

**ASM Handbook of  
Engineering Mathematics**

# **ASM Handbook of Engineering Mathematics**

**By faculty members of  
The Department of Mechanical Engineering,  
The University of Akron**

Mamerto L. Chu, Ph.D.  
Paul Lam, Ph.D.  
Richard J. Gross, Ph.D.  
Benjamin T. F. Chung, Ph.D.

**and by**

Samuel J. Brown, Ph.D.  
President, Quest Engineering &  
Development Corp.

**Editorial Coordinator**

William G. Belding



**American Society for Metals  
Metals Park, Ohio**

Copyright © 1983  
by the  
**AMERICAN SOCIETY FOR METALS**  
All rights reserved

No part of this book may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher.

Nothing contained in this book is to be construed as a grant of any right of manufacture, sale, or use in connection with any method, process, apparatus, product or composition, whether or not covered by letters patent or registered trademark, nor as a defense against liability for the infringement of letters patent or registered trademark.

Library of Congress Cataloging in Publication Data  
Main entry under title:

ASM handbook of engineering mathematics.

*Includes index.*

1. Engineering mathematics—Handbooks, manuals, etc.  
I. Drummond, Jerry. II. Brown, Samuel J. III. Belding,  
William G. IV. University of Akron. Dept. of Mechanical  
Engineering. V. American Society for Metals. VI. Title.  
A. S. M. handbook of engineering mathematics.  
TA332.A79 1983 510'.2462 82-22664  
ISBN 0-87170-157-X

SAN 204-7586

PRINTED IN THE UNITED STATES OF AMERICA

## Preface

This handbook has been compiled to serve as a practical reference for practicing engineers and engineering students who possess basic groundwork in college-level mathematics. In each area of basic mathematics, key equations are presented, without detailed derivations.

A major part of the handbook has been developed by a team of faculty members of the Department of Mechanical Engineering, The University of Akron. The faculty members participating in this program have significant industrial experience, which they bring to the classroom. Thus, this handbook becomes an extension of that enriching quality of experience.

The authors and editors do not claim completeness. The vastness of the field precludes it. But the equations presented here should prove useful to graduate engineers. To serve practical needs, the authors have emphasized practice and subordinated theory.

Because mathematics is an extremely broad field that includes a large and growing body of literature on computers, for example, choices had to be made. The choices represented here have been derived from course work in mechanical engineering, instead of the mathematics of physics or electrical and electronic engineering. Thus, the branches of mathematics emphasized are those useful in the design and manufacturing environment of the typical metalworking company.

Part I of this handbook contains basic equations and theorems of algebra, trigonometry, geometry, analytical geometry, calculus, etc., in ascending order of difficulty, as these subjects are frequently introduced to students.

In Part II, mathematical equations and illustrations present key elements of various disciplines of mechanical engineering. The focus is on those equations that help lead to solutions of practical problems in mechanical analysis and design.

To further aid students and practicing engineers, lists of selected references are presented at the end of each chapter in Part II. It is the sincere hope of the authors and editors that if the precise equation being sought by the reader isn't found in this handbook, our efforts will nevertheless help lead to the answer sought.

# Contents

## Part 1 Basic Mathematics

1 Algebra .....	1
2 Geometry .....	37
3 Mensuration .....	57
4 Trigonometry .....	75
5 Plane Analytic Geometry .....	93
6 Solid Analytic Geometry .....	113
7 Differential Calculus .....	123
8 Integral Calculus .....	145
9 Table of Integrals .....	167
10 Differential Equations .....	203
11 Laplace Transformation .....	219
12 Vector Analysis .....	237
13 Fourier Series and Transforms .....	247
14 Statistics and Probability .....	255

## Part 2 Engineering Mathematics

15 Statics .....	271
16 Vibration Analysis (P. Lam) .....	313
17 Kinematics (M. Chu) .....	333
18 Kinetics (M. Chu) .....	355
19 Stress-Strain (Constitutive) Relationships (S. Brown) .....	391
20 Thermodynamics (R. Gross) .....	441
21 Heat Transfer (B. Chung) .....	477
22 Fluid Dynamics .....	585
Index .....	651

# Contents

## Part 1 Basic Mathematics

<b>1 Algebra</b> .....	<b>1</b>
Fundamental Concepts .....	1
Definitions .....	1
Classification of Numbers .....	2
Notations .....	3
Basic Laws .....	3
Identities .....	4
Powers .....	4
Roots .....	5
Products .....	5
Quotients .....	5
Fractions .....	5
Series .....	6
Common Algebraic Operations .....	6
Partial Fractions .....	8
Ratio and Proportion .....	10
Theorems .....	10
Functions .....	12
Progressions .....	13
Permutations and Combinations .....	13
Probability .....	14
Determinants .....	15
Algebraic Equations .....	16
Quadratic Equations .....	18
Cubic Equations .....	19
Quartic Equations .....	21
General Equations of the $n$ th Degree .....	22
Transcendental Equations .....	23
Horner's Method of Solving Equations .....	25
Newton's Method of Solving Equations .....	26
Steinman's Method of Solving Equations .....	27
Simultaneous Equations .....	28
Inequalities .....	32
Logarithms .....	32
Interest, Annuities and Sinking Funds .....	34

<b>2 Geometry</b> .....	<b>37</b>
Concepts .....	37
Measurement of Plane Angles .....	37
Polygons .....	38
Properties of Triangles .....	39
Circles .....	40
Nonplanar Angles .....	42
Polyhedrons .....	44
Solids with Curved Surfaces .....	45
Constructions .....	47
Lines .....	47
Angles .....	48
Circles .....	48
Polygons .....	52
Ellipse .....	53
Parabola .....	54
Hyperbola .....	55
Cycloid .....	56
Involute of a Circle .....	56
<b>3 Mensuration</b> .....	<b>57</b>
Ratio and Proportion .....	57
Mensuration Formulas .....	59
Plane Rectilinear Figures .....	60
Plane Curvilinear Figures .....	62
Solids with Plane Surfaces .....	67
Solids with Curved Surfaces .....	69
<b>4 Trigonometry</b> .....	<b>75</b>
Plane Trigonometry .....	75
Fundamental Formulas of Trigonometry .....	78
Harmonic Quantities .....	81
Solution of Triangles .....	82
Inverse Trigonometric Functions .....	84
Spherical Trigonometry .....	86
Hyperbolic Trigonometry .....	89
<b>5 Plane Analytic Geometry</b> .....	<b>93</b>
Coordinates .....	93
Straight Lines .....	94
Conic Sections .....	98
Defining Plane Curves Using Differential Calculus .....	105
Reference Curves .....	107
<b>6 Solid Analytic Geometry</b> .....	<b>113</b>
Coordinates .....	113
Transformation of Coordinates .....	114
Points, Lines and Planes .....	115
Surfaces of the Second Degree (Quadric Surfaces) .....	118

<b>7 Differential Calculus</b> .....	<b>123</b>
Basic Concepts .....	123
Indeterminate Forms .....	127
Differentiation Formulas .....	128
Partial Derivatives .....	130
Infinite Series .....	132
Expansion of a Function into a Power Series.....	133
Finding Maxima and Minima.....	139
Principal Theorems of Differential Calculus.....	141
<b>8 Integral Calculus</b> .....	<b>145</b>
Indefinite Integrals .....	145
Definite Integrals.....	151
Fundamental Properties.....	152
Evaluation of Definite Integrals .....	153
Line, Surface, and Volume Integrals .....	156
Applications of Integration .....	160
Applications of Double Integrals.....	165
Application of Triple Integrals.....	166
<b>9 Table of Integrals</b> .....	<b>167</b>
Indefinite Integrals .....	167
Integrals with $(ax + b)$ .....	167
Integrals with $ax^2 + c$ , $ax^n + c$ , $x^2 \pm p^2$ , and $p^2 - x^2$ .....	171
Integrals with $(ax^2 + bx + c)$ .....	174
Integrals with Miscellaneous Algebraic Expressions.....	177
Integrals with the Sine Function where $(a \neq 0)$ .....	177
Integrals with the Cosine Function where $(a \neq 0)$ .....	180
Integrals with Sine and Cosine where $(a \neq 0)$ .....	182
Integrals with Tangent and Cotangent Functions where $(a \neq 0)$ .....	187
Integrals with Hyperbolic Functions where $(a \neq 0)$ .....	188
Integrals with Exponential Functions .....	190
Integrals with Logarithmic Functions where $(a \neq 0)$ .....	192
Integrals with Inverse Trigonometric and Hyperbolic Functions where $(a > 0)$ .....	193
Definite Integrals.....	196
Integrals with Algebraic Functions .....	196
Integrals with Trigonometric Functions .....	197
Integrals with Exponential and Hyperbolic Functions where $(a > 0)$ .....	199
Integrals with Logarithmic Functions .....	201
<b>10 Differential Equations</b> .....	<b>203</b>
Ordinary Differential Equations .....	203
First-Order Equations.....	204
Second-Order Equations.....	207
Pfaffian Differential Equations .....	210



Systems of Linear Differential Equations with Constant Coefficients .....	210
Simultaneous Total Differential Equations .....	211
Partial Differential Equations .....	211
Partial Differential Equations of the First Order .....	212
Partial Differential Equations of the Second Order .....	213
Linear Partial Differential Equations .....	216
Separation of Variables .....	216
<b>11 Laplace Transformation .....</b>	<b>219</b>
Fundamental Transformation Rules .....	219
Theorems .....	221
Applications of Laplace Transforms .....	224
Table of Laplace Transforms .....	228
<b>12 Vector Analysis .....</b>	<b>237</b>
Vector Addition .....	238
Dot or Scalar Product .....	238
Cross or Vector Product .....	239
Derivatives of Vectors .....	241
Gradient .....	242
Integration of Vectors .....	244
Divergence .....	244
Gauss' Theorem .....	244
Green's Theorem .....	245
The Curl of a Vector Field .....	246
Stokes' Theorem .....	246
<b>13 Fourier Series and Transforms .....</b>	<b>247</b>
Basic Case .....	247
<b>14 Statistics and Probability .....</b>	<b>255</b>
Statistical Distributions .....	255
Frequency .....	255
Mean of a Distribution .....	255
Second Moments .....	256
Standard Deviation .....	257
Simple Probability .....	257
Probability Theorems .....	258
Discrete Random Variable .....	259
Permutations and Combinations .....	260
Continuous Random Variable .....	261
Expectation and Moments .....	262
Chebyshev's Inequality .....	263
Binomial Distribution .....	263
Standard Deviation of the Binomial Distribution .....	264
Poisson Distribution .....	265
Normal Distribution .....	265
Standard Deviation of the Mean .....	266
Approximation with Least Squares Method .....	267
Matrix Formulation .....	270

## Part 2 Engineering Mathematics

<b>15 Statics</b> .....	<b>271</b>
Forces and Equilibrium .....	271
Newton's Laws—Foundation of Rigid Body Mechanics .....	271
Resultants of Concurrent Forces .....	272
Resultants in Two Dimensions .....	273
Resultants in Three Dimensions .....	275
Moments and Couples .....	276
Two-Dimensional Moments and Couples .....	276
Three-Dimensional Moments and Couples .....	277
Addition of Moments .....	279
Resultants of Force/Moment Systems .....	279
Replacement of a Force by an Equivalent Force/Couple Combination .....	280
Equilibrium .....	282
General Conditions for Equilibrium .....	282
The Free-Body Diagram .....	283
Adequacy of Constraint .....	284
Structures .....	288
Trusses .....	288
Frames/Machines .....	290
Distributed Force Systems .....	292
Centroids and Centers of Mass .....	292
Beam Loading .....	299
Flexible Cables .....	303
Special (Limiting) Cases of Distributed Loading .....	304
Dry Friction (Coulomb Friction) .....	307
Four Situations Related to the Friction Force .....	308
Angle of Friction .....	309
Wedges .....	309
Screws .....	310
Flexible Belts .....	311
<b>16 Vibration Analysis</b> .....	<b>313</b>
Terminology .....	313
Undamped Free Vibration .....	314
Equivalent Springs and Masses .....	317
Damped Free Vibration (SDOF) .....	317
Forced Harmonic Vibration .....	325
Rotating Unbalanced Forces .....	326
Whirling of Rotating Shafts .....	328
Forced Vibration From Harmonic Ground Motion .....	330
Vibration Transmissibility Isolation .....	331
<b>17 Kinematics</b> .....	<b>333</b>
Kinematics of a Particle .....	333
Curvilinear Motion—Tangential and Normal Components .....	341
Angular Motion .....	342
Relative Motion in a Plane .....	344
Kinematics of a Rigid Body .....	349

<b>18 Kinetics</b> .....	<b>355</b>
Systems of Units .....	355
Kinetics of Particles .....	355
Work, Power, and Energy .....	360
Energy and Momentum Methods .....	361
Kinetics of Rigid Bodies .....	374
Spatial Kinetics of a Rigid Body .....	386
<b>19 Stress-Strain (Constitutive) Relationships</b> .....	<b>391</b>
Elastic Solids .....	391
Constitutive Laws for Anisotropic Elastic Solids .....	393
Isotropic Media .....	401
Newtonian Fluids .....	411
Coordinate Systems .....	416
Plasticity .....	418
Viscoelastic Solids .....	428
<b>20 Thermodynamics</b> .....	<b>441</b>
First Law of Thermodynamics: Conservation of Energy .....	443
Work Equations .....	447
Heat Transfer Equations .....	449
Property Relations .....	452
Second Law of Thermodynamics .....	461
Mixtures and Solutions .....	469
<b>21 Heat Transfer</b> .....	<b>477</b>
Heat Conduction .....	478
Fourier Law, Heat Conduction Equations, and Boundary Conditions .....	479
Thermal Conductivity of Substances .....	480
One-Dimensional Steady-State Heat Conduction .....	480
One-Dimensional Composite Structure at Steady State .....	483
Critical Radius for Insulated Pipes and Spheres .....	488
Steady-State Heat Transfer from Extended Surfaces .....	489
Multidimensional Steady-State Heat Conduction .....	491
Unsteady-State Heat Conduction .....	506
Convection Heat Transfer .....	512
Analogy Between Heat Transfer and Momentum Transfer .....	526
Forced Convection of External Flow .....	528
Forced Convection of Internal Flow .....	534
Free Convection .....	534
Mixed Convection and Convection from Rotating Bodies .....	538
Thermal Radiation .....	546
Blackbody and the Basic Laws of Thermal Radiation .....	551
Radiation Shape Factor .....	557
Radiation View Factors From Elementary Area $dA_1$ Impinging on Another Differential Area $dA_2$ .....	558
Radiation View Factors From Differential Area $dA_1$ to Finite Area $A_2$ .....	559

Radiation View Factor From Finite Area $A_1$ to Finite Area $A_2$ .....	566
Radiation Exchange in Black Enclosure.....	575
Radiation Exchange in Gray Enclosure.....	575
Radiation Shields.....	575

**22 Fluid Dynamics..... 585**

Properties of Fluids.....	585
The Continuum.....	585
Pressure.....	585
Density.....	586
Coefficient of Compressibility.....	586
Viscosity.....	586
Surface Tension.....	587
Vapor Pressure.....	587
Fluid Statics.....	588
Pressure.....	588
Variation of Pressure with Elevation.....	589
Manometers.....	590
Forces on Submerged Surfaces.....	591
Fluid Motion.....	592
Kinematic Relationships—Velocity and Acceleration.....	592
Rotation of Fluid and Vorticity.....	594
Flow Rate Through a Surface.....	594
Streamlines.....	595
Variation of Pressure due to Rigid Body Motion.....	596
Concept of the Control Volume.....	597
Eulerian and Lagrangian Views.....	597
Control Volume Transport Equation.....	597
Laws to Which the Control Volume Concept Is Applied.....	599
Continuity Equation.....	599
Integral Formulation.....	599
Differential Formulation.....	600
Momentum Equation.....	600
Integral Form—Linear Momentum.....	600
Integral Form—Angular Momentum (Moment of Momentum).....	601
Differential Form—Linear Momentum.....	602
Energy Equation.....	604
Integral Form.....	604
Differential Formulation.....	606
Boundary Conditions for the Differential Equations.....	607
Dimensional Analysis.....	609
The Pi Theorem.....	609
Power Product Method.....	609
Nondimensionalization of the Basic Equations.....	613
Flow in Pipes and Ducts.....	614
Flow in a Circular Duct.....	614
General Analysis.....	614
Equations of Motion.....	615
Laminar Flow.....	615
Turbulent Flow.....	616

Non-Circular Ducts .....	621
Flow Between Parallel Plates .....	622
Concentric Circular Annulus .....	623
Additional Cross Sections .....	624
Losses in Pipe Systems Other than Friction .....	624
Piping Systems .....	625
Boundary Layer Flow .....	627
Laminar and Turbulent Boundary Layers .....	627
Equations of Motion—Laminar Boundary Layer .....	628
Laminar Flow Solutions .....	629
Turbulent Boundary Layer .....	631
Inviscid or Ideal Flow .....	633
Results of Inviscid Flow Analysis .....	636
Superposition of Potential Flows .....	638
Drag and Lift on Bodies in a Fluid .....	641
One-Dimensional Compressible Flow .....	643
Total or Stagnation Properties .....	644
Normal Shock Waves .....	645
Isentropic Flow Through a Duct of Varying Area .....	646
Compressible Pipe Flow .....	648

# 1 Algebra

## FUNDAMENTAL CONCEPTS

An algebraic expression includes one or more algebraic quantities (numbers or letters) connected by such signs of operations as  $+$ ,  $-$ ,  $:$ , and  $\sqrt{\quad}$ , with brackets indicating successive operations.

An equality of two algebraic expressions is an identity when the equality holds for all substitutions of arbitrary numerical values for the letters occurring in the expression. An equation is an equality that is valid only for certain definite values.

An identity transformation, the process of obtaining one algebraic expression from another equal expression, can be done in many ways, according to the aim of the transformation. An expression can be given a more compact form suitable for substitution of numerical values or a form suitable for such operations as solving equations, logarithmic calculation, differentiation, or integration.

## DEFINITIONS

**Monomial.** A monomial is a single term:  $a$ ,  $ab$ , or  $x^2y^3$ .

**Binomial.** A binomial defines two terms that are added or subtracted:  $a + b$ ,  $x^2 + y^3$ , or  $ab + x^4y^2$ .

**Polynomial.** A polynomial includes two or more terms added or subtracted:  $a + b + c$  or  $x^2 + 3x - 5xy + y^2$ .

**Rational Term.** A rational term is one not containing the root of a variable:  $x^2$ ,  $1/y$  or  $u$ .

**Irrational Term.** An irrational term contains a root of a variable:  $a^{1/5}$  or  $\sqrt{x}$ .

## 2/BASIC MATHEMATICS

**Integral Term.** An integral term is one in which the variable does not occur in the denominator. For example,  $x/y$  is integral with respect to  $x$  but not with respect to  $y$ .

**Degree of a Term.** The degree of a term means the number of literal factors in a term, and it also equals the sum of the exponents of the literal factors. The degree of  $4a^2b^3$  is  $2 + 3 = 5$ , for example.

**Variable.** A variable is a symbol that represents any value of a given set of elements. For example, in  $A = \pi r^2$ , where  $A$  is the area of a circle with radius  $r$ , and  $\pi = 3.14159$ ,  $A$  and  $r$  are variables. When  $A$  changes as  $r$  is varied, then  $r$  is the independent variable and  $A$  the dependent variable.

**Constant.** A constant is a variable with one element only. An absolute or numerical constant always has the same value. An arbitrary constant, or parameter, has one value under certain conditions and different values under other conditions. Symbols representing numbers 11 or  $\sqrt{2}$  or 3.14159 are absolute constants. In the expression  $e = kP$ , where  $e$  is the elongation of a bar due to a load  $P$ ,  $k$  is a constant that varies with different materials; hence,  $k$  is a parameter.

## CLASSIFICATION OF NUMBERS

**Real Numbers** (positive and negative). Real numbers include rational and irrational numbers. A rational number is expressible as the quotient of two integers, that is, integers such as  $-1$ ,  $2$ ,  $53$ , or fractions, as  $\frac{3}{4}$ ,  $-\frac{5}{2}$ . An irrational number is not expressible as the quotient of two integers, as  $\sqrt{2}$ ,  $\pi$ . The absolute value of a real number is the number itself if the number is positive, and the number with its sign changed if it is negative, as, for example,  $|3| = |-3| = 3$ .

**Imaginary Numbers.** An imaginary number is a product of a real number and the imaginary unit  $i (= \sqrt{-1})$ . Electrical engineers use  $j$  to avoid confusion with  $i$  for current. Example:  $\sqrt{-2} = \sqrt{2}i$ .

**Complex Numbers.** A complex number is a sum of a real number and an imaginary number, as  $a + bi$ , with  $a$  and  $b$  real numbers,  $-3 + 0.5i$ . A real number may be regarded as a complex number in which  $b = 0$ , and an imaginary number as one in which  $a = 0$ . The absolute value of a complex number  $a + bi$  is  $\sqrt{a^2 + b^2}$ , as, for example,  $|-3 + 0.5i| = \sqrt{9 + \frac{1}{4}} = 3.04+$ . Relationships of complex numbers can be expressed in the following forms:

$$i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i, \text{ etc.}$$

$$a + bi = c + di \text{ if and only if } a = c, b = d$$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$$

## NOTATIONS

The main points of separation in a simple algebraic expression are the + and - signs. Thus,  $a + b \times c - d \div x + y$  is interpreted as  $a + (b \times c) - (d \div x) + y$ . The range of operation of the symbols  $\times$  and  $\div$  extends only to the next + or - sign. Between the signs  $\times$  and  $\div$  themselves,  $a \div b \times c$  means  $a \div (b \times c)$ . The  $\div$  sign is the stronger separative. Because this rule is not strictly followed, parentheses should be used to avoid ambiguity.

Exponents and radical signs influence only the next adjacent quantity. Thus  $2ax^3$  means  $2a(x^3)$ , and  $\sqrt{2}ax$  means  $(\sqrt{2})(ax)$ . Instead of  $\sqrt{2}ax$ , it is safer, however, to write  $\sqrt{2} \cdot ax$ , or  $ax\sqrt{2}$ . Any expression within parentheses is to be treated as a single quantity. A horizontal bar serves the same purpose as parentheses.

The notation  $a \cdot b$ , or simply  $ab$ , means  $a \times b$ ; and  $a : b$ , or  $a/b$ , means  $a \div b$ .

**Factorials.** The symbol  $n!$  (when  $n$  is a whole number) means: "n factorial," and means the product of the natural numbers from 1 to  $n$ , inclusive. Thus  $2! = 1 \times 2$ ;  $3! = 1 \times 2 \times 3$  and  $4! = 1 \times 2 \times 3 \times 4$ . The Stirling formula gives approximate values of  $n!$  for large  $n$ :

$$n! \approx n^n e^{-n} \sqrt{2\pi n}$$

## BASIC LAWS

**Existence Law for Addition.** Adding any two numbers  $a$  and  $b$  always gives a single number  $c$ :  $a + b = c$ .

**Commutative Law.** Algebraic numbers can be added or multiplied regardless of order:  $a + b = b + a$ ;  $ab = ba$ .

**Associative Law.** The sum or product of three or more algebraic terms is unaffected by the grouping of the terms:

$$\begin{aligned} a + b + c &= a + (b + c) = (a + b) + c \\ abc &= a(bc) = (ab)c = (ac)b \end{aligned}$$

**Distributive Law.**  $a(b + c) = ab + ac$ .

**Operations with Zero and Negative Numbers.** A number or letter without a preceding sign is assumed to be positive.



## 4/BASIC MATHEMATICS

$$\begin{array}{ll} a + 0 = a & 0 - a = -a \\ a + (-a) = 0 & -(-a) = a \\ a \cdot 0 = 0 & a(-b) = -ab \\ 0/a = 0, \text{ if } a \neq 0 & (-a)(-b) = ab \end{array}$$

If  $ab = 0$ , then either  $a = 0$  or  $b = 0$  or  $a = b = 0$ .  $a/0$  is undefined.

**Order Relationships.** If  $a$  and  $b$  are real numbers, then either  $a < b$  or  $a = b$  or  $a > b$ . If  $c$  is a third real number, and if  $a < b$  and  $b < c$ , then  $a < c$ .

**Axioms.** The following relationships apply in algebraic transformations:

- If equals are added to equals, the sums are equal
- If equals are subtracted from equals, the differences are equal
- If equals are multiplied by equals, the products are equal
- If equals are divided by equals (except zero), the quotients are equal
- Like powers or like roots of equals are equal
- Numbers or terms equal to the same number or equal numbers are equal
- The whole equals the sum of its parts

## IDENTITIES

An identity is a statement in symbolic form that holds for all values of the variables involved. For example,  $(a + b)(a - b) = a^2 - b^2$  is true regardless of the values substituted for  $a$  and  $b$ . Common identities are listed below.

## POWERS

$$\begin{array}{l} (-a)^n = a^n, \text{ if } n \text{ is even} \\ (-a)^n = -a^n, \text{ if } n \text{ is odd} \\ a^m \cdot a^n = a^{m+n} \end{array}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = \left(\frac{b}{a}\right)^{-n} = \frac{b^{-n}}{a^{-n}}$$

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1; 0^n = 0; 0^0 \text{ is meaningless}$$