ASM Handbook of Engineering Mathematics

ASM Handbook of Engineering Mathematics

By faculty members of
The Department of Mechanical Engineering,
The University of Akron

Mamerto L. Chu, Ph.D.
Paul Lam, Ph.D.
Richard J. Gross, Ph.D.
Benjamin T. F. Chung, Ph.D.

and by

Samuel J. Brown, Ph.D.
President, Quest Engineering & Development Corp.

Editorial Coordinator

William G. Belding

American Society for Metals
Metals Park, Ohio

Library of Congress Cataloging in Publication Data
Main entry under title:

ASM handbook of engineering mathematics.

Includes index.
1. Engineering mathematics—Handbooks, manuals, etc.
I. Drummond, Jerry. II. Brown, Samuel J. III. Belding, William G. IV. University of Akron. Dept. of Mechanical Engineering. V. American Society for Metals. VI. Title: A.S.M. handbook of engineering mathematics.
TA332.A79 1983 510'.2462 82-22664
ISBN 0-87170-157-X

SAN 204-7586

PRINTED IN THE UNITED STATES OF AMERICA

Preface

This handbook has been compiled to serve as a practical reference for practicing engineers and engineering students who possess basic groundwork in college-level mathematics. In each area of basic mathematics, key equations are presented, without detailed derivations.

A major part of the handbook has been developed by a team of faculty members of the Department of Mechanical Engineering, The University of Akron. The faculty members participating in this program have significant industrial experience, which they bring to the classroom. Thus, this handbook becomes an extension of that enriching quality of experience.

The authors and editors do not claim completeness. The vastness of the field precludes it. But the equations presented here should prove useful to graduate engineers. To serve practical needs, the authors have emphasized practice and subordinated theory.

Because mathematics is an extremely broad field that includes a large and growing body of literature on computers, for example, choices had to be made. The choices represented here have been derived from course work in mechanical engineering, instead of the mathematics of physics or electrical and electronic engineering. Thus, the branches of mathematics emphasized are those useful in the design and manufacturing environment of the typical metalworking company.

Part I of this handbook contains basic equations and theorems of algebra, trigonometry, geometry, analytical geometry, calculus, etc., in ascending order of difficulty, as these subjects are frequently introduced to students.

In Part II, mathematical equations and illustrations present key elements of various disciplines of mechanical engineering. The focus is on those equations that help lead to solutions of practical problems in mechanical analysis and design.

To further aid students and practicing engineers, lists of selected references are presented at the end of each chapter in Part II. It is the sincere hope of the authors and editors that if the precise equation being sought by the reader isn't found in this handbook, our efforts will nevertheless help lead to the answer sought.

Contents

Contents

Part 2 Engineering Mathematics

1
Algebra

FUNDAMENTAL CONCEPTS

An algebraic expression includes one or more algebraic quantities (numbers or letters) connected by such signs of operations as +, −, :, and $\sqrt{\ }$, with brackets indicating successive operations.

An equality of two algebraic expressions is an identity when the equality holds for all substitutions of arbitrary numerical values for the letters occurring in the expression. An equation is an equality that is valid only for certain definite values.

An identity transformation, the process of obtaining one algebraic expression from another equal expression, can be done in many ways, according to the aim of the transformation. An expression can be given a more compact form suitable for substitution of numerical values or a form suitable for such operations as solving equations, logarithmic calculation, differentiation, or integration.

DEFINITIONS

Monomial. A monomial is a single term: a, ab, or x^2y^3.

Binomial. A binomial defines two terms that are added or subtracted: $a + b$, $x^2 + y^3$, or $ab + x^4y^2$.

Polynomial. A polynomial includes two or more terms added or subtracted: $a + b + c$ or $x^2 + 3x - 5xy + y^2$.

Rational Term. A rational term is one not containing the root of a variable: x^2, $1/y$ or u.

Irrational Term. An irrational term contains a root of a variable: $a^{1/5}$ or $\sqrt{x}$.

Integral Term. An integral term is one in which the variable does not occur in the denominator. For example, x/y is integral with respect to x but not with respect to y.

Degree of a Term. The degree of a term means the number of literal factors in a term, and it also equals the sum of the exponents of the literal factors. The degree of $4a^2b^3$ is $2 + 3 = 5$, for example.

Variable. A variable is a symbol that represents any value of a given set of elements. For example, in $A = \pi r^2$, where A is the area of a circle with radius r, and $\pi = 3.14159$, A and r are variables. When A changes as r is varied, then r is the independent variable and A the dependent variable.

Constant. A constant is a variable with one element only. An absolute or numerical constant always has the same value. An arbitrary constant, or parameter, has one value under certain conditions and different values under other conditions. Symbols representing numbers 11 or $\sqrt{2}$ or 3.14159 are absolute constants. In the expression $e = kP$, where e is the elongation of a bar due to a load P, k is a constant that varies with different materials; hence, k is a parameter.

CLASSIFICATION OF NUMBERS

Real Numbers (positive and negative). Real numbers include rational and irrational numbers. A rational number is expressible as the quotient of two integers, that is, integers such as -1, 2, 53, or fractions, as $\frac{3}{4}$, $-\frac{5}{2}$. An irrational number is not expressible as the quotient of two integers, as $\sqrt{2}$, π. The absolute value of a real number is the number itself if the number is positive, and the number with its sign changed if it is negative, as, for example, $|3| = |-3| = 3$.

Imaginary Numbers. An imaginary number is a product of a real number and the imaginary unit $i(=\sqrt{-1})$. Electrical engineers use j to avoid confusion with i for current. Example: $\sqrt{-2} = \sqrt{2}i$.

Complex Numbers. A complex number is a sum of a real number and an imaginary number, as $a + bi$, with a and b real numbers, $-3 + 0.5i$. A real number may be regarded as a complex number in which $b = 0$, and an imaginary number as one in which $a = 0$. The absolute value of a complex number $a + bi$ is $\sqrt{a^2 + b^2}$, as, for example, $|-3 + 0.5i| = \sqrt{9 - \frac{1}{4}} = 3.04+$. Relationships of complex numbers can be expressed in the following forms:

$$i = \sqrt{-1},\ i^2 = -1,\ i^3 = -i,\ i^4 = 1,\ i^5 = i, \text{ etc.}$$
$$a + bi = c + di \text{ if and only if } a = c,\ b = d$$
$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i$$

NOTATIONS

The main points of separation in a simple algebraic expression are the + and − signs. Thus, $a + b \times c - d \div x + y$ is interpreted as $a + (b \times c) - (d \div x) + y$. The range of operation of the symbols × and ÷ extends only to the next + or − sign. Between the signs × and ÷ themselves, $a \div b \times c$ means $a \div (b \times c)$. The ÷ sign is the stronger separative. Because this rule is not strictly followed, parentheses should be used to avoid ambiguity.

Exponents and radical signs influence only the next adjacent quantity. Thus $2ax^3$ means $2a(x^3)$, and $\sqrt{2}ax$ means $(\sqrt{2})(ax)$. Instead of $\sqrt{2}ax$, it is safer, however, to write $\sqrt{2} \cdot ax$, or $ax\sqrt{2}$. Any expression within parentheses is to be treated as a single quantity. A horizontal bar serves the same purpose as parentheses.

The notation $a \cdot b$, or simply ab, means $a \times b$; and $a:b$, or a/b, means $a \div b$.

Factorials. The symbol $n!$ (when n is a whole number) means: "n factorial," and means the product of the natural numbers from 1 to n, inclusive. Thus $2! = 1 \times 2$; $3! = 1 \times 2 \times 3$ and $4! = 1 \times 2 \times 3 \times 4$. The Stirling formula gives approximate values of $n!$ for large n:

$$n! \approx n^n e^{-n}\sqrt{2\pi n}$$

BASIC LAWS

Existence Law for Addition. Adding any two numbers a and b always gives a single number c: $a + b = c$.

Commutative Law. Algebraic numbers can be added or multiplied regardless of order: $a + b = b + a$; $ab = ba$.

Associative Law. The sum or product of three or more algebraic terms is unaffected by the grouping of the terms:

$$a + b + c = a + (b + c) = (a + b) + c$$
$$abc = a(bc) = (ab)c = (ac)b$$

Distributive Law. $a(b + c) = ab + ac$.

Operations with Zero and Negative Numbers. A number or letter without a preceding sign is assumed to be positive.

$$a + 0 \equiv a \qquad 0 - a \equiv -a$$
$$a + (-a) \equiv 0 \qquad -(-a) \equiv a$$
$$a \cdot 0 \equiv 0 \qquad a(-b) \equiv -ab$$
$$0/a \equiv 0, \text{ if } a \neq 0 \qquad (-a)(-b) \equiv ab$$

If $ab = 0$, then either $a = 0$ or $b = 0$ or $a = b = 0$. $a/0$ is undefined.

Order Relationships. If a and b are real numbers, then either $a < b$ or $a = b$ or $a > b$. If c is a third real number, and if $a < b$ and $b < c$, then $a < c$.

Axioms. The following relationships apply in algebraic transformations:

- If equals are added to equals, the sums are equal
- If equals are subtracted from equals, the differences are equal
- If equals are multiplied by equals, the products are equal
- If equals are divided by equals (except zero), the quotients are equal
- Like powers or like roots of equals are equal
- Numbers or terms equal to the same number or equal numbers are equal
- The whole equals the sum of its parts

IDENTITIES

An identity is a statement in symbolic form that holds for all values of the variables involved. For example, $(a + b)(a - b) = a^2 - b^2$ is true regardless of the values substituted for a and b. Common identities are listed below.

POWERS

$(-a)^n = a^n$, if n is even

$(-a)^n = -a^n$, if n is odd

$a^m \cdot a^n = a^{m+n}$

$$\frac{a^m}{a^n} = a^{m-n}$$

$(ab)^n = a^n b^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = \left(\frac{b}{a}\right)^{-n} = \frac{b^{-n}}{a^{-n}}$$

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$$

$(a^m)^n = a^{mn}$

$a^0 = 1$; $0^n = 0$; 0^0 is meaningless