Ming-Yang Kao
Xiang-Yang Li (Eds.)

LNCS 4508

Algorithmic Aspects in Information and Management

Third International Conference, AAIM 2007
Portland, OR, USA, June 2007
Proceedings

Springer

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Library of Congress Control Number: 2007927664

CR Subject Classification (1998): F.2.1-2, E.1, G.1-3, J.1

LNCS Sublibrary: SL 3 – Information Systems and Application, incl. Internet/Web and HCI

ISSN 0302-9743
ISBN-10 3-540-72868-6 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-72868-9 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media

springer.com

Printed in Germany

Typesetting: Camera-ready by author, data conversion by Scientific Publishing Services, Chennai, India
Printed on acid-free paper SPIN: 12071777 06/3180 5 4 3 2 1 0

Preface

The papers in this volume were presented at the 3rd International Conference on Algorithmic Aspects in Information and Management (AAIM 2007), held June 6–8, 2007 in Portland, Oregon, USA. This conference is intended for original algorithmic research on immediate applications and/or fundamental problems pertinent to information management and management science, broadly construed.

Submissions to the conference this year were conducted electronically. A total of 120 papers were submitted, of which 40 were initially accepted, and 1 paper was withdrawed later. The papers were evaluated by an international Program Committee consisting of Hee-Kap Ahn, Takao Asano, Amotz Bar-Noy, Hans Bodlaender, Peter Brucker, Leizhen Cai, Gruia Calinescu, Jianer Chen, Siu-Wing Cheng, Marek Chrobak, Yang Dai, Rudolf Fleischer, Jie Gao, Joachim Gudmundsson, Bhaskar DasGupta, Gregory Gutin, Wen-Lian Hsu, Giuseppe F. Italiano, Ming-Yang Kao, Sanjiv Kapoor, Tak-Wah Lam, Erran Li Li, Jing Li, Xiang-Yang Li, Peter Bro Miltersen, Seffi Naor, Chung Keung Poon, Kirk Pruhs, Rajeev Raman, Paul Spirakis, Zheng Sun, Wing Kin Sung, Jan van Leeuwen, Jie Wang, Lusheng Wang, Weizhao Wang, Yu Wang, JinHui Xu, Yinfeng Xu, and Binhai Zhu.

The submitted papers to AAIM 2007 were from Algeria, Canada, Chile, China (mainland and Taiwan), Germany, Hong Kong, India, Italy, Japan, Mexico, Netherlands, Portugal, South Korea, Spain, Sweden, Switzerland, Turkey, Ukraine, UK, and USA.

Each paper was evaluated by at least two Program Committee members and most papers were actually evaluated by at least three Program Committee members, asisted in some cases by external reviews and comments. In addition to these selected papers, the conference also includeed three invited keynote talks by Anna Karlin from the University of Washington, Tuomas Sandholm from CMU, and Shang-Hua Teng from Boston University.

We thank all the people who made this meeting possible: the authors for submitting their papers to AAIM 2007, the Program Committee members and external reviewers (listed on the pages that follow) for their excellent work, and the three invited keynote speakers. Finally, we thank the Washington State University, Vancouver campus, for their support and the local organizers and our colleagues for their assitance.

June 2007

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Invited Lecture

Solving Generalized Maximum Dispersion with Linear Programming

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Abstract. The GENERALIZED MAXIMUM DISPERSION problem asks for a partition of a given graph into p vertex-disjoint sets, each of them having at most k vertices. The goal is to maximize the total edge-weight of the induced subgraphs. We present the first LP-based approximation algorithm.

Keywords: Approximation Algorithms, Randomized Algorithms, Generalized Maximum Dispersion.

1 Introduction

Let $G = (V, E)$ be an undirected graph ($n = |V|, m = |E|$) with non-negative edge weights w_{ij} for $(i,j) \in E$ (for convenience, we set $w_{ij} = 0$ for $(i,j) \notin E$, implicitly assuming that G is a complete graph). The weight $\omega(S)$ of a subgraph of G induced by $S \subset V$ is the sum $\sum_{i \in S, j \in S} w_{ij}$.
For non-negative integers p, k with $pk \leq n$ the GENERALIZED MAXIMUM DISPERSION problem is to find p disjoint induced subgraphs of G, each with at most k vertices, such that the sum of the edge weights of the subgraphs is maximum.

For a motivation of this problem consider the following problem: A large scale manufacturer wants to expand in a new region and can use n locations. He wants to expand in exactly p business areas (e.g. restaurants, groceries, home-improvement markets etc.). For each such area he is allowed to use up to k locations, where $pk \leq n$. Furthermore assume, that all n locations are ready, i.e. do not cause any extra costs. Shops of the same area are attractive for the manufacturer, if they are as far as possible from each other (which means that for one customer only one shop comes into question). Thus the manufacturer wishes

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to maximize the sum over all distances between locations of the same area, i.e. he has to solve exactly our GENERALIZED MAXIMUM DISPERSION problem.

GENERALIZED MAXIMUM DISPERSION is $\mathcal{NP}$-hard, which can be easily seen by a reduction from the MAXIMUM CLIQUE problem. As DENSE SUBGRAPH is a special case, it follows from [16] that there is even no PTAS for GENERALIZED MAXIMUM DISPERSION.

A promising and often successful approach to cope with the hardness of a combinatorial optimization problem is to design polynomial-time approximation algorithms. Given an instance I of a maximization problem and an (approximation) algorithm A the *approximation factor* $r_A(I)$ is defined by $r_A(I) = A(I)/OPT(I) \leq 1$.

Previous Work. To the best of our knowledge, the GENERALIZED MAXIMUM DISPERSION problem is only considered by Hassin, Rubinstein and Tamir [14]. They provide a polynomial-time algorithm with approximation factor $\frac{1}{2-1/\lceil k/2 \rceil}$ in graphs where the edge weights satisfy the triangle inequality.

As expressed in the name, the GENERALIZED MAXIMUM DISPERSION is a natural generalization of the MAXIMUM DISPERSION problem (which is also denoted by MAX-k-DENSE-SUBGRAPH problem). For a given weighted graph the MAXIMUM DISPERSION problem chooses one vertex set (i.e. $p = 1$) with exactly k vertices, where the total edge-weight of the induced subgraph is to be maximized. The MAXIMUM DISPERSION problem is also $\mathcal{NP}$-hard and remains $\mathcal{NP}$-hard even when the weights satisfy the triangle inequality [18]. For further results about this problem see [3,4,5,6,7,8,12,13,15,20].

A similar problem, called MAX-p-Section, is considered by Andersson [1]. He provides an approximation algorithm based on semidefinite programming, which is a natural generalization of the approximation algorithm for the special MAX-Bisection problem given by Frieze and Jerrum [9]. Many of the techniques used in this paper – as relaxation and randomized rounding – base on the pioneer paper of Goemans and Williamson [11] who applied semidefinite programming on MAX-CUT.

The Results. We present a randomized rounding algorithm for the GENERALIZED MAXIMUM DISPERSION problem which achieves for every $0 < \delta \leq 1/2$ and $0 < \varepsilon < 1$ with probability at least $\frac{\varepsilon k}{12n}$ a solution with value at least $\frac{(1-\delta)^2(1-\varepsilon)k}{2n-pk} W$, provided that $k \geq \frac{3(1-\delta)}{\delta^2} \ln(\frac{36np}{\varepsilon k})$, where W is the value of an optimal solution. We also show how this algorithm can be derandomized. By iterating the randomized algorithm we obtain the same approximation guarantee even under the weaker condition $k \geq \frac{3(1-\delta)}{\delta^2} \ln(4p)$. A key point in our algorithm is that it can be viewed as a combination of direct randomized rounding and random sampling in the following sense: Instead of rounding the fractional solution directly by taking the fractional solution as rounding probabilities, we take a convex sum of the fractional solution and a certain probability for uniformly distributing the vertices among the subgraphs.

The paper is organized as follows. In Section 2 we introduce a linear relaxation for the GENERALIZED MAXIMUM DISPERSION problem. Depending on this relaxation, we analyze a randomized algorithm in Section 3 and a deterministic algorithm in Section 4.

2 A Linear Relaxation for Generalized Maximum Dispersion

In GENERALIZED MAXIMUM DISPERSION the task is to construct disjoint subsets $S_1, \ldots, S_p \subset V$ maximizing the total weight of the subgraphs induced by the sets S_ℓ, $\ell = 1, \ldots, p$. Let W denote the value of an optimal solution for the given instance. An integer program is given below. For each vertex $i \in V$ we introduce a p-variate vector $x_{i1}, \ldots, x_{ip}$, where $x_{i\ell} = 1$ is interpreted as $i \in S_\ell$. The constraints (2) enforce that each vertex belongs to at most one of the subgraphs, (3) mirrors the cardinality constraints. $z_{ij\ell} = 1$ and (1) imply that both, i and j, are part of the same subset S_ℓ, and the weight of the corresponding edge contributes to the objective function.

$$\begin{aligned}
\text{maximize} \quad & \sum_{(i,j)\in E} w_{ij} \sum_{\ell=1}^{p} z_{ij\ell} & & \\
\text{subject to } & 0 \le z_{ij\ell} \le x_{i\ell},\ 0 \le z_{ij\ell} \le x_{j\ell} & \text{for } (i,j) \in E,\ \ell = 1, \ldots, p & \quad (1)\\
& \sum_{\ell=1}^{p} x_{i\ell} \le 1 & \text{for } i = 1, \cdots, n & \quad (2)\\
& \sum_{i=1}^{n} x_{i\ell} \le k & \text{for } \ell = 1, \ldots, p & \quad (3)\\
& x_{i\ell}, z_{ij\ell} \in \{0,1\} & \text{for } i, j = 1, \cdots, n,\ \ell = 1, \cdots, p & \quad (4)
\end{aligned}$$

When we relax the integrality constraints (4), an optimal fractional solution $x^*_{i\ell}, z^*_{ij\ell} \in [0,1]$ can be computed in polynomial time using standard linear programming techniques. We now round the fractional solution to an integer. Our rounding scheme is a mixture of direct LP-based randomized rounding (with the fractional solution $x^*_{i\ell}$) and distributing the vertices among the sets uniformly at random with probability k/n. We define the following probabilities

$$p_{i\ell} := c x^*_{i\ell} + (1-c)k/n \quad \text{for} \quad i = 1, \ldots, n,\ \ell = 1, \ldots, p.$$

$$p_i := \sum_{\ell=1}^{p} p_{i\ell} \quad \text{for} \quad i = 1, \ldots, n.$$

$c \in [0,1]$ is a constant, depending on k and n, which we will specify later so as to obtain a good approximation factor.

For $\ell = 1, \ldots, p$ let $e_\ell \in \mathbb{N}^p$ be the $0/1$-vector whose ℓ-th component is 1 and the other entries are 0, and let $e_{p+1} \in \mathbb{N}^p$ be the zero vector.

Let $0 < \delta \leq 1/2$ and let X_i, $1 \leq i \leq n$, be n mutually independent random variables with values in $\{e_\ell \mid \ell = 1, \ldots, p+1\}$ with

$$\Pr[X_i = e_\ell] = \begin{cases} (1-\delta)p_{i\ell} & \text{for } \ell = 1, \ldots, p \\ 1-(1-\delta)p_i & \text{for } \ell = p+1 \end{cases}$$

We can think of the X_i's as n mutually independent $(p+1)$-faced dice.

Algorithm RANDDISP

1. For $\ell = 1, \ldots, p$ define

$$x_{i\ell} = \begin{cases} 1 & \text{if } X_i = e_\ell \\ 0 & \text{else} \end{cases}$$

If $X_i = e_{p+1}$, then $x_{i\ell} = 0$ for $\ell = 1, \ldots, p$.

2. For $i, j \in \{1, \ldots, n\}$ and $\ell \in \{1, \ldots, p\}$ define

$$z_{ij\ell} = x_{i\ell}x_{j\ell}$$

RANDDISP generates a random 0/1 assignment for the $x_{i\ell}$'s and the $z_{ij\ell}$'s. We show that this is a feasible solution for GENERALIZED MAXIMUM DISPERSION with non-zero probability.

3 A Randomized Algorithm for Generalized Maximum Dispersion

Let $\omega := \sum_{(i,j)\in E} w_{ij} \sum_{\ell=1}^{p} z_{ij\ell}$ be the weight resulting from the above assignment.

Theorem 1. *Let $0 < \delta \leq 1/2$, $0 < \varepsilon < 1$ and $c_\delta = \frac{3(1-\delta)}{\delta^2}$. If $k \geq c_\delta \ln(\frac{36np}{\varepsilon k})$, then with probability at least $\frac{\varepsilon k}{12n}$ the $z_{ij\ell}$, $x_{i\ell}$, $i, j \in \{1, \ldots, n\}$, $\ell \in \{1, \ldots, p\}$ build a feasible solution for* GENERALIZED MAXIMUM DISPERSION *and*

$$\omega \ \geq \ (1-\delta)^2(1-\varepsilon)\frac{k}{2n-pk}W.$$

PROOF Let $A_0, A_1, \ldots, A_p$ be the following events. A_0 is the event $\omega < (1-\delta)^2(1-\varepsilon)\frac{k}{2n-pk}W$. For $\ell = 1, \ldots, p$, A_ℓ is the event $\sum_{i=1}^{n} x_{i\ell} > k$.

We will derive upper bounds for $\Pr[A_\ell]$, $\ell = 0, \ldots, p$.
Let $a(n) = 1/(1 + \frac{\varepsilon k}{8n})$ and let $b(n) = 1 - a(n)$.

Claim 1: $\Pr[A_0] \ \leq \ a(n)$