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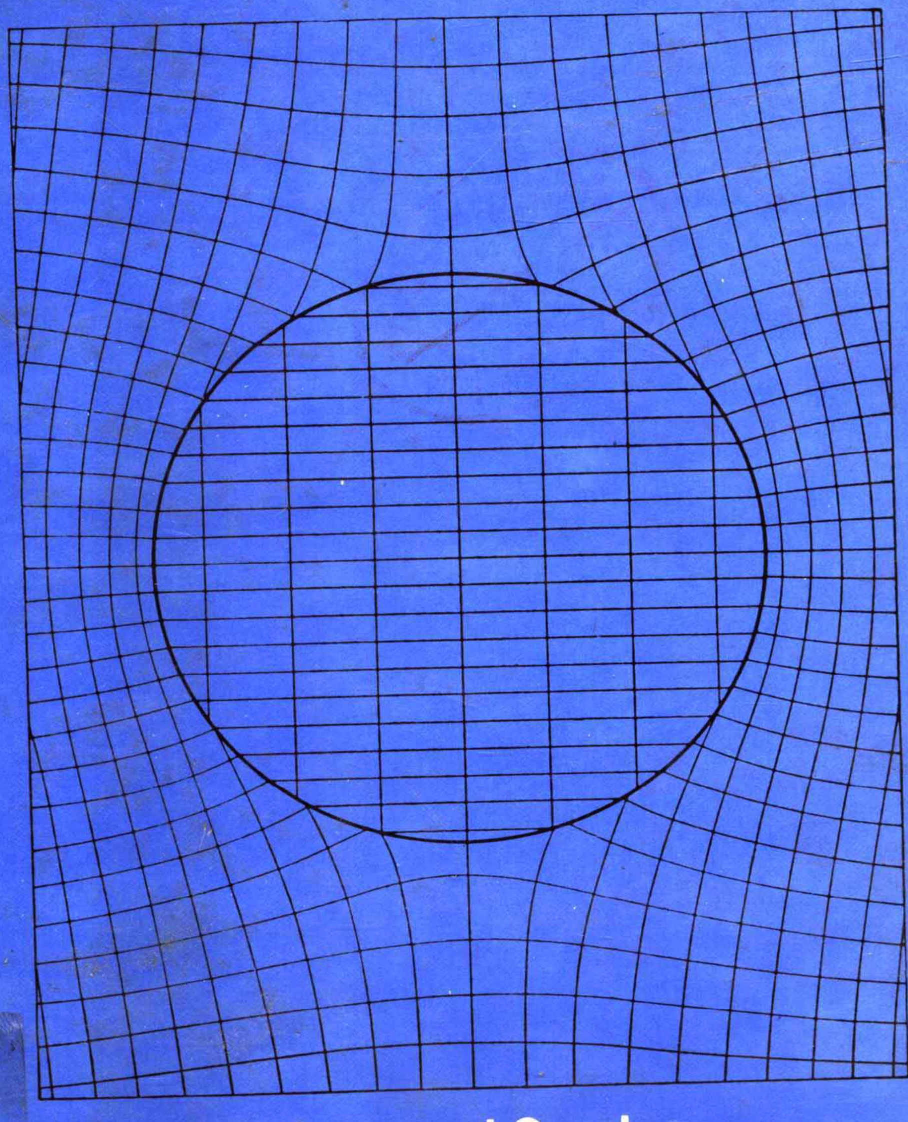
Electromagnetics

Classical and Modern Theory and Applications



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ELECTROMAGNETICS

Classical and Modern Theory and Applications

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PREFACE

The objectives of this book are more ambitious than those of the usual book on electromagnetics. Not only does it seek to provide the essential introductory aspects of electricity and magnetism, but it also seeks to provide examples of how these basic principles are employed in an understanding and description of a wide range of electromagnetic applications. Because many of these important applications overlap electromagnetics with other fields, we have included certain details of acoustics, thermal, plasmas, elasticity, physical and geometrical optics. Such applications often require extensive mathematical sophistication in their description, and we have attempted to provide sufficient mathematical material, either within the text or in appendixes, to permit the reader to follow the developments. Moreover, we also recognize that exact solutions to certain problems are often not possible, and we have included some material to show how one would approach the solution of problems using graphical, numerical or approximate analytical methods.

In the development of the text material, we proceed from basic concepts to particular specialized viewpoints. Thus we devote considerable attention to fields and field configurations from graphical numerical and analytical viewpoints before we seek to apply these to a traditional discipline. Because of subsequent needs, we seek ideas and examples from a wide variety of disciplines in these early chapters and find them in thermal, acoustic, mechanical, and fluids as well as in electricity. It is not until we have explored the general properties of fields do we turn our attention to a detailed study of electricity and magnetism. Once we have discussed these areas and have developed an understanding of electromagnetism and the important aspects of Maxwell's equations, we then turn to the various important areas of application of such fields. These studies range from plane waves in free space, in bounded space, and in confined regions such as in waveguides and in resonators. As one changes the wavelength of the fields, one is led from radiation problems from simple configurations to a more extensive discussion of simple and multiple radiators and the broad aspects of diffraction. The physical optics problems, and these are often referred to as Fourier Optics, are brought into focus through a variety of applications including optical filtering and signal processing, holography, and some aspects of fiber optics.

The introduction of anisotropic mediums into the fields, either through birefrigent crystals or by introducing plasmas with and without applied magnetic fields (as in the ionosphere), leads to questions of the generation of ordinary and extraordinary waves, the polarization of waves, and dispersion, as inevitable consequences.

It must be understood, however, that it is not our purpose to discuss in detail every aspect of every field that we introduce. Specifically, we are not trying to include a complete discussion of many disciplines within one pair of covers. We are trying to show some of the logical directions which the broad field of electromagnetics has impacted, and to present enough content so that the broad concepts are understood. We recognize that the mathematical sophistication often becomes more demanding with more detailed studies, and in consequence we might not consider all of the practical aspects of a given field. This occurs, for example, when we discuss problems having rectangular configurations but do little with cylindrical configurations. Hence, our discussion of cylindrical waveguides, cylindrical resonators, and cylindrical fibers is limited. It is our expectation that if we can provide an understanding in the mathematically less complicated cases that the reader will be able to develop an understanding of the comparable situations with different geometrical constraints.

With objectives such as these, some material is introductory and other is rather advanced. The use of the book will be dictated by the objectives of a particular course of study. The instructor is provided with a wide choice of topics, and the demands on the student will be dictated by his course objectives. Thus the book can be used in fields courses at the junior level, and it can also be used in more specialized courses at more advanced levels. In addition, practicing engineers will greatly benefit from this book because of its wide scope and coverage.

The authors wish to express their appreciation to Mrs. Ada Willis for her patience in typing the original manuscript and its numerous revisions, and to Mrs. Katherine MacDougall for her expert typing of the final manuscript.

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CHAPTER 1

POTENTIAL-FLOW FIELDS

This chapter introduces classes of field problems which are characterized as potential-flow fields. The physical conditions which lead to such classes of behavior are considered to constitute a phenomenological class. The features of such potential-flow fields, which are classified as conservative, irrotational or curl-free fields are described by Laplace's equation.

1-1. Field Versus Lumped Parameter Description

The study of lumped, linear systems analysis proceeds by modeling the system elements in terms of through (flow) and across (potential) variables. Linear physical elements which constitute such systems fall into three classes when described in mathematical terms relating the terminal through and across variables: proportional, derivative, integral. When an array of elements are interconnected, the equilibrium equations of the connected array are deduced by imposing known physical laws: Kirchoff's laws for electrical networks; conservation of energy for thermal systems, conservation of mass for fluid systems. The formulations prove to be analogs of each other, and the general physical considerations often parallel each other sufficiently well to allow understanding in one field to aid in an understanding in an analogous field. Often this understanding is improved by performing experiments in an analogous system since experiments may be more easily carried out and interpreted in one system than another. To elaborate on this point, we shall consider some familiar examples.

Example 1-1.1. Develop the equations and the network representation of a linear system consisting of a spring, a mass, and a dashpot interconnected as shown in Fig. 1-1.1. The dashpot consists of a piston immersed in a viscous fluid.

Solution. The equation of motion for an applied force f, is written as follows:

$$M \frac{dv}{dt} + Dv + K \int v \, dt = f$$

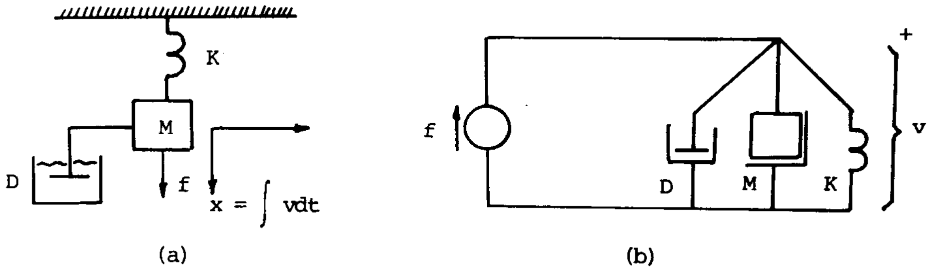

Figure 1-1.1. Representations of a spring, mass, dashpot system. (a) Physical system. (b) Equivalent network representation.

where K is the spring constant and D is the damping constant. This equation may be written explicitly in terms of x, since dx/dt = v. Thus

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + Kx = f \tag{1-1.2}$$

Solution of one or the other of these equations, subject to two specified initial conditions, will serve to determine uniquely the motion of the mass as a function of time.

ΔΔΔ

The lumped parameter analysis discussed in the example is concerned only with the bulk or overall behavior of the system. It does not concern itself with questions such as:

1. How does the spring constant depend on the material and construction of the spring?
2. Under what conditions will the elongation of the spring be directly proportional to the displacement?
3. What is the maximum elongation of the spring before it weakens?
4. How does the viscous force produced by the dashpot depend on the physical properties of the fluid used?
5. In what way should the dimensions of the dashpot be changed in order to double the viscous force?
6. Under what conditions will the viscous force be proportional to the velocity?

These questions, and others of a similar nature, are those that field analysis attempts to answer. In other words, field analysis is the link between the physical properties of all of the elements of a system and the lumped parameter model that described the gross behavior of the system.

A complete field analysis of the spring would give the relation between the stress and strain at each point in the spring as a function of the external elongation of the spring. The field analysis of the dashpot would provide a solution for the velocity field in the fluid as a function of piston velocity and position. In terms of this velocity field and the viscosity of the fluid of the field the overall damping force could be obtained.

Clearly, field analysis describes fields that are functions of the spatial coordinates as well as time. As a result, the description is given as partial differential equations as contrasted with the ordinary type of differential equation that occurs in a lumped parameter analysis.

Example 1-1.2. Develop the equilibrium equation for a parallel RLC circuit as shown in Fig. 1-1.2.

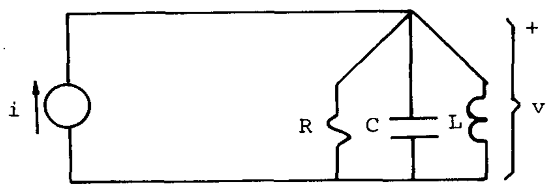

Figure 1-1.2. A parallel RLC circuit.

Solution. For ideal elements, the current-voltage terminal relations for three elements are:

$$i = \frac{v}{R} \; ; \quad i = C \frac{dv}{dt} \; ; \quad i = \frac{1}{L} \int v \, dt \tag{1-1.3}$$

By an application of the Kirchoff current law to the node or junction, the equation giving the relation between the applied current and the voltage across the elements, is:

$$C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int v \, dt = i \tag{1-1.4}$$

The exact parallelism between Figs. 1-1.1 and 1-1.2, and Eqs. (1-1.1) and (1-1.4) for the mechanical and electrical systems clearly illustrates the analogy between the two systems.

ΔΔΔ

Just as in the mechanical MDK system, the lumped parameter analysis of the CRL electrical circuit does not provide answers to many questions of fundamental importance. Among these unanswered questions are:

1. Under what conditions can circuit elements such as resistors, inductors and capacitors be assumed to have an electrical behavior according to the current-voltage terminal relations given in Eq. (1-1.3)?
2. Do the circuit parameters, resistance R, inductance L, and capacitance C, depend on frequency or are they truly constants?

3. What is the maximum voltage that can be applied across the capacitor before dielectric breakdown occurs?
4. How should an inductor be constructed in order to obtain a maximum value of inductance L in a given volume?
5. Why is the current in an inductor related to the terminal voltage according to the expression $i = (1/L) \int v\, dt$?
6. Is the lumped parameter description of the circuit valid for frequencies greater than 10^{10} Hz?
7. Will the circuit radiate electromagnetic waves?
8. For a resistor with a non-uniform cross section, what is the temperature distribution throughout the resistor? Will hot spots develop?
9. How can the capacitance C be increased?

All of these questions and many more of similar type are of utmost importance not only to the engineer concerned with the design of resistors, inductors and capacitors, but also to the circuit applications engineer. An analysis to determine the electric and magnetic fields existing in the space around and within these elements will permit answers to all of the questions posed. In addition, the studyh of the fields associated with these devices is necessary for a proper understanding of the operation of circuit elements and the limitations inherent in the lumped parameter model. Again, the importance of field theory in providing the link between the detailed physical phenomena taking place in a circuit and the lumped parameter model used to describe the terminal behavior is apparent.

1-2. Definition of a Physical Field

We have noted that the lumped parameter system is intimately related to and an approximation to the more general distributed parameter case, and that an analogy exists among different physical systems (mechanical, electrical, fluid, thermal, elastic). However, no short and concise definition of what constitutes a physical field can be given. A general understanding and feeling for the field concept is perhaps best obtained from a consideration of a number of typical fields and the attributes that they have in common. However, because we shall be interested in a description of the behavior of the system at all points in the field, the description must be given in terms of *point* variables and not in terms of *terminal* variables.

Let us consider a certain volume or region in which the state of a physical system can be described by a function $f(x,y,z,t)$ of the spatial rectangular coordinates x,y,z and the time t. The function f is a point function; that is, it has a definite value at each point in the region and represents a field quantity. As an example, the temperature distribution throughout a body is a field quantity; likewise the pressure distribution in a fluid is a field quantity. Thus one important attribute of a field is that it has "extent," i.e., it describes a physical property as a continuous function of space and time coordinates. If the physical property is independent of time, the corresponding describing function represents a *static* or *time independent* field quantity. On the other hand, if the property being described varies with time, then the field is a *dynamic* or *time dependent* field.

When the property in question, be it temperature, pressure, density, etc. can be uniquely described at each point in space at a given time by a single number or scalar quantity, the collection of these scalar numbers defines a *scalar field*. Not all physical fields are scalar fields. Many of the properties of physical systems require not only a magnitude but also a direction to be specified, e.g., the velocity distribution in a fluid. In this case the property in question is described by a vector point function $\overline{F}(x,y,z,t)$ and the field is a *vector field*. Note that both the magnitude and direction of a vector field changes from point to point throughout the region of interest and, in general, also with time. A vector field may always be decomposed into three scalar fields. For example, the velocity field which describes the flow of a fluid can be decomposed into three scalar fields that give the components of the velocity in the x,y,z directions at each point.

Although vectors are used in a geometrical sense in static mechanics and in geometry, such vectors do not constitute a vector field. A vector field has extent and is an infinite collection of vectors, one at each point, describing a physical property throughout a region of space rather than, for example, a single force applied at a specific point along a truss or beam. Thus one might pictorially show a field by associating a number (to denote the magnitude or strength of the field at a point) and an arrow (to denote the direction of the field at a point) at every point in the field. In a less general case one might consider a three dimensional grid structure. At each point of intersection of the grid there would be both a number and a unit vector to specify the magnitude and direction of the field at that point.

In addition to scalar and vector fields, there are further generalizations that lead to what is termed a *tensor field*. An example of a tensor field is the stress distribution throughout an elastic solid. In an elastic body forces may exist that tend to elongate or compress the body along each coordinate direction x,y,z at each point. In addition, shear forces may exist that tend to shear the body along different planes. The complete description of the state of stress in the body thus requires more than three scalar quantities (if there were no shear stress, 3 scalar quantities would suffice) in order to describe it. In this particular case a 3 by 3 matrix, or nine scalar quantities are required, and the resultant matrix with elements that are point functions of the coordinates represents a tensor field of order two. On this general basis, scalars are tensors of order zero while vectors are tensors of order one.

1-3. Occurrence of Physical Fields in Engineering and Science

There is virtually no branch of engineering that is not concerned, to some extent, with field phenomena. The vibration of strings, beams, membranes, etc., is described by field quantities that give the displacement from equilibrium and the velocity at each point throughout the body. The flow of any fluid or gas is described by various field quantities such as velocity, pressure, density, etc. The temperature distribution and flow of heat is a field problem. Likewise any physical process depending on a diffusion mechanism requires a field description. Examples of diffusion are those that occur in filtration problems, in chemical mixing problems, in the flow of electrons and holes in semiconductors, and the diffusion of neutrons in reactors.

The propagation of waves or wave phenomena is intimately associated with fields. There are many examples of waves occurring in nature, among which are: the common acoustic or sound waves in air, water waves on the ocean, electro-magnetic waves, waves on strings and beams, ultrasonic waves in material bodies, magnetohydrodynamic waves in conducting fluids, etc. All wave problems have many features in common so that the detailed study of one or more types helps to provide a background for the understanding of other kinds of waves as well.

Many of the fields that occur in engineering are qualitatively familiar to us because of their common occurrence and our ability to detect their presence with our physical senses. An intuitive feeling for the nature of the velocity field in a flowing fluid or the temperature field throughout a room is held by most students in engineering or science. On the other hand, there are many fields that are a good deal more abstract in nature and even at times might be regarded more as mathematical inventions rather than as having physical reality. For example, Newton's law of gravitation states that two bodies attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between their centers. From this basic law has arisen the concept of a gravitational field. That is, the ability of one mass to attract another is attributed to the existence of a force field set up by the first mass and vice versa. Any other mass situated in the force field of the first mass will then experience a force, with this force being communicated to it through the gravitational force field. We could regard the notion of this gravitational force field that pervades all of space around a given mass as a fictitious field. However, even if we adhered to this viewpoint, we would still have to admit that the concept was an extremely useful one, and simplifies the solution of problems such as evaluating the flight trajectory of an interplanetary space ship, or the motions of the planets.

In many branches of modern physics such abstract fields are introduced in order to describe natural physical processes. To the extent that the field can be endowed with all the necessary properties to describe correctly all of the observed physical phenomena, it can be said to have physical reality, even though our senses