THREE-DIMENSIONAL PROBLEMS OF THE MATHEMATICAL THEORY OF ELASTICITY AND THERMOELASTICITY

edited by:

V.D. KUPRADZE

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PREFACE

In the best books on the theory of elasticity the investigation of three-dimensional boundary value problems has been so far limited to bodies of special shape (a half-space, a sphere, some cases of axially symmetrical bodies and so on). The greatest attention has been given to static problems, less attention to oscillation problems and still less to problems of general dynamics. Such a situation might be well expected – it reflects the historical background of the theory of elasticity which during the entire preceding period was concentrated on bodies of particular profiles and was above all interested in problems of static equilibrium.

It would be wrong to attribute this situation only to the importance of the above-mentioned problems for technology and engineering. The true reason is that the methods of classical elasticity were inadequate for developing a rigorous and sufficiently complete theory of three-dimensional boundary value problems.

Unlike the three-dimensional problems, the theory of the plane problem worked out mainly by the classical methods (the theory of analytic functions, Fredholm's theory of integral equations and, later, the theory of one-dimensional singular integral equations) has been extensively developed and found its perfect expression in I.N. Muskhelishvili's book "Some Basic Problems of the Mathematical Theory of Elasticity" the first edition of which appeared in 1933.

The situation is currently changing. The theory of three-dimensional problems may now be worked out by a variety of means. We shall just mention two of the possibilities: on the one hand, it is the modern theory of generalized solutions of differential equations (the method of Hilbert spaces, variational methods), on the other hand – the theory of multi-dimensional singular potentials and singular integral equations.

The first trend – based on the ideas of the modern functional analysis which are novel to the classical mechanics – is characterized by great generality involving the case of variable coefficients and boundary manifolds of the general type. Owing to such generality, it may be employed in the first place for proving theorems on the existence of non-classical solutions, requiring additional, sometimes essential, restrictions when used for classical solutions.

A fine, though concise, treatment of these topics may be found in G. Fichera's papers "Existence Theorems in Elasticity" and "Boundary Value Problems of Elasticity with Unilateral Constraints", Handbuch der Physik, VIa/2, Springer Verlag, 1972, and in C. Dafermos' paper "On the Existence of Asymptotic Stability of Solutions to the Equations of Thermoelasticity", Arch. Rat. Mech. Anal. 29, 4, 1968.

The second trend based on the rapidly developing theory of singular integrals and integral equations is a direct extension of the concepts of the theory of potentials and Fredholm equations which are, as known, the prevailing concepts of the classical mechanics. This approach, being not so general as the first one, allows to investigate in detail cases most important for the theory and application, retaining the efficiency of the methods of the classical mechanics of continua.

The present book has adopted the second trend. It is an attempt to develop – apparently for the first time with adequate completeness and at the modern level of mathematical rigour – the general theory of three-dimensional problems of statics, oscillation and dynamics for linear equations with constant and piecewise-constant coefficients of classical elasticity, thermoelasticity and couple-stress elasticity.

Much space in the book is assigned to general problems (existence and uniqueness theorems, an analysis of differential properties of solutions, the continuous dependence on the data of a problem etc.). A great deal of attention is also given to questions of the actual construction of solutions in a form allowing to express them numerically under very general conditions.

With this end in view the solutions are represented as generalized Fourier series to construct which there is no need to know the eigenfunctions and the eigenvalues of any auxiliary boundary value problems. New representations of solutions by quadratures have been found for some particular cases.

We think that the simple construction of solutions and the representation of elementary structures by explicit invertible operators, together with a detailed analysis of the smoothness of solutions, may serve in the conditions of modern computing facilities as the basis for obtaining convenient algorithms of numerical computations and for estimating approximations.

The book reproduces the monograph of the same authors published by the Tbilisi University Press in 1968. It was favourably received by readers and sold out within a short time. In 1971 the first edition was awarded the State Prize of the Georgian SSR.

When a second edition was called for, the book was extensively revised and enlarged. To make the book accessible to a wider circle of people the authors rewrote nearly all the chapters, simplified a number of proofs, corrected the noted errata.

The chapters of the book are divided into sections, the sections into articles; each section has its own numeration of formulas; the formula number is denoted by two figures enclosed in brackets; for example, (5.9) means the ninth formula in the fifth section. When reference is made to a formula, the number of a chapter is added to the number of the formula; thus, (VIII, 3.6) means the sixth formula in the third section of the eighth chapter. However, if reference is made to a formula within a given chapter, the chapter number is omitted.

Theorems, lemmas, definitions and notes are numerated in the same manner but without brackets. Theorem V, 2.10 therefore means the tenth theorem in the second section of the fifth chapter. Again, if reference is made within a given chapter, the chapter number is left out. All the chapters, except the first one, are supplemented with problems some of which may be used as a subject of independent research.

The bibliography consists of those titles which were available to the authors at the time of writing the book. It does not claim to bibliographic irreproachability and does not include the books published after 1972.

Tbilisi The Authors

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