DYNAMIC ANALYSIS
AND FAILURE MODES OF SIMPLE STRUCTURES

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PREFACE

Mechanical analysis of structures is required to ensure that they maintain their integrity and provide predictable and acceptable performance and dynamic response throughout the specified lifetime profile of loads and acceleration environments. This analysis includes vibration- and shock-induced displacements and stresses, isolation of sensitive components, and the evaluation of elastic instability, fatigue, and fracture as potential failure modes. In aerospace applications, as in other applications, proper design of vents is required to maintain safe pressure differentials in venting air from a compartment. When the structure contains electronics, thermal analysis may be utilized to achieve adequate heat transfer and avoid temperature-related performance degradation or failure of these components. This book addresses these technical areas in a manner most useful to the engineer who is involved in a mechanical design and requires timely quantitative answers to the problems that arise.

The organization of the subject matter is in a logical sequence, introducing the concepts of loads and failure modes in the first chapter, and dealing with the natural frequency of components and structures in the next two chapters. With an estimate of natural frequency, the specified acceleration environment (vibration, Chapter 4, and shock, Chapter 5) may be utilized to obtain displacements and stresses, and isolation may be designed where required (Chapter 6). The acceleration-induced stresses may then be evaluated in terms of fatigue, fracture, elastic instability, and material strength limitations (Chapters 7 through 10). Chapters 11 and 12 cover venting and thermal analysis.

The approach taken is to describe the problem in physical terms, defining the known (given) parameters and the unknown quantities that must be evaluated. The methodology is then provided for obtaining quantitative estimates for these unknowns and relating them to specified limiting values and failure modes to obtain an understanding of the acceptability of the design. The methodology used may include equations, data tables, and figures, and can be rapidly implemented with the use of a pocket calculator. This type of analysis is particularly helpful in the early design configuration stage, in the comparison of different design approaches, and in monitoring the results of more massive and detailed computer analyses. It will provide the engineer with a better understanding of the system being designed and will pave the way for the development of an effective finite element model for computer analysis.

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CONTENTS

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MECHANICAL LOADS AND FAILURE MODES

1.1 FORCES

This chapter describes and characterizes the types of mechanical loads, deformations, stresses, and failure modes that will be addressed in subsequent chapters. The mechanical loads are the forces that act on a structure to produce a change in the deformation and stress in the structure. These forces are specified by four characteristics: their point of application on the structure, their direction, their magnitude, and their time dependence. Because forces have both magnitude and direction, they are defined as vector quantities, in contrast to scalar quantities such as mass, which have only magnitude and no direction.

The ways in which these four characteristics of a force affect a structure are illustrated as follows:

1. *Point of Application*. A force of constant magnitude acting in a fixed direction normal to a slender cantilever beam and applied at the free end of the beam will produce twice the stress and strain in the cantilever material at the fixed end of the beam than will be produced when the force is applied to the midpoint of the cantilever.

2. *Direction*. A force of constant magnitude applied normal to the free end of a horizontal cantilever beam stressed by its own weight will result in a greater total stress and displacement in the cantilever when the force is pointing down in the direction of the gravitational force than when it is pointing up.

3. *Magnitude*. A force applied at a fixed point on a cantilever beam and acting in a fixed direction normal to the beam will cause a stress and strain in

the cantilever material that increases directly as the magnitude of the force increases, within the elastic limits of the cantilever material.

4. *Time Dependence*. If a force of given magnitude, applied in a given direction normal to the end of a cantilever beam, is applied suddenly, the maximum transient stress and displacement in the cantilever is twice that produced by the same force applied slowly. If the same force produces stresses that are below the yield strength of the cantilever material and cannot cause failure, repeated application of this force over a period of many cycles may cause the cantilever to fail (rupture) by fatigue or by fracture.

Static (time-independent) forces acting on a structure can produce static (constant) stresses and strains in the structure unless the structure contains materials that creep. Creep is a slow and continuous increase in deformation under constant or decreasing stress, or a slow decrease in stress when the deformation is maintained constant (relaxation). Time-dependent forces can be caused by time-dependent accelerations imposed on a structure. The acceleration results in inertial forces acting throughout the structure. These inertial forces are opposite to the direction of acceleration, are applied everywhere throughout the mass of the structure, and are proportional to the product of the material mass density and the magnitude of the acceleration. These inertial forces are distributed forces, which act on every particle of mass in the structure, since, ideally, every part of the structure undergoes the same acceleration. As the acceleration increases beyond 1 g (g = earth's gravity acceleration = 386 in./sec^2), the situation is equivalent to uniformly increasing the weight of the structure and orienting it so that "down" is opposite to the direction of acceleration. In the treatment of accelerations and inertial forces in later chapters, the propagation through a structure of stress/strain waves, due to time-dependent accelerations of a mounting surface supporting the structure, is ignored. This simplification assumes that the entire structure accelerates instantaneously. From a design viewpoint, the most important time-dependent inertial forces acting on a structure are due to vibration and shock.

Vibration (Chapter 4) is an oscillating stress and deformation of the structure defined by either a fixed frequency (sinusoidal vibration) or a time-varying set of frequencies and amplitudes conforming to an amplitude/frequency envelope (a spectrum). The latter phenomenon is called random vibration, and the spectrum is specified in terms of a power spectral density (PSD). It is called random vibration because the frequencies and amplitudes are not predictable except in terms of probabilities, as defined by the PSD. Major sources of random vibration are ground transportation and launch operations. Although the random-vibration environment due to surface transportation has the longest duration, often lasting hours, its amplitude is normally less severe than that due to launch operations. Space-system launch vehicles use rocket-engine firings that last for minutes and present the most severe random-vibration environment to which an aerospace system will be exposed. Since the type of acceleration time history due to rocket-engine firings is difficult to reproduce in the laboratory, an equivalent approach is taken. A shaker table is controlled to yield the same time-averaged acceleration, or power per unit mass (PSD), at every frequency as would result from a time average of the complete rocket-motor firing. The shaker table produces its own random-vibration time history, and over a very short time interval its average power versus frequency may be quite different from the average values for the rocket motor; but over a period of several minutes the shaker table can be controlled to yield an average power spectrum within a few decibels of the rocket-motor time-averaged spectrum.

Shock (Chapter 5) is a sudden, severe, and transient acceleration of the structure. In aerospace applications shock can be caused by pyrotechnic separation of a system from a supporting structure with the additional sudden release of stored energy in the previously strained system or supporting structure. Shock can also occur when a vehicle lands with a payload still on board. Pyrotechnic shocks last milliseconds or less and can produce accelerations of thousands of g's. The acceleration time history of a shock represents the actual acceleration versus time in the structural member that contains the pyrotechnic device or that transmits the shock to another structure. This acceleration time history is a complex, nonanalytical function of time. For purposes of analysis, specification, and testing, the shock acceleration time history is converted to a shock acceleration response spectrum. This spectrum represents, at each frequency, the magnitude of the acceleration in g units that would be experienced by a simple harmonic oscillator at that frequency when exposed to the shock. Any specified amount of damping may be incorporated into the shock acceleration response spectrum. This spectrum is much easier to use in design and analysis than the shock acceleration time history. It is also much easier to specify a laboratory shock test using a very simple half-sine or sawtooth shock acceleration time history to approximate the actual shock acceleration response spectrum than it is to approximate the actual shock acceleration time history in the laboratory.

In design and analysis it is usually assumed that the acceleration environment (limit load, vibration, or shock) may be imposed on a structure with the acceleration vector acting in either direction along any three orthogonal axes. This is because the acceleration environment is not always specified as to directionality with respect to the structure, and subsystem structures sometimes change orientation with respect to the primary structure between successive flight units. For the same reason, laboratory tests usually require random vibration along three orthogonal axes and shocks in both directions along three orthogonal axes. The test axes are usually chosen to be parallel to the subsystem housing walls in the case of rectangular housings.

1.2 STRESSES

The application of forces to a structure produces deformations and stresses. These stresses in the structural material may be sufficiently severe in magni-

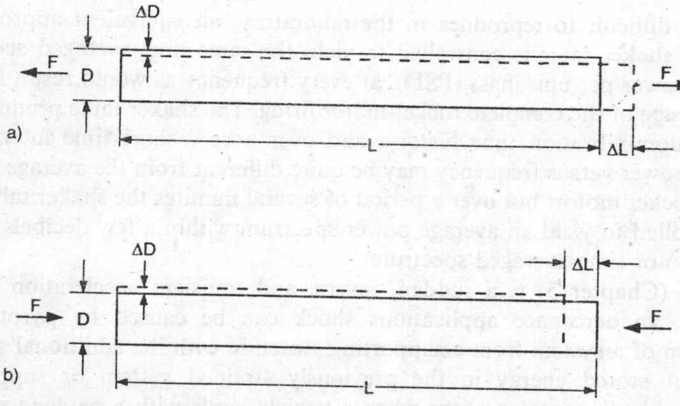

FIGURE 1.1 Tension and compression. In (*a*), the bar under tension undergoes axial elongation and lateral contraction. In (*b*), the bar under compression undergoes axial contraction and lateral expansion.

tude, duration, or repetition to cause the structure to fail. In Chapter 4 analysis techniques are provided for estimating material stresses based on the structural design and the load environments to which the structure is subjected. The stresses may be tension, compression, or shear, or a combination of them. The stresses will also reflect the time-varying behavior of the loads that cause them, sometimes resulting in alternating tensile and compressive stresses and other time-varying combinations. A brief review of some of the simple relationships among forces, elastic constants, and stresses follows.

A straight, homogeneous bar of length L in. having a uniform circular cross section of diameter D in. is subjected to a tensile force of F lbf parallel to its long axis and uniformly distributed over the bar cross section. See Fig. 1.1*a*. The tensile stress in the bar is

$$\sigma = F/A \text{ psi (pounds force per square inch)} \tag{1-1}$$

where $A = (\pi/4)D^2$ in.^{2}, the cross-sectional area of the bar. Equation (1-1) shows that the stress is proportional to the load (force) and is also a function of structural geometry, in this case the cross-sectional area A. Equation (1-1) is usually applied only when the magnitude of the stress is in the elastic range of the bar material, that is, σ does not exceed the yield stress of the bar material. In other cases, more complex geometric parameters such as moments of inertia may be involved. The axial elongation of the bar under tension is

$$\Delta L = \varepsilon L \text{ in.} \tag{1-2}$$

where ε is the strain in the bar (dimensionless) and

$$\varepsilon = \sigma/E. \tag{1-3}$$

where E is the modulus of elasticity, or Young's modulus, of the bar material (psi). Equation (1-3) shows that the strain is proportional to the stress and inversely proportional to the modulus of elasticity. Equation (1-3) is usually applied only when the magnitude of σ is less than the yield stress of the bar material. In other cases other elastic moduli may be involved.

Combining the preceding relationships yields

$$\Delta L = FL/AE \text{ in.} \tag{1-4}$$

As long as the material stress is in the elastic range, the elongation of the bar is directly proportional to the length and the tensile force and inversely proportional to the cross-sectional area and modulus of elasticity. This relationship is independent of the shape of the cross section, provided the cross section is constant over the length of the bar.

When the bar under tension undergoes axial elongation, it also undergoes lateral contraction and a decrease in cross-sectional area. The linear contraction at right angles to the load, for the round bar of diameter D, is

$$\Delta D = -\nu\varepsilon D \text{ in.} \tag{1-5}$$

where

$$\nu = -(\Delta D/D)/\varepsilon = -(\Delta D/D)/(\Delta L/L) \equiv \text{Poisson's ratio} \tag{1-6}$$

Poisson's ratio is the ratio of the unit lateral contraction to the unit axial elongation. The new cross-sectional area of the loaded bar is

$$A' = (1 - \nu\varepsilon)^2 A \text{ in.}^2 \tag{1-7}$$

and its new volume is

$$V' = (1 + \varepsilon)(1 - \nu\varepsilon)^2 V \text{ in.}^3 \tag{1-8}$$

where V is the original volume of the unloaded bar. Neglecting the second and third powers of ε, the unit volume expansion of the bar is

$$(V' - V)/V = \varepsilon(1 - 2\nu) \tag{1-9}$$

Since it is unlikely that materials of interest will diminish in volume when under tension, ν must be less than $\frac{1}{2}$. Most structural metals have a value of ν

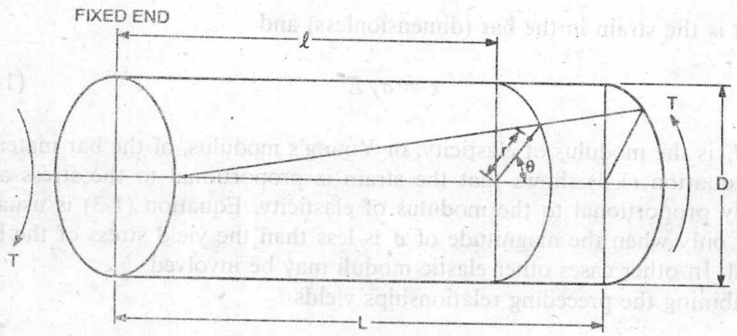

FIGURE 1.2 Pure shear in a twisted bar. The twisting moment T causes the bar to twist through an angle θ at distance l from the fixed end.

in the range 0.28–0.35, while plastics used in printed wire boards have lower values, in the neighborhood of 0.1.

When the bar is subjected to compressive forces as in Fig. 1.1b, it undergoes a decrease in length and an increase in lateral dimensions, causing a change in algebraic sign in Eqs. (1-2), (1-4), and (1-5) and resulting in a unit volume contraction of

$$(V' - V)/V = -\varepsilon(1 - 2\nu) \tag{1-10}$$

For most materials of interest, the modulus of elasticity and Poisson's ratio are the same for tension and compression.

A pure shear stress may be produced by subjecting the same bar to equal and opposite twisting couples (torsion), T in.-lb, at its ends in planes normal to its longitudinal axis, as in Fig. 1.2. The bar will twist, with every section (i.e., every plane perpendicular to the axis) rotating about the axis through an angle

$$\theta = (Tl)/JG \text{ rad} \tag{1-11}$$

where l = distance along the axis from the fixed end of the bar (in.)
J = polar moment of inertia
= 2(moment of inertia about central axis)
$= 2I = 2(\pi/64)D^4$ (in.^{4})
G = shear modulus
= modulus of rigidity of the bar material
$= E/2(1 + \nu)$ psi.

The shear stress at a radius r from the axis of the bar is

$$\tau = Tr/J \text{ psi} \tag{1-12}$$