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PREFACE

The eleventh volume of the series, *Control and Dynamic Systems: Advances in Theory and Applications*, continues in the purpose of this serial publication in bringing together diverse information on important progress in the field of control and dynamic systems theory and applications as achieved and presented by leading contributors. As pointed out in the two previous volumes, the retitling of this series reflects the growing emphasis on applications to large scale systems and decision making in addition to the more traditional, but still more important areas of endeavor, in this very broad field.

This volume begins with a contribution by Patrick L. Smith which explores a number of important issues with respect to the modeling of a dynamic system, the beginning point for the resolution of the system synthesis problem. Issues with respect to the utilization of the Kalman filter as a concise model for the identification of a large class of dynamic systems are explored. Computational and convergence issues are defined with a view to reducing computational requirements. The application of the techniques in this contribution to nonlinear system representations is explored.

Computer aided design techniques have been applied to a number of areas of engineering. It is most appropriate to do so also to control engineering problems. The second contribution by Page and Stear deals with this broad issue. In order to do so a nonlinear functional which is a function of all the required system specifications is formulated, and functional minimization techniques are applied to it in order to design the feedback control system automatically. The various issues with respect to functional minimization techniques as they relate to the computer aided control design problem are addressed and conclusions reached. The power and utility of this rather generic approach are then illustrated by its application to several examples.

In the earlier phases of modern control technology the applicational issues tended to be rather simplistic when compared to the more complex systems control technology is asked to address today. With this trend toward more complex systems, there follows a requirement for the development of more efficient algorithmic techniques for the analysis and synthesis of these more complex classes of systems. The next contribution by Ronald D. Sugar deals with some rather powerful techniques in this direction, multilevel systems optimization techniques. Multilevel systems techniques may be used to decompose a large or complex system into a collection of smaller interrelated sybsystems, and then coordinate the solutions of the individual subsystems in such a way as to achieve optimal performance for the overall system. The power and utility of these techniques are illustrated by their application to a rather complex systems problem.

There has been an enormous amount of effort on the international scene devoted to system filtering techniques used in the design of a control system. A problem of considerable import is the determination of just how well a given control system design really does perform, and one of the techniques here is system smoothing methods. In particular, in order to develop the ultimate in precision in the analysis of a given control system design, one must resort to nonlinear smoothing techniques. The next contribution by John B. Peller addresses this significant and complex area. The derivation of the dynamic equations for nonlinear smoothing is developed and reduced to the linear smoother case, confirming results obtained earlier. Illustrative applications and practical approximation techniques are presented.

The field of differential game theory used to describe the competitive situations which abound in society is still in its infancy. Yet because of its essential importance in the dynamic decision making process involved in these many instances, the evolutionary development of technology in this important field will be highly motivated. The next contribution by L. C. Westphal embodies a number of fundamental issues in this broad field and presents numerous new basic results. It represents in its totality an important step forward in the development of techniques in this most important and challenging field.

This volume closes with an overview of the evolutionary growth of Soviet contributions to control theory as viewed by a man who played a vital role in so many of these developments in the Soviet Union, Alexander Ya. Lerner.

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CONTENTS

Fitting Multistage Models to Input/Output Data

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NOMENCLATURE

b	Covariance of the measurement residual
$\hat{b}$	Estimate of b
$F(i), L(i)$	$3n-1$ by n matrices
$f_1(u(i);\underline{\alpha})$ $f_2(z(i);\underline{\beta})$	No memory nonlinear functions of the input $u(i)$ and the output $z(i)$
$\underline{h}$	Output matrix of dimension $n \times 1$
I_n	Identity matrix of dimension $n \times n$
$J(\underline{\theta})$	Constrained least-squares cost functional
$\underline{k}$	Steady-state Kalman filter gain of dimension $n \times 1$
$\underline{k}'$	Reduced Kalman gain vector of dimension $(n-1) \times 1$

M	Steady-state covariance of $\hat{\underline{x}}'(i)$
$M(1)$	Covariance of $\underline{x}(1)$
m	Integer number of delays on input
N	Total number of measurements
$N_n(\underline{m}, Q)$	Multivariate normal distribution with mean $\underline{m}$ and covariance Q
n	Dimension of the state vector $\underline{x}(i)$
P	Steady-state covariance of $\hat{\underline{x}}'(i)$
Q	Covariance of $\underline{r}(i)$
R	Measurement error covariance
$\underline{r}(i)$	Random input vector of dimension n
$\underline{s}(k+1)$	$3n-1$ vector
T	Transition matrix for the adjoint process
$u(i)$	Measurement of the input
$\underline{v}$	Composite vector of measurement innovations
$v(i)$	Measurement innovation
$w(i)$	Measurement error
$\hat{\underline{x}}'(i)$	One-step-ahead predicted estimate of the state vector
$\hat{\underline{x}}(i)$	Filtered estimate of the state vector
$\underline{x}(i)$	$n \times 1$ vector of state variables
$z(i)$	Measurement of the output
$\underline{\alpha}, \underline{\beta}$	Unknown parameters in f_1 and f_2
$\underline{\gamma}$	Input matrix of dimension $n \times 1$
$\underline{\theta}$	Unknown parameter vector of dimension $(3n-1) \times 1$
$\hat{\underline{\theta}}$	Estimate of $\underline{\theta}$
$\underline{\lambda}(i)$	Lagrange multiplier of dimension $n \times 1$
$\hat{\rho}_j$	Estimate of the lag j autocorrelation coefficient of measurement residuals
$p(\underline{v} \mid \underline{\theta})$	Probability density function of the innovations
Φ	State transition matrix
$\underline{\varphi}$	State transition parameter vector of dimension n

$\underline{\varphi}'$	Reduced state transition parameter vector of dimension n - 1
$\omega(i)$	Random input for Box and Jenkins model in the example

I. INTRODUCTION

It is assumed in this study that the ultimate objective of modeling a dynamic system is to predict or control the output of the system by observing or manipulating the inputs. In concrete terms the model is a digital computer program, which, when supplied the measurements of the past and present input and output, computes the predicted future output of the system. The random nature of the problem is considered in developing the model, but the model itself is a completely deterministic system. System characterization and system identification are the principal aspects of modeling. System characterization is concerned with defining a class of mathematical models and system identification with the determination of the specific model belonging to this preselected class which best fits the observations.

The class of models examined in this study are linear stationary multistage processes. The usefulness and convenience of linear models are well known and many techniques have been proposed to fit linear models to input/output data ([1] to [7], for example). In fact, because of the many publications in this area, the main contributions of this study are listed below:

(a) The class of Kalman filter models developed by Mehra [7] for free linear systems is extended to forced linear systems and to the specific problem of fitting models to input/output data.

(b) A recursive form for the gradient of the likelihood function is derived which greatly reduces the computer memory requirements.

(c) The numerical problems resulting from a singularity in the gradient of the likelihood function for the Kalman filter representation are eliminated by rescaling the likelihood function.

(d) The direct application of the results obtained in this study to a class of nonlinear system representations is shown.

II. LINEAR MODELS

The following is a list of comments and assumptions which describe the class of models examined in this study:

(a) N simultaneous measurements of a scalar input sequence and scalar output sequence of an isolated system are made at uniformly spaced instants of time and are denoted $\{u(i): i=1,\ldots,N\}$ and $\{z(i): i=1,\ldots,N\}$, respectively.

(b) The measurements are assumed to be error-free.

(c) The measurements are assumed to be generated by a multistage time-invariant linear process of order n which is driven by both the measured inputs and unmeasured inputs.

(d) The unmeasured inputs are assumed to be mutually independent random variables which are identically distributed and independent of the measured input and output.

(e) The prediction ability of the model may be degraded for any of the following reasons:

1. Actual random input disturbances may be present.
2. The system may not be a linear, time-invariant multistage process, as assumed in Paragraph (c).
3. There may be measurement errors.
4. There may be errors in identifying the parameters