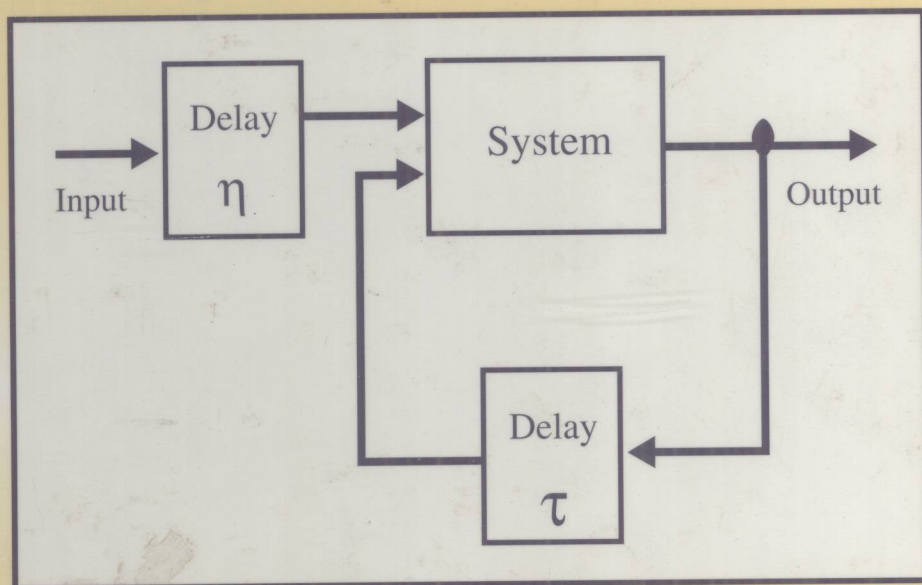


Control Engineering Series

ROBUST CONTROL AND FILTERING FOR TIME-DELAY SYSTEMS



Magdi S. Mahmoud

TP13
M215

ROBUST CONTROL AND FILTERING FOR TIME-DELAY SYSTEMS

Magdi S. Mahmoud
*Kuwait University
Safat, Kuwait*



E200000404



MARCEL DEKKER, INC.

NEW YORK • BASEL

Library of Congress Cataloging-in-Publication Data

Mahmoud, Magdi S.

Robust control and filtering for time-delay systems / Magdi S. Mahmoud.

p. cm. -- (Control engineering : 5)

Includes bibliographical references and index.

ISBN: 0-8247-0327-8

1. Robust control. 2. Time delay systems. I. Title. II. Control engineering (Marcel Dekker) ; 5.

TJ217.2 M34 2000

629.8'312--dc21

99-054346

This book is printed on acid-free paper.

Headquarters

Marcel Dekker, Inc.

270 Madison Avenue, New York, NY 10016

tel: 212-696-9000; fax: 212-685-4540

Eastern Hemisphere Distribution

Marcel Dekker AG

Hutgasse 4, Postfach 812, CH-4001 Basel, Switzerland

tel: 41-61-261-8482; fax: 41-61-261-8896

World Wide Web

<http://www.dekker.com>

The publisher offers discounts on this book when ordered in bulk quantities. For more information, write to Special Sales/Professional Marketing at the headquarters address above.

Copyright © 2000 by Marcel Dekker, Inc. All Rights Reserved.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage and retrieval system, without permission in writing from the publisher.

Current printing (last digit):

10 9 8 7 6 5 4 3 2 1

PRINTED IN THE UNITED STATES OF AMERICA

**ROBUST CONTROL
AND FILTERING FOR
TIME-DELAY SYSTEMS**

CONTROL ENGINEERING

A Series of Reference Books and Textbooks

Editor

NEIL MUNRO, PH.D., D.SC.

Professor

Applied Control Engineering

University of Manchester Institute of Science and Technology
Manchester, United Kingdom

1. Nonlinear Control of Electric Machinery, *Darren M. Dawson, Jun Hu, and Timothy C. Burg*
2. Computational Intelligence in Control Engineering, *Robert E. King*
3. Quantitative Feedback Theory: Fundamentals and Applications, *Constantine H. Houpis and Steven J. Rasmussen*
4. Self-Learning Control of Finite Markov Chains, *A. S. Poznyak, K. Najim, and E. Gómez-Ramírez*
5. Robust Control and Filtering for Time-Delay Systems, *Magdi S. Mahmoud*
6. Classical Feedback Control: With MATLAB, *Boris J. Lurie and Paul J. Enright*

Additional Volumes in Preparation

To the biggest S's of my life:
my mother **SAKINA** and my wife **SALWA**
for their unique style, devotion,
and overwhelming care

MSM

Series Introduction

Many textbooks have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever-increasing complex problems faced by practicing engineers. However, few of these books fully address the applications aspects of control engineering. It is the intention of this series to redress this situation.

The series will stress applications issues, and not just the mathematics of control engineering. It will provide texts that present not only both new and well-established techniques, but also detailed examples of the application of these methods to the solution of real-world problems. The authors will be drawn from both the academic world and the relevant applications sectors.

There are already many exciting examples of the application of control techniques in the established fields of electrical, mechanical (including aerospace), and chemical engineering. We have only to look around in today's highly automated society to see the use of advanced robotics techniques in the manufacturing industries; the use of automated control and navigation systems in air and surface transport systems; the increasing use of intelligent control systems in the many artifacts available to the domestic consumer market; and the reliable supply of water, gas, and electrical power to the domestic consumer and to industry. However, there are currently many challenging problems that could benefit from wider exposure to the applicability of control methodologies, and the systematic systems-oriented basis inherent in the application of control techniques.

This series will present books that draw on expertise from both the academic world and the applications domains, and will be useful not only as academically recommended course texts but also as handbooks for practitioners in many applications domains.

Professor Mahmoud is to be congratulated for another outstanding contribution to the series.

Neil Munro

0.1 Preface

In many physical, industrial and engineering systems, **delays** occur due to the finite capabilities of information processing and data transmission among various parts of the system. Delays could arise as well from inherent physical phenomena like mass transport flow or recycling. Also, they could be by-products of computational delays or could intentionally be introduced for some design consideration. Such delays could be constant or time-varying, known or unknown, deterministic or stochastic depending on the system under consideration. In all of these cases, the time-delay factors have, by and large, counteracting effects on the system behavior and most of the time lead to poor performance. Therefore, the subject of **Time-Delay Systems (TDS)** has been investigated as functional differential equations over the past three decades. This has occupied a separate discipline in mathematical sciences falling between differential and difference equations. For example, the books by Hale [1], Kolmanovskii and Myshkis [2], Gorecki et al [3] and Hale and Lunel [4] provide modest coverage on the fundamental mathematical notions and concepts related to TDS; the book by Malek-Zavarei and Jamshidi [5] presents different topics of modeling and control related to TDS with constant delay and the book by Stepan [6] gives a good account of classical stability methods of TDS.

Due to the fact that almost all existing systems are subject to uncertainties, due to component aging, parameter variations or modeling errors, the concepts of **robustness**, **robust performance** and **robust design** have recently become common phrases in engineering literature and constitute integral part of control systems research. In turn, this has naturally brought into focus an important class of systems: **Uncertain Time-Delay Systems (UTDS)**. During the last decade, we have witnessed increasingly growing interest on the subject of UTDS and numerous results have appeared in conferences and/or published in technical journals. Apart from these scattered results and the volume edited very recently by Dugard and Verriest [7] however, there is no single book written exclusively on the analysis, design, filtering and control of uncertain time-delay systems. It is therefore believed that a book that aims at bridging this gap is certainly needed.

This book is about UTDS. It is directed towards providing a pool of methods and approaches that deal with uncertain time-delay systems. In so doing, it is intended to familiarize the reader with various aspects of the control and filtering of different uncertain time-delay systems. This will range from linear to some classes of nonlinear, from continuous-time to discrete-time and from time-invariant to time-varying systems. Throughout the book, I have endeavored to stress mathematical formality in a way to spring intuitive understanding and to explain how things work. I hope that this approach will attract the attention of a wide spectrum of readership.

The book consists of ten chapters and is organized as follows. Chapter 1 is an introduction to UTDS. It gives an overview of the related issues in addition to some systems examples. The remaining nine chapters are divided into two major parts. Part I deals with **robust control** and consists of Chapters 2 through 7. Part II treats **robust filtering** and is divided into Chapters 8 to 10. The book is supplemented by appendices containing some standard lemmas and mathematical results that are repeatedly used throughout the different chapters.

The material included makes it adequate for use as a text for one-year (two-semesters) courses at the graduate level in Engineering. The prerequisites are linear system theory, modern control theory and elementary matrix theory. As a textbook, it does not purport to be a compendium of all known results on the subject. Rather, it puts more emphasis on the recent robust results of control and filtering of time-delay systems.

Outstanding features of the book are:

- (1) It brings together the recent ideas and methodologies of dealing with uncertain time-delay systems.
- (2) It adopts a state-space approach in the system representation and analysis throughout.
- (3) It provides a unification of results on control design and filtering.
- (4) It presents the material systematically all the way from stability analysis, stabilization, control synthesis and filtering.
- (5) It includes the treatment of continuous-time and discrete-time systems side-by-side.

Magdi S. Mahmoud

Bibliography

- [1] Hale, J., "**Theory of Functional Differential Equations**," Springer-Verlag, New York, 1977.
- [2] Kolomanovskii, V. and A. Myshkis, "**Applied Theory of Functional Differential Equations**," Kluwer Academic Pub., New York, 1992.
- [3] Gorecki, H., S. Fuska, P. Garbowski and A. Korytowski, "**Analysis and Synthesis of Time-Delay Systems**," J. Wiley, New York, 1989.
- [4] Hale, J. and S. M. V. Lunel, "**Introduction to Functional Differential Equations**," vol. 99 , Applied Math. Sciences, Springer-Verlag, New-York, 1991.
- [5] Malek-Zavarei, M. and M. Jamshidi, "**Time-Delay Systems: Analysis Optimization and Applications**," North-Holland, Amsterdam, 1987.
- [6] Stepan, G., "**Retarded Dynamical Systems: Stability and Characteristic Functions**," Longman Scientific & Technical, Essex, 1989.
- [7] Dugard, L. and E. I. Verriest (Editors), "**Stability and Control of Time-Delay Systems**," Springer-Verlag, New York, 1997.

0.2 Acknowledgments

In writing this book on time-delay systems that aims at providing a unified view of a large number of results obtained over two decades or more, I faced the difficult problem of acknowledging the contributions of the individual researchers. After several unsuccessful attempts and barring the question of priority, I settled on the approach of referring to papers and/or books which I believed taught me a particular approach and then adding some notes at the end of each chapter to shed some light on the various papers. I apologize, in advance, in case I committed some injustices and assure the researchers that the mistake was unintentional. Although the book is an outgrowth of my academic activities for more than twelve years, most of the material has been compiled while I was on sabbatical leave from Kuwait University (KU), KUWAIT and working as a visiting professor at Nanyang Technological University (NTU), SINGAPORE. I am immensely pleased for such an opportunity which generated the proper environment for producing this volume. In particular, I am gratefully indebted to the excellent library services provided by KU and NTU.

Over the course of my career, I have enjoyed the opportunity of interacting with several colleagues who have stimulated my thinking and research in the systems engineering field. In some cases, their technical contributions are presented explicitly in this volume; in other cases, their influence has been more subtle. Among these colleagues are Professors A. A. Kamal and A. Y. Bilal (Cairo University), Professor M. I. Younis (National Technology Program, EGYPT), Professors W. G. Vogt and M. H. Mickle (University of Pittsburgh), Professor M. G. Singh (UK), Professor M. Jamshidi (University of New Mexico), Professor A. P. Sage (George Mason University), Professor H. K. Khalil (Michigan State University), Dr. M. Zribi, Dr. L. Xie, Dr. A. R. Leyman and Dr. A. Yacin (NTU) and Dr. S. Kotob (Kuwait Institute for Scientific Research, KUWAIT). I have also enjoyed the encouragement and patience of my family (Salwa, Medhat, Monda and Mohamed) who were very supportive, as time working on this book was generally time spent away from them. Finally, I owe a measure of gratitude to Cairo University (EGYPT)

and Kuwait University (KUWAIT) for providing the intellectual environment that encourages me to excel further in the area of systems engineering.

Magdi S. Mahmoud

Contents

0.1	Preface	v
0.2	Acknowledgments	ix
1	Introduction	1
1.1	Notations and Definitions	3
1.1.1	Notations	3
1.1.2	Definitions	5
1.2	Time-Delay Systems	8
1.3	Uncertain Time-Delay Systems	9
1.4	System Examples	11
1.4.1	Stream Water Quality	11
1.4.2	Vehicle Following Systems	12
1.4.3	Continuous Stirred Tank Reactors with Recycling . .	13
1.4.4	Power Systems	14
1.4.5	Some Biological Models	15
1.5	Discrete-Time Delay Systems	16
1.5.1	Example 1.1	17
1.5.2	Example 1.2	18
1.6	Outline of the Book	18
1.7	Notes and References	20
I	ROBUST CONTROL	
2	Robust Stability	27
2.1	Stability Results of Time-Delay Systems	27
2.1.1	Stability Conditions of Continuous-Time Systems . .	28
2.1.2	Example 2.1	31
2.1.3	Example 2.2	34

2.1.4	Stability Conditions of Discrete-Time Systems	35
2.1.5	Example 2.3	37
2.1.6	Example 2.4	41
2.2	Robust Stability of UTDS	42
2.2.1	Stability Conditions for Continuous-Time Systems	42
2.2.2	Example 2.5	44
2.2.3	Example 2.6	49
2.2.4	Example 2.7	49
2.2.5	Stability Conditions for Discrete-Time Systems	49
2.2.6	Example 2.8	53
2.3	Stability Tests Using \mathcal{H}_∞ -norm	53
2.4	Stability of Time-Lag Systems	54
2.4.1	Example 2.9	58
2.5	Stability of Linear Neutral Systems	61
2.6	Stability of Multiple-Delay Systems	65
2.7	Stability Using Lyapunov-Razumikhin Theorem	68
2.7.1	Example 2.10	70
2.8	Stability Using Comparison Principle	71
2.9	Notes and References	74
3	Robust Stabilization	75
3.1	Introduction	75
3.2	Time-Delay Systems	76
3.2.1	Problem Description	76
3.2.2	State Feedback Synthesis	77
3.2.3	Two-Term Feedback Synthesis	85
3.2.4	Static Output Feedback Synthesis	89
3.2.5	Dynamic Output Feedback Synthesis	91
3.3	Simulation Examples	95
3.3.1	Example 3.1	95
3.3.2	Example 3.2	96
3.3.3	Example 3.3	97
3.3.4	Example 3.4	97
3.3.5	Example 3.5	98
3.3.6	Example 3.6	98
3.3.7	Example 3.7	99
3.3.8	Example 3.8	99
3.4	Uncertain Time-Delay Systems	99
3.4.1	Problem Statement and Definitions	100

3.4.2	Closed-Loop System Stability	101
3.5	Nominal Control Synthesis	103
3.5.1	Example 3.9	105
3.6	Uncertainty Structures	106
3.6.1	Control Synthesis for Matched Uncertainties	107
3.6.2	Example 3.10	109
3.6.3	Control Synthesis for Mismatched Uncertainties	111
3.6.4	Example 3.11	114
3.6.5	Control Synthesis for Norm-Bounded Uncertainties	115
3.6.6	Example 3.12	118
3.7	Notes and References	119
4	Robust \mathcal{H}_∞ Control	121
4.1	Linear Uncertain Systems	122
4.1.1	Problem Statement and Preliminaries	122
4.1.2	Robust \mathcal{H}_∞ Control	124
4.2	Nonlinear Systems	126
4.2.1	Problem Statement and Preliminaries	127
4.2.2	Robust \mathcal{H}_∞ -Performance Results	135
4.3	Discrete-Time Systems	137
4.3.1	Problem Description and Preliminaries	138
4.3.2	Robust \mathcal{H}_∞ Control	141
4.4	Multiple-Delay Systems	143
4.4.1	Problem Description	144
4.4.2	State Feedback \mathcal{H}_∞ -Control	145
4.4.3	Example 4.1	152
4.4.4	Problem Description with Uncertainties	152
4.4.5	Example 4.2	157
4.5	Linear Neutral Systems	158
4.5.1	Robust Stabilization	159
4.5.2	Robust \mathcal{H}_∞ Performance	161
4.6	Notes and References	165
5	Guaranteed Cost Control	167
5.1	Continuous-Time Systems	167
5.1.1	Uncertain State-Delay Systems	167
5.1.2	Robust Performance Analysis I	168
5.1.3	Robust Performance Analysis II	173
5.1.4	Synthesis of Guaranteed Cost Control I	176

5.1.5	Synthesis of Guaranteed Cost Control II	180
5.2	Discrete-Time Systems	184
5.2.1	Problem Formulation	184
5.2.2	Robust Performance Analysis III	185
5.2.3	Synthesis of Guaranteed Cost Control III	190
5.3	Observer-Based Control	193
5.3.1	Problem Description	194
5.3.2	Closed-Loop System	195
5.3.3	Robust Performance Analysis IV	196
5.3.4	Synthesis of Observer-Based Control	200
5.3.5	A Computational Algorithm	205
5.3.6	Example 5.1	206
5.4	Notes and References	207
6	Passivity Analysis and Synthesis	209
6.1	Introduction	209
6.2	Continuous-Time Systems	210
6.2.1	A Class of Uncertain Systems	210
6.2.2	Conditions of Passivity: Delay-Independent Stability	211
6.2.3	Conditions of Passivity: Delay-Dependent Stability	213
6.2.4	μ -Parameterization	216
6.2.5	Observer-Based Control Synthesis	220
6.3	Discrete-Time Systems	223
6.3.1	A Class of Discrete-Delay Systems	223
6.3.2	Conditions of Passivity: Delay-Independent Stability	225
6.3.3	Conditions of Passivity: Delay-Dependent Stability	227
6.3.4	Parameterization	236
6.3.5	State-Feedback Control Synthesis	240
6.3.6	Output-Feedback Control Synthesis	242
6.4	Notes and References	245
7	Interconnected Systems	247
7.1	Introduction	247
7.2	Problem Statement and Definitions	248
7.2.1	Uncertainty Structures	248
7.3	Decentralized Robust Stabilization I	250
7.4	Decentralized Robust H_∞ Performance	257
7.4.1	Example 7.1	263
7.5	Decentralized Robust Stabilization II	265

7.5.1 Problem Statement and Preliminaries 265

7.6 Decentralized Stabilizing Controller 268

7.6.1 Adjustment Procedure 273

7.6.2 Example 7.2 273

7.7 Notes and References 276

II ROBUST FILTERING

8 Robust Kalman Filtering 303

8.1 Introduction 303

8.2 Continuous-Time Systems 304

8.2.1 System Description 304

8.2.2 Robust Filter Design 305

8.2.3 A Riccati Equation Approach 307

8.2.4 Steady-State Filter 311

8.2.5 Example 8.1 314

8.3 Discrete-Time Systems 315

8.3.1 Uncertain Discrete-Delay Systems 315

8.3.2 Robust Filter Design 316

8.3.3 A Riccati Equation Approach 319

8.3.4 Steady-State Filter 323

8.3.5 Example 8.2 326

8.4 Notes and References 326

9 Robust \mathcal{H}_∞ Filtering 331

9.1 Introduction 331

9.2 Linear Uncertain Systems 332

9.2.1 Problem Description and Preliminaries 332

9.2.2 Robust \mathcal{H}_∞ Filtering 334

9.2.3 Worst-Case Filter Design 339

9.3 Nonlinear Uncertain Systems 342

9.3.1 Problem Description and Assumptions 342

9.3.2 Robust \mathcal{H}_∞ Filtering Results 348

9.4 Linear Discrete-Time Systems 354

9.4.1 Problem Description 354

9.4.2 \mathcal{H}_∞ -Estimation Results 357

9.5 Linear Parameter-Varying Systems 365

9.5.1 Discrete-Time Models 366