

MARK DUGOPOLSKI

INTERMEDIATE

Algebra

3RD EDITION

www.mhhe.com/dugopolski

FREE
Student Online
Learning Center
Available

P/N 236883-7

supported by

ALEKSTM

www.highed.aleks.com

FREE CD
INCLUDED





Intermediate Algebra

Intermediate Algebra

E D I T I O N

3

Mark Dugopolski

Southeastern Louisiana University



Boston Burr Ridge, IL Dubuque, IA Madison, WI
New York San Francisco St. Louis Bangkok
Bogotá Caracas Lisbon London Madrid Mexico City Milan
New Delhi Seoul Singapore Sydney
Taipei Toronto

McGraw-Hill Higher Education

A Division of The McGraw-Hill Companies

INTERMEDIATE ALGEBRA, THIRD EDITION

Copyright © 2000 by The McGraw-Hill Companies, Inc. All rights reserved. Previous edition © 1996, 1992 by Addison-Wesley Publishing Company, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

This book is printed on acid-free paper.

6 7 8 9 0 VNH/VNH 0 9 8 7 6 5 4 3

ISBN 0-07-229466-3

ISBN 0-07-229484-1 (AIE)

Vice president and editorial director: *Kevin T. Kane*

Publisher: *JP Lenney*

Sponsoring editor: *William K. Barter*

Developmental editor: *Erin Brown*

Marketing manager: *Mary K. Kittell*

Project manager: *Vicki Krug*

Production supervisor: *Enboge Chong*

Senior designer: *Sabrina Dupont*

Photo research coordinator: *Jodi Banowetz*

Senior supplement coordinator: *Audrey A. Reiter*

Compositor: *Interactive Composition Corporation*

Typeface: *10 1/2/12 Times Roman*

Printer: *Von Hoffmann Press, Inc.*

Cover/interior designer: *Amanda Kavanagh*

Photo credits: Page 27, p. 53, p. 73: *US Army Corps of Engineers*; p. 121: *Paul Conklin*; p. 199, p. 206, p. 333, p. 387, p. 400, p. 447, p. 627: *Susan Van Etten*; p. 257: *Herb Snitzer/Stock Boston*; p. 537: *Molly McCallister*; p. 571: *Courtesy of British Airways*; p. 596: *Frederick von Huene/von Huene Workshop*. All other photos © *PhotoDisc, Inc.* (with exception of pages 158, 317, 369, 460, 504, 631).

Library of Congress has cataloged Algebra for College Students as follows:

Dugopolski, Mark.

Algebra for college students / Mark Dugopolski. — 2nd ed.

p. cm.

Includes index.

ISBN 0-07-232399-X

1. Algebra. I. Title.

QA152.2.D828 2000

512.9—dc21

99-40279

CIP

INDEX OF SELECTED APPLICATIONS

NOT FOR RESALE

Biology/Health/Life Sciences

Basic energy requirement, 386
Building fitness, 163
Cigarette usage, 72
Female target heart rate, 30
Hazardous to her health, 103
Heart rate, 193
Male target heart rate, 30
Protein and carbohydrates, 235
Staying fit, 170
Waist-to-hip ratio, 163, 193

Business

Advertising dollars, 163
Allocating resources, 163, 249
Annual bonus, 282
Average cost, 295
Average profit, 295
Bonus and taxes, 208
Budget planning, 162
Choosing a selling price, 82
Corporate taxes, 208
Cost, 120, 129, 170, 208
Depreciation, 171
Dividing the estate, 83
Earned income, 170
Economic impact, 633
Fortune 500 profits, 83
Free market, 209
Going bananas, 626
Increasing revenue, 490
Las Vegas vacation, 378
Less impact, 633, 651
Making circuit boards, 150
Marginal cost, 282
Marginal profit, 282
Maritime losses, 371
Maximum profit, 591
Maximum revenue, 590
Negative income tax, 332
Net worth of a bank, 22
Phase I advertising, 248
Processing, 357
Profit function, 129, 482, 498
Printing costs, 171
Recovering an investment, 464
Rental costs, 129, 193
Rose Bowl bound, 342
Selling, 358
Shipping restrictions, 163
Textbook case, 209
Ticket prices, 490
Total economic impact, 637, 651

Total spending, 637
Toy sales, 83
Wedding bells, 342
White-water rafting, 378
Year-end bonus, 118

Chemistry

Acid solutions, 82
Chlorine bleach, 82
Gas laws, 516
Increasing acidity, 82
Increasing the percentage, 83
Ions for breakfast, 562
Ions in your veins, 562
Mixture, 82, 83, 84, 216, 236
Neuse River pH, 547
Radioactive decay, 540
Roanoke River pH, 548
Stomach acid, 547
Three solutions, 254
Tomato juice, 547

Construction

Area of an inscribed circle, 498
Building a patio, 579
Dealing in gravel, 170
Diagonals, 427
Dimensions of a frame, 81
Doorway dimensions, 81
Fencing, 72
Great Chicago flood, 72, 323
Guy wire, 443
Heavy penalties, 643
House plans, 288
Length of a road, 428
Modern art, 37
Painting, 358, 378
Perimeter of a lot, 81
Pricing plastic, 516
Reinforcing rods, 516
Roofing, 357
Shawn's shed, 84
Teamwork, 50

Consumer Applications

Assessed for repairs, 73
Camaro Z28 depreciation, 129
Camaro Z28 inflation, 129
Carpeting costs, 51, 509
Car shopping, 93
Charitable contributions, 116
Comparing copiers, 256
Constant increase, 633

Cost, 330
Dealer discounts, 116
Depreciating Monte Carlo, 139
Distance between streets, 73
Fast cat, 171
Health food mix, 83
High cost of nursing care, 31
Increasing salary, 642
Inflationary spiral, 51
Listing a house, 82
Long distance charges, 129
Measuring risk, 129
Mixing investments, 254
Mustang Sally, 82
Net worth of a family, 22
Pricing the Crown Victoria, 139
Saving for retirement, 30, 274
Selling-price range, 103
Seven years of salary, 642
Soaring cost of nursing care, 31
Social security, 198
Student loan, 31
Vehicle cost, 509

Design

Approach speed, 456
Cubic coating, 288
Displacement-length ratio, 498
Energy efficient, 288
Fabric design, 633
Flute reproduction, 599
Golden rectangle, 472
House of seven gables, 578
House plans, 323
Landing a Piper Cheyenne, 406
Landing speed and weight, 407, 444
Landscape design, 330
Manufacturing a box, 443
Maximum area, 590, 621
Open-top box, 472
Overflow pan, 288
Pleasing painting, 371
Sail area-displacement ratio, 499
Shipping parts, 443
Spillway capacity, 444
Swimming pool design, 488
Volume of a flute, 599

Environment

An increasing problem, 267
Air pollution, 183
Available habitat, 288
Bitter cold, 22

Capture-recapture method, 371
Carbon dioxide emission, 150
Cleaning up the river, 371
Depth and flow, 150
Diversity index, 555, 562
Factoring in the wind, 406
Finding river flow, 568
Going with the flow, 562
Infestation, 632
Probability of rain, 216
Record flood, 562
Recycling progress, 64
Solid waste, 64, 267, 342
Sonic boom, 610
Thinning eggshells, 183
Wildlife management, 567
World energy use, 150

Geometry

Angle, 236, 243
Area, 30, 288, 295, 321, 413
Cardboard box, 71
Diagonal, 396, 427, 464
Diameter, 72, 267
Fish tank, 71
Height, 71, 72, 443
Ice sculpture, 71
Parallelogram, 45
Parthenon, 45
Perimeter, 30, 81, 170, 208, 623
Radius, 72, 183, 396
Rectangle, 71, 81
Reflecting pool, 71
Second base, 71
Surface area of a cube, 428
Square, 45, 170, 525
Tale of two circles, 623
Triangle, 45, 81, 413
Volume, 295, 304, 413

Investment

Average annual return, 428
Best bond fund, 397
Best stock fund, 397
Big saver, 651
Buying stock, 151
Chocolate bars, 547
Comparing investments, 274
Compound interest, 539, 567
Diversification, 84, 223, 249
Doubling time, 547, 567
Financial independence, 84
Finding time, 71, 561

Golden years, 562
Growth rate, 555
Interest, 71, 208
Investing, 82
Outstanding performance, 539
Overdue loan payment, 397
Partial year, 540
Retirement fund, 651
Saving, 31, 274
Stocks, 274
Top stock, 547
Wealth-building portfolio, 372
World's largest mutual fund, 651

Science

Accident reconstruction, 509
Arecibo Observatory, 591
Altitude of a satellite, 72
Below sea level, 22
Bitter cold, 22
Comparing wind chills, 406
Distance to the sun, 267
Estimating armaments, 72, 372
Falling objects, 442, 516, 525
Female femurs, 118
Heating water, 150
Highs and lows, 22
Kepler's third law, 428
Marine navigation, 609
Measuring ocean depths, 570
Musical tones, 633
Orbit of Venus, 428
Orbits of the planets, 396
Popping corn, 446
Radius of the earth, 72
Resistance, 378, 516
Seacoast artillery, 482
Shock absorbers, 516
Siege and garrison artillery, 482
Skeletal remains, 118
Sound level, 548
Space travel, 267
Time, 267, 456
Using leverage, 516
World's largest telescope, 591

Sports

America's Cup, 406
Bicycle gear ratio, 516
Boxing match, 487
Cross-country cycling, 472
Decathlon champion, 488
Diving time, 406

First Super Bowl, 83
Flying high, 482
Football, 72, 632
Foul ball, 464
Maximum height, 183, 590
Maximum sail area, 396
Maximum sailing speed, 406
Mixed doubles, 83
Ping pong, 488
Pole vaulting, 456
Putting the shot, 482
Sailboat speed, 428
Sailboat stability, 427
Sky diving, 406
Super Bowl contender, 216
Tennis, 82, 322
Time of flight, 488
Velocity of a pop up, 130
World records, 111

Statistics/Demographics

Above the poverty level, 562
AIDS, 384
Average price, 364
Average speed, 364
Bachelor's degrees, 93
Below the poverty level, 562
Big family, 651
California growin', 397
Campaigning for governor, 371
Civilian labor force, 570
Explosive situation, 364
Fastest airliner, 37
Golden years, 330
Higher education, 103, 343
Imports and exports, 567
Life expectancy, 274, 282, 330
Logistic growth, 548
Master's degrees, 94
Population growth, 444, 540, 548
Predicting heights of preschoolers, 663
Public school enrollment, 64, 349
Racial balance, 364
Rising costs of health care, 443
Senior citizens, 103
Teacher's average salary, 64
The golden state, 349
Total construction, 171
Weighted average, 94

*To my wife and daughters,
Cheryl, Sarah, and Alisha*

P R E F A C E

Intermediate Algebra is designed to provide students with the algebra background needed for further college-level mathematics courses. The unifying theme of this text is the development of the skills necessary for solving equations and inequalities, followed by the application of those skills to solving applied problems. My primary goal in writing the third edition of *Intermediate Algebra* has been to retain the features that made the second edition so successful while incorporating the comments and suggestions of second-edition users. In addition, I have provided many new features that will help instructors to reach the goals that they have set for their students. As always, I endeavor to write texts that students can read, understand, and enjoy, and at the same time gain confidence in their ability to use mathematics. Although a complete development of each topic is provided in *Intermediate Algebra*, the text *Elementary Algebra* in this series would be more appropriate for students with no prior experience in algebra.

Content Changes

While the essence of the text remains, the topics have been rearranged and new features added to reflect the current needs of instructors who are teaching intermediate algebra courses.

- Graphing is covered earlier in the text. Graphing linear equations in two variables is now in Chapter 3 immediately following linear equations in one variable. An introduction to functions and their graphs is also included in Chapter 3.
- Functions are also covered earlier in the text. The phrase, “is a function of” can be found in Section 3.3. The definition of a function as an object is then given in Section 3.5.
- Systems of equations and inequalities (Chapter 4) is also covered earlier in the third edition.
- More emphasis is given to reading and understanding graphs. Exercises that involve graphs have been added to many sections of the text along with conceptual questions relating to the graphs.

In addition to these changes, the text and exercise sets have been carefully revised where necessary. Many new, applied examples have been added to the text and a large number of new, applied exercises included in the exercise sets. Particular care has been given to achieving an appropriate balance of problems that progressively increase in difficulty from routine exercises in the beginning of the set to more challenging exercises at the end of the set. As in earlier editions, fractions and decimals are used in the exercises and throughout the text discussions to help reinforce the basic arithmetic skills that are required for success in algebra.

Features

- Each chapter begins with a Chapter Opener that discusses a real application of algebra. The discussion is accompanied by a photograph, and in most cases by a real-data application graph that helps students to visualize algebra and more fully

to understand the concepts discussed in the chapter. In addition, each chapter contains a Math at Work feature, which profiles a real person and the mathematics that he or she uses on the job. These two features have corresponding real-data exercises.

- **NEW!** An increased emphasis on real-data applications that involve graphs is a focus for the third edition. Applications have been added throughout the text to help demonstrate concepts, to motivate students, and to give students practice using new skills. Many of the real-data exercises contain data obtained from the Internet. Internet addresses are provided as a resource for both students and teachers. Because Internet addresses frequently change, a list of addresses will also be available on the web site. An Index of Applications listing applications by subject matter is included at the front of the text.
- Every section begins with In This Section, a list of topics that shows the student what will be covered. Because the topics correspond to the headings within each section, students will find it easy to locate and study specific concepts.
- Important ideas, such as definitions, rules, summaries, and strategies, are set apart in boxes for quick reference. Color is used to highlight these boxes as well as other significant points in the text.
- **NEW!** The third edition contains three new margin features that appear throughout the text:

Calculator Close-ups give students an idea of how and when to use a graphing calculator. Some Calculator Close-ups simply introduce the features of a graphing calculator, whereas others enhance the understanding of algebraic concepts. For this reason many of the Calculator Close-ups will benefit even those students who do not use a graphing calculator. A graphing calculator is not required for studying from this text.

Study Tips are included in the margins throughout the text. These short tips are meant to reinforce continually good study habits and to keep reminding students that it is never too late to make improvements in the manner in which they study.

Helpful Hints are short comments that enhance the material in the text, provide another way of approaching a problem, or clear up misconceptions.

- At the end of every section there are Warm-up exercises, a set of ten simple statements that are to be answered true or false. These exercises are designed to provide a smooth transition between the ideas and the exercise sets. They help students to understand that every statement in mathematics is either true or false. They are also good for discussion or group work.
- **NEW!** Every section-ending exercise set in the third edition generally begins with six simple writing exercises. These exercises are designed to get students to review the definitions and rules of the section before doing more traditional exercises. For example, the student might be simply asked what properties of equality were discussed in this section.
- The end-of-section Exercises follow the same order as the textual material and contain exercises that are keyed to examples, as well as numerous exercises that are not keyed to examples. This organization allows the instructor to deal with only part of a section if necessary and to easily determine which exercises are appropriate to assign. The keyed exercises give the student a place to start practicing and

building confidence, whereas the nonkeyed exercises are designed to “wean” the student from following examples in a step-by-step manner. Getting More Involved exercises are designed to encourage writing, discussion, exploration, and cooperative learning. Graphing Calculator Exercises require a graphing calculator and are identified with a graphing calculator logo. Exercises for which a scientific calculator would be helpful are identified with a scientific calculator logo.

- Every chapter ends with a four-part Wrap-up, which includes the following:

The Chapter Summary lists important concepts along with brief illustrative examples.

NEW!

Enriching Your Mathematical Word Power appears at the end of each chapter and consists of multiple-choice questions in which the important terms are to be matched with their meanings. This feature emphasizes the importance of proper terminology.

The Review Exercises contain problems that are keyed to the sections of the chapter as well as numerous miscellaneous exercises.

The Chapter Test is designed to help the student assess his or her readiness for a test. The Chapter Test has no keyed exercises, thus enabling the student to work independently of the sections and examples.

- At the end of each chapter there is a Collaborative Activities feature that is designed to encourage interaction and learning in groups. Instructions and suggestions for using these activities and answers to all problems can be found in the Instructor’s Solutions Manual.
- The Making Connections exercises at the end of Chapters 2–12 are designed to help students review and synthesize the new material with ideas from previous chapters, and in some cases to review material necessary for success in the upcoming chapter. Every Making Connections exercise set includes at least one applied exercise that requires ideas from one or more of the previous chapters.

Coverage

For those who wish to cover more on functions, Chapter 9 can be covered after functions are introduced in Chapter 3. For those who wish to cover less on functions, Sections 3.5 and 3.6 can be omitted. Some or all of Chapter 4 can be omitted for those who desire a less extensive treatment of systems of linear equations. However, if you have a graphing calculator to do the determinants, Cramer’s rule with three variables is rather fun.

Supplements for the Instructor

ANNOTATED INSTRUCTOR’S EDITION

This ancillary includes answers to all exercises and tests. Each answer is printed next to each problem on the page where the problem appears. The answers are printed in a second color for ease of use by instructors.

PRINT AND COMPUTERIZED TEST BANK

The testing materials provide an array of formats that allow the instructor to create tests using both algorithmically generated test questions and those from a standard test bank. This testing system enables the instructor to choose questions either

manually or randomly by section, question type, difficulty level, and other criteria. Testing is available for IBM, IBM compatible, and Macintosh computers. Instructors can edit questions in the testing system as well if they seek a degree of customization. The print version of the test bank is softcover and provides questions found in the computerized version along with answer keys. Each chapter of the print version contains three different tests. Additionally, the print test bank contains four different, comprehensive final exams.

INSTRUCTOR'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, this supplement contains detailed, worked solutions to all of the exercises in the text. The solutions are done by the techniques used in the text. Instructions and suggestions for using the Collaborative Activities feature in the text are also included in the Instructor's Solutions Manual.

Supplements for the Student

STUDENT'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, the Student's Solutions Manual contains complete worked-out solutions to all of the odd-numbered exercises in the text. It also contains solutions for all exercises in the Chapter Tests. It may be purchased by your students from McGraw-Hill.

DUGOPOLSKI VIDEO SERIES

The video tape series contains instructional material and presents opportunities for students to work problems and to check their results. The tapes are text-specific and cover all chapters of the text. The tapes are facilitated by instructors who introduce topics and work through examples. Students are encouraged to work examples on their own and to check their results with those provided.

DUGOPOLSKI TUTORIAL CD-ROM

This interactive CD-ROM is a self-paced tutorial specifically linked to the text and reinforces topics through unlimited opportunities to review concepts and to practice problem solving. The CD-ROM contains text-, chapter-, and section-specific tutorials, multiple-choice questions with feedback, as well as algorithmically-generated questions. It requires virtually no computer training on the part of students and supports IBM and Macintosh computers.

In addition, a number of other technology and Web-based ancillaries are under development; they will support the ever-changing technology needs in developmental mathematics. For further information about these or any supplements, please contact your local McGraw-Hill sales representative.

Acknowledgments

First I thank all of the students and professors who used the previous editions of this text, for without their support there would not be a third edition. I sincerely appreciate the efforts of the reviewers who made many helpful suggestions:

Gisela Acosta, *Valencia Community College, East*
Francisco E. Alarcon, *Indiana University of Pennsylvania*

Dimos Arsenidis, *C.S.U.L.B.*
Luis A. Beltran, *Miami-Dade Community College*
Don Bigwood, *Bismarck State College*
Ray F. Brinker, *Western Illinois University*
Mary Jean Brod, *University of Montana*
Deborah Bryant, *Santa Rosa Junior College*
Kathleen A. Cantone, *Onondaga Community College*
Jack Carson, Jr., *Madison Area Technical College*
Oiyin Pauline Chow, *Harrisburg Area Community College*
Al Coons, *Pima Community College*
Victor M. Cornell, *Mesa Community College*
Diane Daniels, *Mississippi State University*
Gregory Davis, *University of Wisconsin, Green Bay*
Joe DiCostanzo, *Johnson County Community College*
Elizabeth L. Doane, *Housatonic Community Technical College*
Lacey P. Echols, *Butler University*
Sue W. Fader, *Delaware Technical and Community College, Stanton Campus*
Cynthia Fleck, *Wright State University*
Dr. Jeanette W. Glover, *University of Memphis*
Jacqueline R. Grace, *SUNY New Paltz*
Roberta Grenz, *Community College of Southern Nevada*
Patricia L. Hirschy, *Asnuntuck Comm-Tech College*
Daniel L. Hostetler, *Univ. College/University of Cincinnati*
Heidi A. Howard, *Florida Community College at Jacksonville*
Milia Ison, *DeAnza College*
Doris J. Jones, *Langston University*
Giles Wilson Maloof, *Boise State University*
John Martin, *Santa Rosa Junior College*
Kenneth J. Mead, *Genesee Community College*
Aaron Montgomery, *Purdue University, North Central*
Christina Morian, *Lincoln University*
Daniel P. Munton, *Santa Rosa Junior College*
Jim Neary, *Ivy Tech State College*
Masood Poorandi, *Bethune Cookman College*
Christopher P. Reisch, *SUNY Buffalo*
Togba C. Sapolucia, *Northeast Houston Community College*
Fred Schineller, *Arizona State University*
Patricia L. Schulte, *Penn State University*
Mark Serebransky, *Camden County College*
Ann Sitomer, *Grossmont College*
Lourdes Triana, *Humboldt State University*
Paul J. Welsh, *Pima Community College-East Campus*
Albert E. White, *Lincoln University*
Jackie Wing, *Angelina Jr. College*

Dr. Judith B. Wood, *Central Florida Community College*
Vivian J. Zabrocki, *Montana State University, Billings.*

I would also like to thank those who reviewed the second edition.

Nancy Angle, *Cerritos College*
Chris Barker, *DeAnza College*
Richard Basic, *Lakeland Community College*
Richard A. Butterworth, *Massasoit Community College*
Nancy Carpenter, *Johnson County Community College*
Florence Chambers, *Joliet Junior College*
Irene Doo, *Austin Community College*
David Dudley, *Phoenix College*
James Fryxell, *College of Lake County*
Terry Fung, *Kean College*
Jane Hammontree, *Tulsa Junior College*
Robert A. Hawes, *Northern Essex Community College*
Dale Hughes, *Johnson County Community College*
Kathy Kepner, *Paducah Community College*
Judith Lenk, *Ocean County College*
Mitchel Levy, *Broward Community College*
Joy McMullen, *Lakeland Community College*
Gael Mericle, *Mankato State University*
Jane Morrison, *South Suburban Community College*
Linda Padilla, *Joliet Junior College*
Sue Parsons, *Cerritos College*
Joanne Peebles, *El Paso Community College*
Rose L. Pugh, *Bellevue Community College*
Scott Reed, *College of Lake County*
Fred Russell, *Charles County Community College*
Howard Sorken, *Broward Community College*
Patricia Stanley, *Ball State University*
Eric Stietzel, *Foothill College*
Diane Tesar, *South Suburban College*
Charles Waiveris, *Central Connecticut State University*

I also thank Edgar Reyes of Southeastern Louisiana University for his help with the CD-ROM and Rebecca Muller of Southeastern Louisiana University for her work on the printed test bank. Finally, my thanks go to Laurel Technical Services for error checking the manuscript. I thank the staff at McGraw-Hill for all of their help and encouragement throughout the revision process. Special thanks go to Bill Barter and Erin Brown. I also want to express my sincere appreciation to my wife, Cheryl, for her invaluable patience and support.

Hammond, Louisiana

M.D.

Rational Expressions



Information is everywhere—in the newspapers and magazines we read, the televisions we watch, and the computers we use. And now people are talking about the Information Superhighway, which will deliver vast amounts of information directly to consumers' homes. In the future the combination of telephone, television, and computer will give us on-the-spot health care recommendations, video conferences, home shopping, and perhaps even electronic voting and driver's license renewal, to name just a few. There is even talk of 500 television channels!

Some experts are concerned that the consumer will give up privacy for this technology. Others worry about regulation, access, and content of the enormous international computer network.

Whatever the future of this technology, few people understand how all their electronic devices work. However, this vast array of electronics rests on physical principles, which are described by mathematical formulas. In Exercises 49 and 50 of Section 6.6 we will see that the formula governing resistance for receivers connected in parallel involves rational expressions, which are the subject of this chapter.

Chapter Opener

Each **chapter opener** features a real-world situation that can be modeled using mathematics. Each chapter contains exercises that relate back to the chapter opener.

42. Trimming hedges. Lourdes can trim the hedges around her property in 8 hours by using an electric hedge trimmer. Rafael can do the same job in 15 hours by using a manual trimmer. How long would it take them to trim the hedges working together?

43. Filling the tub. It takes 10 minutes to fill Alisha's bathtub and 12 minutes to drain the water out. How long would it take to fill it with the drain accidentally left open?

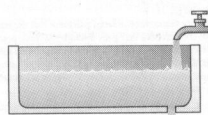


FIGURE FOR EXERCISE 43

44. Eating machine. Charles can empty the cookie jar in $1\frac{1}{2}$ hours. It takes his mother 2 hours to bake enough cookies to fill it. If the cookie jar is full when Charles comes home from school, and his mother continues baking and restocking the cookie jar, then how long will it take him to empty the cookie jar?

45. Filing the invoices. It takes Gina 90 minutes to file the monthly invoices. If Hilda files twice as fast as Gina does, how long will it take them working together?

46. Painting alone. Julie can paint a fence by herself in 12 hours. With Betsy's help, it takes only 5 hours. How long would it take Betsy by herself?

47. Buying fruit. Molly bought \$5.28 worth of oranges and \$8.80 worth of apples. She bought 2 more pounds of oranges than apples. If apples cost twice as much per pound as oranges, then how many pounds of each did she buy?

48. Raising rabbits. Luke raises rabbits and raccoons to sell for meat. The price of raccoon meat is three times the price of rabbit meat. One day Luke sold 160 pounds of meat, \$72 worth of each type. What is the price per pound of each type of meat?

49. Total resistance. If two receivers with resistances R_1 and R_2 are connected in parallel, then the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

relates the total resistance for the circuit R with R_1 and R_2 . Given that R_1 is 3 ohms and R is 2 ohms, find R_2 .

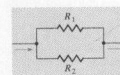


FIGURE FOR EXERCISE 49

50. More resistance. Use the formula from Exercise 49 to find R_1 and R_2 given that the total resistance is 1.2 ohms and R_1 is 1 ohm larger than R_2 .

51. Las Vegas vacation. Brenda of Horizon Travel has arranged for a group of gamblers to share the \$24,000 cost of a charter flight to Las Vegas. If Brenda can get 40 more people to share the cost, then the cost per person will decrease by \$100.

a) How many people were in the original group?
b) Write the cost per person as a function of the number of people sharing the cost.

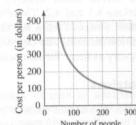


FIGURE FOR EXERCISE 51

52. White-water rafting. Adventures, Inc. has a \$1,500 group rate for an overnight rafting trip on the Colorado River. For the last trip five people failed to show, causing the price per person to increase by \$25. How many were originally scheduled for the trip?

53. Doggie bag. Muffy can eat a 25-pound bag of dog food in 28 days, whereas Missy eats a 25-pound bag in 23 days. How many days would it take them together to finish a 50-pound bag of dog food.

54. Rodent food. A pest control specialist has found that 6 rats can eat an entire box of sugar-coated breakfast cereal in 13.6 minutes, and it takes a dozen mice 34.7 minutes to devour the same size box of cereal. How long would it take all 18 rodents, in a cooperative manner, to finish off a box of cereal?

Margin Notes

Margin notes include **Helpful Hints**, **Study Tips**, and **Calculator Close-ups**. The *Helpful Hints* point out common errors or reminders. The *Study Tips* provide practical suggestions for improving study habits. The optional *Calculator Close-ups* provide tips on using a graphing calculator to aid in your understanding of the material. They also include insightful suggestions for increasing calculator proficiency.

calculator

close-up

These Calculator Close-ups are designed to help reinforce the concepts of algebra, not replace them. Do not rely too heavily on your calculator or use it to replace the algebraic methods taught in this course.

Graphing on the Number Line

To construct a number line, we draw a straight line and label any convenient point with the number 0. Now we choose any convenient length and use it to locate points to the right of 0 as points corresponding to the positive integers and points to the left of 0 as points corresponding to the negative integers. See Fig. 1.4. The numbers corresponding to the points on the line are called the **coordinates** of the points. The distance between two consecutive integers is called a **unit**, and it is the same for any two consecutive integers. The point with coordinate 0 is called the **origin**. The numbers on the number line increase in size from left to right. When we compare the size of any two numbers, the larger number lies to the right of the smaller one on the number line.

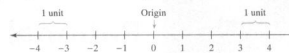


FIGURE 1.4

It is often convenient to illustrate sets of numbers on a number line. The set of integers, J , is illustrated or **graphed** as in Fig. 1.5. The three dots to the right and left on the number line indicate that the integers go on indefinitely in both directions.



FIGURE 1.5

EXAMPLE 2

Graphing on the number line

List the elements of each set and graph each set on a number line.

- $\{x \mid x \text{ is a whole number less than } 4\}$
- $\{a \mid a \text{ is an integer between } 3 \text{ and } 9\}$
- $\{y \mid y \text{ is an integer greater than } -3\}$

Solution

- The whole numbers less than 4 are 0, 1, 2, and 3. Figure 1.6 shows the graph of this set.



FIGURE 1.6

- The integers between 3 and 9 are 4, 5, 6, 7, and 8. The graph is shown in Fig. 1.7.



FIGURE 1.7

- The integers greater than -3 are -2, -1, 0, 1, and so on. To indicate the continuing pattern, we use a series of dots on the graph in Fig. 1.8.



FIGURE 1.8

- Three-digit number.** The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the old number. If the hundreds digit plus twice the tens digit is equal to the units digit, then what is the number?
- Working overtime.** To make ends meet, Ms. Farnsby works three jobs. Her total income last year was \$48,000. Her income from teaching was just \$6,000 more than her income from house painting. Royalties from her textbook sales were one-seventh of the total money she received from teaching and house painting. How much did she make from each source last year?
- Pocket change.** Harry has \$2.25 in nickels, dimes, and quarters. If he had twice as many nickels, half as many dimes, and the same number of quarters, he would have \$2.50. If he has 27 coins altogether, then how many of each does he have?

GETTING MORE INVOLVED

- Exploration.** Draw diagrams showing the possible ways to position three planes in three-dimensional space.
- Discussion.** Make up a system of three linear equations in three variables for which the solution set is $\{(0, 0, 0)\}$. A system with this solution set is called a **homogeneous** system. Why do you think it is given that name?
- Cooperative learning.** Working in groups, do parts (a)–(d) below. Then write a report on your findings.
 - Find values of a , b , and c so that the graph of $y = ax^2 + bx + c$ goes through the points $(-1, -2)$, $(1, 0)$, and $(2, 7)$.
 - Arbitrarily select three ordered pairs and find the equation of the parabola that goes through the three points.
 - Could more than one parabola pass through three given points? Give reasons for your answer.
 - Explain how to pick three points for which no parabola passes through all of them.

4.4 SOLVING LINEAR SYSTEMS USING MATRICES

In this section

- Matrices
- The Augmented Matrix
- The Gaussian Elimination Method
- Inconsistent and Dependent Equations

You solved linear systems in two variables by substitution and addition in Sections 4.1 and 4.2. Those methods are done differently on each system. In this section you will learn the Gaussian elimination method, which is related to the addition method. The Gaussian elimination method is performed in the same way on every system. We first need to introduce some new terminology.

Matrices

A **matrix** is a rectangular array of numbers. The **rows** of a matrix run horizontally, and the **columns** of a matrix run vertically. A matrix with m rows and n columns has **order** $m \times n$ (read “ m by n ”). Each number in a matrix is called an **element** or **entry** of the matrix.

EXAMPLE 1

Order of a matrix

Determine the order of each matrix.

- $\begin{bmatrix} -1 & 2 \\ 5 & \sqrt{2} \\ 0 & 3 \end{bmatrix}$
- $\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 0 & 2 \end{bmatrix}$
- $[1 \ 3 \ 6]$

Solution

Because matrix (a) has 3 rows and 2 columns, its order is 3×2 . Matrix (b) is a 2×2 matrix, matrix (c) is a 3×3 matrix, and matrix (d) is a 1×3 matrix.

The Augmented Matrix

The solution to a system of linear equations such as

$$\begin{aligned} x - 2y &= -5 \\ 3x + y &= 6 \end{aligned}$$

study tip

As soon as possible after class, find a quiet place and work on your homework. The longer you wait, the harder it is to remember what happened in class.

8.1 FACTORING AND COMPLETING THE SQUARE

In this section

- Review of Factoring
- Review of the Even-Root Property
- Completing the Square
- Miscellaneous Equations
- Imaginary Solutions

Factoring and the even-root property were used to solve quadratic equations in Chapters 5, 6, and 7. In this section we first review those methods. Then you will learn the method of completing the square, which can be used to solve any quadratic equation.

Review of Factoring

A quadratic equation is a second-degree polynomial equation of the form

$$ax^2 + bx + c = 0,$$

where a , b , and c are real numbers with $a \neq 0$. If the second-degree polynomial on the left-hand side can be factored, then we can solve the equation by breaking it into two first-degree polynomial equations (linear equations) using the following strategy.

Strategy for Solving Quadratic Equations by Factoring

1. Write the equation with 0 on the right-hand side.
2. Factor the left-hand side.
3. Use the zero factor property to set each factor equal to zero.
4. Solve the simpler equations.
5. Check the answers in the original equation.

EXAMPLE 1

Solving a quadratic equation by factoring

Solve $3x^2 - 4x = 15$ by factoring.

Solution

Subtract 15 from each side to get 0 on the right-hand side:

$$\begin{array}{rcl} 3x^2 - 4x - 15 & = & 0 \\ (3x + 5)(x - 3) & = & 0 \quad \text{Factor the left-hand side.} \\ 3x + 5 = 0 & \text{or} & x - 3 = 0 \quad \text{Zero factor property} \\ 3x = -5 & \text{or} & x = 3 \\ x = -\frac{5}{3} & & \end{array}$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$. Check the solutions in the original equation. ■

Review of the Even-Root Property

In Chapter 7 we solved quadratic equations by using the even-root property.

Strategy Boxes

The **strategy boxes** generally provide a numbered list of concepts from a section or a set of steps to follow in problem solving. They can be used by students who prefer a more structured approach to problem solving or used as a study tool to review important points within sections.

Math at Work

The **Math at Work** feature that appears in each chapter explores the careers of individuals who use the mathematics presented in the chapter in their work. Students are referred to exercises that directly relate to the occupation highlighted in Math at Work.

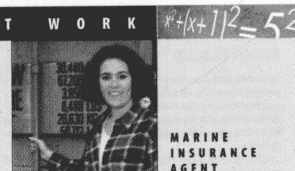
6.5 Solving Equations Involving Rational Expressions

(6-37) 369

MATH AT WORK

Cargo has been lost, or the hull of a ship has been damaged. What is the amount of money that should be paid to the insured party? Lisa M. Paccione, Ocean Marine Claim Representative for the St. Paul Insurance Company, investigates, evaluates, resolves, and pays these types of claims. Ms. Paccione does this by gathering data, occasionally doing a visual inspection, interviewing witnesses, and negotiating with attorneys.

Decisions about losses are based on the insured party's individual policy as well as traditional marine practices and maritime law. When consignees suffer a cargo loss, they not only are compensated for the actual amount of the damaged goods, but also receive an additional "advance" in the settlement. Customarily, the advance is 10% over the value of the goods. The amount that St. Paul pays the insured party for a valid claim is computed by using a proportion. In Exercises 59 and 60 of this section you will solve problems involving this proportion.



EXAMPLE 8 Ratios and proportions

The ratio of men to women at a football game was 4 to 3. If there were 12,000 more men than women in attendance, then how many men and how many women were in attendance?

Solution

Let x represent the number of men in attendance and $x - 12,000$ represent the number of women in attendance. Because the ratio of men to women was 4 to 3, we can write the following proportion:

$$\begin{aligned} \frac{4}{3} &= \frac{x}{x - 12,000} \\ 4x - 48,000 &= 3x \\ x &= 48,000 \end{aligned}$$

So there were 48,000 men and 36,000 women at the game. ■



WARM-UPS

True or false? Explain.

1. In solving an equation involving rational expressions, multiply each side by the LCD for all of the denominators.
2. To solve $\frac{1}{x} + \frac{1}{2x} = \frac{1}{3}$, first change each rational expression to an equivalent rational expression with a denominator of $6x$.
3. Extraneous roots are not real numbers.
4. To solve $\frac{1}{x-2} + 3 = \frac{1}{x+2}$, multiply each side by $x^2 - 4$.
5. The solution set to $\frac{1}{3x+4} - \frac{6}{2x+1} = \frac{7}{5}$ is $\left\{-\frac{4}{3}, -\frac{1}{2}\right\}$.

Warm-ups

Warm-ups appear before each set of exercises at the end of every section. They are true or false statements that can be used to check conceptual understanding of material within each section.

Because there are no real even roots of negative numbers, the expressions

$$a^{1/2}, x^{-3/4}, \text{ and } y^{1/6}$$

are not real numbers if the variables have negative values. To simplify matters, we sometimes assume the variables represent only positive numbers when we are working with expressions involving variables with rational exponents. That way we do not have to be concerned with undefined expressions and absolute value.

EXAMPLE 8 Expressions involving variables with rational exponents

Use the rules of exponents to simplify the following. Write your answers with positive exponents. Assume all variables represent positive real numbers.

a) $x^{2/3}x^{4/3}$

b) $\frac{a^{1/2}}{a^{3/4}}$

c) $(x^{1/2}y^{-3})^{1/2}$

d) $\left(\frac{x^2}{y^{1/3}}\right)^{-1/2}$

Solution

a) $x^{2/3}x^{4/3} = x^{6/3}$
 $= x^2$

Use the product rule to add the exponents.
 Reduce the exponent.

b) $\frac{a^{1/2}}{a^{3/4}} = a^{1/2-3/4}$
 $= a^{-1/4}$

Use the quotient rule to subtract the exponents.

c) $(x^{1/2}y^{-3})^{1/2} = (x^{1/2})^{1/2}(y^{-3})^{1/2}$
 $= x^{1/4}y^{-3/2}$
 $= \frac{x^{1/4}}{y^{3/2}}$

Simplify.

Power of a product rule

Power of a power rule

Definition of negative exponent

d) Because this expression is a negative power of a quotient, we can first find the reciprocal of the quotient, then apply the power of a power rule:

$$\left(\frac{x^2}{y^{1/3}}\right)^{-1/2} = \left(\frac{y^{1/3}}{x^2}\right)^{1/2} = \frac{y^{1/6}}{x} = \frac{1}{x} \cdot \frac{1}{y^{1/6}}$$

WARM-UPS

True or false? Explain.

1. $4^{-1/2} = \frac{1}{2}$

2. $16^{1/2} = 8$

3. $(3^{2/3})^3 = 9$

4. $8^{-2/3} = -4$

5. $2^{1/2} \cdot 2^{1/2} = 2$

6. $\left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$

7. $\frac{3}{3^{1/2}} = 3^{1/2}$

8. $(2^2)^{1/2} = 2^3$

9. $3^{1/3} \cdot 6^{1/3} = 18^{2/3}$

10. $2^{3/4} \cdot 2^{1/4} = 4$

COLLABORATIVE ACTIVITIES

Beorg's Business

In manufacturing or other businesses in which time is money and tasks are easily shared, problems involving work appear. An owner or manager who wants to know how to bid a job often develops a table of times needed to complete the job as determined by how much work is required and who could be assigned to the job.

Beorg owns a kaleidoscope-manufacturing company with two employees, Scott and Salina. It takes Scott one hour to make one kaleidoscope, and it takes Salina $\frac{1}{2}$ hour to make one kaleidoscope. Beorg wants to know how long it would take to complete a certain number of kaleidoscopes. Using the information given and answering the questions below, fill in the following table for Beorg.

Name of Employee	Time for one kaleidoscope	Time for 20 kaleidoscopes
Scott	1 hr	
Salina	$\frac{1}{2}$ hr	
Scott & Salina		
Sammy		
Scott & Sammy	$\frac{3}{4}$ hr	
Salina & Sammy		
Scott, Salina, & Sammy		

Grouping: Four students per group

Topic: Applications of work problems

- How long will it take Scott and Salina working together to make one kaleidoscope?
- Beorg hires a third person, Sammy, and has him and Scott make one kaleidoscope. Working together, it takes them $\frac{1}{2}$ hour to make one kaleidoscope. How long would it take Sammy by himself to make one kaleidoscope?
- How long would it take Salina and Sammy working together to make one kaleidoscope? How long would it take for all three working together?
- Now Beorg wants to finish his time table. He would like to have 20 kaleidoscopes completed each day.

Extension: Is Sammy in the combination or combinations you found in the last question? Is it worth having Sammy work? Remember that when someone is starting a new job, he or she may work more slowly until he or she learns how to do the job more efficiently. Find out how fast Sammy would need to work for production to double (40 kaleidoscopes in an 8-hour day).

WRAP-UP

CHAPTER 6

SUMMARY

Rational Expressions

Rational expression The ratio of two polynomials with the denominator not equal to zero

Domain of a rational expression The set of all possible numbers that can be used as replacements for the variable

Examples

$$\frac{x^2 - 1}{2x - 3}$$

$$D = \left\{ x \mid x \neq \frac{3}{2} \right\}$$

Collaborative Activities

Collaborative Activities appear at the end of each chapter. The activities are designed to encourage interaction and learning in a group setting.