Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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I. W. Knowles Y. Saitō (Eds)

Differential Equations and Mathematical Physics

Proceedings, Birmingham, Alabama 1986

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Proceedings of an International Conference held in Birmingham, Alabama, USA, March 3–8, 1986

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This volume is respectfully dedicated
to Professor Tosio Kato on the occasion
of his seventieth birthday.

PREFACE

This volume forms a permanent record of lectures given at the International Conference on Differential Equations and Mathematical Physics held at the University of Alabama at Birmingham during March 3-8, 1986.

The conference was supported by about 250 mathematicians from the following countries: Belgium, Canada, Czechoslovakia, Denmark, Egypt, Finland, France, Hungary, India, Ireland, Japan, Kuwait, Nigeria, Norway, P.R. of China, South Africa, Sweden, Switzerland, The Netherlands, the U.K., the U.S.A., and West Germany. Its main purpose was to provide a forum for the discussion of recent developments in the theory of ordinary and partial differential equations, both linear and non-linear, with particular reference to work relating to the equations of mathematical physics. Invited one-hour lectures were given by P. Deift, R. DiPerna, W.N. Everitt, C. Foias, T. Kato, S. Kotani, A. Majda, J. Mawhin, J. McLaughlin, J. McLeod, C. Morawetz, R. Newton, R. Phillips, M. Reed, I. Sigal, and B. Simon. The remainder of the program consisted of invited one-half hour lectures.

On behalf of the participants, the conference directors acknowledge, with gratitude, the generous financial support provided by the U.S. National Science Foundation, under grant number DMS-8516772, the Department of Mathematics and the Graduate School, University of Alabama at Birmingham, and the College of Arts and Sciences, the Graduate School and the Office of Academic Affairs, University of Alabama. We acknowledge also the valuable support provided by the other members of the conference committee: Robert Kauffman, Roger Lewis, and Fred Martens from UAB, and Richard Brown and James Ward from UA. As always, the committee is much indebted to the faculty, staff, and graduate students of the Department of Mathematics at UAB for their manifold contributions; here, we wish to make particular mention of Mrs. Eileen Schauer for undertaking the onerous task of typing much of the conference material, including many of the the papers appearing in this volume.

Ian W. Knowles
Yoshimi Saito
Conference Directors

LECTURES NOT APPEARING IN THE PROCEEDINGS

A Nonlinear Eigenvalue Problem in Astrophysical Magnetohydrodynamics

John A. Adam (Old Dominion University, U.S.A.)

Existence of Non-Trivial Periodic Solutions of a Certain Third-Order Non-Linear Differential Equation

Anthony Uyi Afuwape (University of Ife, NIGERIA)

Stabilization of Solutions for a Class of Degenerate Equations in Divergence Form in One Space Dimension

N. Alikakos (University of Tennessee, U.S.A.)

Spectral Properties of Indefinite Elliptic Problems

W. Allegretto (University of Alberta, CANADA)

Quasilinear Parabolic Systems

H. Amann (Universitat Zurich, SWITZERLAND)

Convergence Properties of Strongly-Damped Semilinear Wave Equations

Joel D. Avrin (University of North Carolina-Charlotte, U.S.A.)

On Smoothness of Solutions of Elliptic Equations in n-Dimensional Nonsmooth Domains

A. Azzam (Kuwait University, KUWAIT)

Singular Elliptic Operators with Discrete Spectra

J. V. Baxley (Wake Forest University, U.S.A.) *and R. O. Chapman* (Oxford University, U.K.)

Numerical Solution of Nonlinear Parabolic Variational and Quasi-Variational Inequalities

S. A. Belbas (University of Alabama, U.S.A.)

The Kolomogoroff-Arnold-Moses Theorem in Schrodinger's Equation

Jean Bellissard (California Institute of Technology, U.S.A.)

The Limiting Absorption Principle for Differential Operators with Short-Range Perturbations

Matania Ben-Artzi (University of California, Los Angeles, U.S.A.)

Random Wave Operators

Marc A. Berger (Georgia Institute of Technology, U.S.A.)

Singular Linear Differential and Difference Operators in the Complex Plane and in B^*-Algebras

C. E. Billigheimer (McMaster University, CANADA)

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Erkki Brandas and Erik Engdahl (Uppsala University, SWEDEN)

CONTENTS

ANALYTICAL SOLUTIONS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

G. Adomian[1]

Our objective is to address the need for realistic solution of the nonlinear stochastic systems of equations in space and time which arise in the modeling of frontier problems in physics. What is meant by realistic solution is solution of the problem as it is rather than forcing it into an oversimplified mold to make it easily solvable. For a wide range of problems, of course, it is adequate to use perturbation, linearization, etc., but generally assumptions of weak nonlinearity, small fluctuations, and convenient but unphysical stochastic processes may be unjustified, and we resort to them only when no other approach is possible. (For some nonlinear systems, exact linearization is possible by clever transformations of variables to make the equations linear and solvable. However, this is not generally possible and one resorts to ad hoc or perturbative methods.) In systems involving stochastic parameters, e.g., differential equations with stochastic process coefficients - the stochastic operator case - usual analyses employ perturbation or hierarchy methods which require that fluctuations be small. Another common restrictive assumption is an assumed special nature or behavior for the processes - for mathematical rather than physical reasons. The literature abounds with unrealistic unphysical assumptions and approximations such as white noise, monochromatic approximation, local independence, etc.

These limitations and assumptions are made for mathematical tractability and use of well known theory. Yet our final objective must not be simply the satisfaction of quoting theorems and stating an abstruse

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