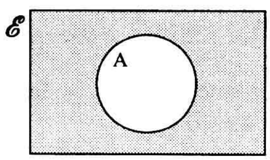

Figure 1.4 *The shaded area is the complement,* A′, *of the set* A.

Example 1.6

$\mathscr{E} = \mathbb{N}$

$A = \{x \mid x > 5\}$

then $A' = \{1, 2, 3, 4, 5\}$