

# **CONTROL AND DYNAMIC SYSTEMS**

*Advances in Theory  
and Applications*

**Volume 11**

# CONTROL AND DYNAMIC SYSTEMS

ADVANCES IN THEORY  
AND APPLICATIONS

*Edited by*

C. T. LEONDES

DEPARTMENT OF ENGINEERING  
UNIVERSITY OF CALIFORNIA  
LOS ANGELES, CALIFORNIA

VOLUME 11 1974



ACADEMIC PRESS New York and London

A Subsidiary of Harcourt Brace Jovanovich, Publishers

COPYRIGHT © 1974, BY ACADEMIC PRESS, INC.

ALL RIGHTS RESERVED.

NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS, INC.

111 Fifth Avenue, New York, New York 10003

*United Kingdom Edition published by*

ACADEMIC PRESS, INC. (LONDON) LTD.

24/28 Oval Road, London NW1

**LIBRARY OF CONGRESS CATALOG CARD NUMBER: 64-8027**

**PRINTED IN THE UNITED STATES OF AMERICA**

## CONTRIBUTORS

**Alexander Ya. Lerner** (491), Department of Applied Mathematics, Weizmann Institute of Science, Rehovot, Israel

**J. A. Page** (25), Department of Electrical Engineering, Loyola Marymount University, Los Angeles, California

**John B. Peller** (255), North American Rockwell Corporation, 12214 Lakewood Boulevard, Downey, California

**Patrick L. Smith** (1), The Aerospace Corporation, El Segundo, California

**E. B. Stear** (25), Department of Electrical Engineering, University of California, Santa Barbara, California

**Ronald D. Sugar** (145), Hughes Aircraft Company, Space and Communications Group, El Segundo, California

**L. C. Westphal** (389), School of Engineering and Applied Science, University of California, Los Angeles, California 90024

## PREFACE

The eleventh volume of the series, *Control and Dynamic Systems: Advances in Theory and Applications*, continues in the purpose of this serial publication in bringing together diverse information on important progress in the field of control and dynamic systems theory and applications as achieved and presented by leading contributors. As pointed out in the two previous volumes, the retitling of this series reflects the growing emphasis on applications to large scale systems and decision making in addition to the more traditional, but still more important areas of endeavor, in this very broad field.

This volume begins with a contribution by Patrick L. Smith which explores a number of important issues with respect to the modeling of a dynamic system, the beginning point for the resolution of the system synthesis problem. Issues with respect to the utilization of the Kalman filter as a concise model for the identification of a large class of dynamic systems are explored. Computational and convergence issues are defined with a view to reducing computational requirements. The application of the techniques in this contribution to nonlinear system representations is explored.

Computer aided design techniques have been applied to a number of areas of engineering. It is most appropriate to do so also to control engineering problems. The second contribution by Page and Stear deals with this broad issue. In order to do so a nonlinear functional which is a function of all the required system specifications is formulated, and functional minimization techniques are applied to it in order to design the feedback control system automatically. The various issues with respect to functional minimization techniques as they relate to the computer aided control design problem are addressed and conclusions reached. The power and utility of this rather generic approach are then illustrated by its application to several examples.

In the earlier phases of modern control technology the applicational issues tended to be rather simplistic when compared to the more complex systems control technology is asked to address today. With this trend toward more complex systems, there follows a requirement for the development of more efficient algorithmic techniques for the analysis and synthesis of these more complex classes of systems. The next contribution by Ronald D. Sugar deals with some rather powerful techniques in this direction, multilevel systems optimization techniques. Multilevel systems techniques may be used to decompose a large or complex system into a collection of smaller interrelated subsystems, and then coordinate the solutions of the individual subsystems in such a way as to achieve optimal performance for the overall system. The power and utility of these techniques are illustrated by their application to a rather complex systems problem.

## PREFACE

There has been an enormous amount of effort on the international scene devoted to system filtering techniques used in the design of a control system. A problem of considerable import is the determination of just how well a given control system design really does perform, and one of the techniques here is system smoothing methods. In particular, in order to develop the ultimate in precision in the analysis of a given control system design, one must resort to nonlinear smoothing techniques. The next contribution by John B. Peller addresses this significant and complex area. The derivation of the dynamic equations for nonlinear smoothing is developed and reduced to the linear smoother case, confirming results obtained earlier. Illustrative applications and practical approximation techniques are presented.

The field of differential game theory used to describe the competitive situations which abound in society is still in its infancy. Yet because of its essential importance in the dynamic decision making process involved in these many instances, the evolutionary development of technology in this important field will be highly motivated. The next contribution by L. C. Westphal embodies a number of fundamental issues in this broad field and presents numerous new basic results. It represents in its totality an important step forward in the development of techniques in this most important and challenging field.

This volume closes with an overview of the evolutionary growth of Soviet contributions to control theory as viewed by a man who played a vital role in so many of these developments in the Soviet Union, Alexander Ya. Lerner.

## CONTENTS OF PREVIOUS VOLUMES

### Volume 3

Guidance and Control of Reentry and Aerospace Vehicles

*Thomas L. Gunckel, II*

Two-Point Boundary-Value-Problem Techniques

*P. Kenneth and R. McGill*

The Existence Theory of Optimal Control Systems

*W. W. Schmaedeke*

Application of the Theory of Minimum-Normed Operators to Optimum-Control-System Problems

*James M. Swiger*

Kalman Filtering Techniques

*H. W. Sorenson*

Application of State-Space Methods to Navigation Problems

*Stanley F. Schmidt*

Author Index—Subject Index

### Volume 4

Algorithms for Sequential Optimization of Control Systems

*David Isaacs*

Stability of Stochastic Dynamical Systems

*Harold J. Kushner*

Trajectory Optimization Techniques

*Richard E. Kopp and H. Gardner Moyer*

Optimum Control of Multidimensional and Multilevel Systems

*R. Kulikowski*

Optimal Control of Linear Stochastic Systems with Complexity Constraints

*Donald E. Johansen*

Convergence Properties of the Method of Gradients

*Donald E. Johansen*

Author Index—Subject Index

## CONTENTS OF PREVIOUS VOLUMES

### Volume 1

On Optimal and Suboptimal Policies in Control Systems

*Masanao Aoki*

The Pontryagin Maximum Principle and Some of Its Applications

*James J. Meditch*

Control of Distributed Parameter Systems

*P. K. C. Wang*

Optimal Control for Systems Described by Difference Equations

*Hubert Halkin*

An Optimal Control Problem with State Vector Measurement Errors

*Peter R. Schultz*

On Line Computer Control Techniques and Their Application to Reentry  
Aerospace Vehicle Control

*Francis H. Kishi*

Author Index—Subject Index

### Volume 2

The Generation of Liapunov Functions

*D. G. Schultz*

The Application of Dynamic Programming to Satellite Intercept and Rendezvous  
Problems

*F. T. Smith*

Synthesis of Adaptive Control Systems by Function Space Methods

*H. C. Hsieh*

Singular Solutions in Problems of Optimal Control

*C. D. Johnson*

Several Applications of the Direct Method of Liapunov

*Richard Allison Nesbit*

Author Index—Subject Index

## CONTENTS OF PREVIOUS VOLUMES

### Volume 5

Adaptive Optimal Steady State Control of Nonlinear Systems

*Allan E. Pearson*

An Initial Value Method for Trajectory Optimization Problems

*D. K. Scharmack*

Determining Reachable Regions and Optimal Controls

*Donald R. Snow*

Optimal Nonlinear Filtering

*J. R. Fischer*

Optimal Control of Nuclear Reactor Systems

*D. M. Wiberg*

On Optimal Control with Bounded State Variables

*John McIntyre and Bernard Paiewonsky*

Author Index—Subject Index

### Volume 6

The Application of Techniques of Artificial Intelligence to Control System Design

*Jerry M. Mendel and James J. Zapalac*

Controllability and Observability of Linear, Stochastic, Time-Discrete Control Systems

*H. W. Sorenson*

Multilevel Optimization Techniques with Application to Trajectory Decomposition

*Edward James Bauman*

Optimal Control Theory Applied to Systems Described by Partial Differential Equations

*William L. Brogan*

Author Index—Subject Index

## CONTENTS OF PREVIOUS VOLUMES

### Volume 7

Computational Problems in Random and Deterministic Dynamical Systems

*Michael M. Connors*

Approximate Continuous Nonlinear Minimal-Variance Filtering

*Lawrence Schwartz*

Computational Methods in Optimal Control Problems

*J. A. Payne*

The Optimal Control of Systems with Transport Lag

*Roger R. Bate*

Entropy Analysis of Feedback Control Systems

*Henry L. Weidemann*

Optimal Control of Linear Distributed Parameter Systems

*Elliot I. Axelband*

Author Index—Subject Index

### Volume 8

Method of Conjugate Gradients for Optimal Control Problems with State Variable Constraint

*Thomas S. Fong and C. T. Leondes*

Final Value Control Systems

*C. E. Seal and Allen Stubberud*

Final Value Control System

*Kurt Simon and Allen Stubberud*

Discrete Stochastic Differential Games

*Kenneth B. Bley and Edwin B. Stear*

Optimal Control Applications in Economic Systems

*L. F. Buchanan and F. E. Norton*

Numerical Solution of Nonlinear Equations and Nonlinear, Two-Point Boundary-Value Problems

*A. Miele, S. Naqvi, A. V. Levy, and R. R. Iyer*

Advances in Process Control Applications

*C. H. Wells and D. A. Wismer*

Author Index—Subject Index

## CONTENTS OF PREVIOUS VOLUMES

### Volume 9

Optimal Observer Techniques for Linear Discrete Time Systems

*Leslie M. Novak*

Application of Sensitivity Constrained Optimal Control to National Economic Policy Formulation

*D. L. Erickson and F. E. Norton*

Modified Quasilinearization Method for Mathematical Programming Problems and Optimal Control Problems

*A. Miele, A. V. Levy, R. R. Iyer, and K. H. Well*

Dynamic Decision Theory and Techniques

*William R. Osgood and C. T. Leondes*

Closed Loop Formulations of Optimal Control Problems for Minimum Sensitivity

*Robert N. Crane and Allen R. Stubberud*

Author Index—Subject Index

### Volume 10

The Evaluation of Suboptimal Strategies Using Quasilinearization

*R. G. Graham and C. T. Leondes*

Aircraft Symmetric Flight Optimization

*Michael Falco and Henry J. Kelley*

Aircraft Maneuver Optimization by Reduced-Order Approximation

*Henry J. Kelley*

Differential Dynamic Programming—A Unified Approach to the Optimization of Dynamic Systems

*David Q. Mayne*

Estimation of Uncertain Systems

*Jack O. Pearson*

Application of Modern Control and Optimization Techniques to Transportation Systems

*Daniel Tabak*

Integrated System Identification and Optimization

*Yacov Y. Haimes*

Author Index—Subject Index

# CONTENTS

CONTRIBUTORS . . . . .	vii
PREFACE . . . . .	ix
CONTENTS OF PREVIOUS VOLUMES . . . . .	xi

## Fitting Multistage Models to Input/Output Data

*Patrick L. Smith*

I. Introduction . . . . .	3
II. Linear Models . . . . .	4
III. Identification of the Kalman Filter Model . . . . .	12
IV. Example . . . . .	18
V. Extensions . . . . .	20
VI. Summary and Conclusions . . . . .	22
References . . . . .	22

## Computer Aided Control Systems Design Techniques

*J. A. Page and E. B. Stear*

I. Introduction . . . . .	29
II. Computational Techniques of Function Minimization . . . . .	37
III. Control System Design . . . . .	92
IV. Summary and Conclusions . . . . .	130
V. Appendices . . . . .	134
References . . . . .	141

## Multilevel Optimization of Multiple Arc Trajectories

*Ronald D. Sugar*

I. Introduction . . . . .	146
II. Background . . . . .	150
III. General Formulation of Trajectory Decomposition . . . . .	164
IV. A Low Thrust Interplanetary Swingby Example . . . . .	179
V. Computational Aspects of the Low Thrust Swingby Example . . . . .	199
VI. Conclusions and Future Work . . . . .	237
References . . . . .	242

## CONTENTS

### Nonlinear Smoothing Techniques

*John B. Peller*

I.	Introduction . . . . .	256
II.	Exact Differential Equations for the Smoothing Problem . . . . .	264
III.	The Approximation Problem . . . . .	306
IV.	Linear Gaussian Case . . . . .	308
V.	Approximations for the Nonlinear Case . . . . .	319
VI.	Maximum Likelihood Smoothing . . . . .	363
VII.	Extensions, Summary, and Areas for Future Study . . . . .	366
	Bibliography . . . . .	372
	Appendix: Summary of Principal Results . . . . .	378

### Toward the Synthesis of Solutions of Dynamic Games

*L. C. Westphal*

I.	Introduction . . . . .	390
II.	Problem Statement and Overview . . . . .	392
III.	The Solution of Separable Static Games . . . . .	400
IV.	Applications of Dual Cones to Dynamic Games . . . . .	423
V.	Examples . . . . .	451
VI.	Summary and Conclusions . . . . .	486
	References . . . . .	487

### A Survey of Soviet Contributions to Control Theory

*Alexander Ya. Lerner*

I.	Introduction . . . . .	491
II.	Dynamics of Linear Systems . . . . .	493
III.	Non-Linear Systems . . . . .	498
IV.	Optimal Control . . . . .	502
V.	Learning Systems . . . . .	507
VI.	Perspectives . . . . .	510
	References . . . . .	511

SUBJECT INDEX . . . . .	515
-------------------------	-----

# Fitting Multistage Models to Input / Output Data

PATRICK L. SMITH  
The Aerospace Corporation  
El Segundo, California

I. INTRODUCTION. . . . .	3
II. LINEAR MODELS . . . . .	4
III. IDENTIFICATION OF THE KALMAN FILTER MODEL . . . . .	12
IV. EXAMPLE . . . . .	18
V. EXTENSIONS. . . . .	20
VI. SUMMARY AND CONCLUSIONS . . . . .	22
REFERENCES. . . . .	22

## NOMENCLATURE

$b$	Covariance of the measurement residual
$\hat{b}$	Estimate of $b$
$F(i), L(i)$	$3n-1$ by $n$ matrices
$f_1(u(i); \underline{\alpha})$	No memory nonlinear functions of the input $u(i)$
$f_2(z(i); \underline{\beta})$	and the output $z(i)$
$\underline{h}$	Output matrix of dimension $n \times 1$
$I_n$	Identity matrix of dimension $n \times n$
$J(\underline{\theta})$	Constrained least-squares cost functional
$\underline{k}$	Steady-state Kalman filter gain of dimension $n \times 1$
$\underline{k}'$	Reduced Kalman gain vector of dimension $(n-1) \times 1$

$M$	Steady-state covariance of $\hat{\underline{x}}'(i)$
$M(1)$	Covariance of $\underline{x}(1)$
$m$	Integer number of delays on input
$N$	Total number of measurements
$N_n(\underline{m}, Q)$	Multivariate normal distribution with mean $\underline{m}$ and covariance $Q$
$n$	Dimension of the state vector $\underline{x}(i)$
$P$	Steady-state covariance of $\hat{\underline{x}}'(i)$
$Q$	Covariance of $\underline{r}(i)$
$R$	Measurement error covariance
$\underline{r}(i)$	Random input vector of dimension $n$
$\underline{s}(k+1)$	$3n-1$ vector
$T$	Transition matrix for the adjoint process
$u(i)$	Measurement of the input
$\underline{v}$	Composite vector of measurement innovations
$v(i)$	Measurement innovation
$w(i)$	Measurement error
$\hat{\underline{x}}'(i)$	One-step-ahead predicted estimate of the state vector
$\hat{\underline{x}}(i)$	Filtered estimate of the state vector
$\underline{x}(i)$	$n \times 1$ vector of state variables
$z(i)$	Measurement of the output
$\underline{\alpha}, \underline{\beta}$	Unknown parameters in $f_1$ and $f_2$
$\underline{\gamma}$	Input matrix of dimension $n \times 1$
$\underline{\theta}$	Unknown parameter vector of dimension $(3n-1) \times 1$
$\hat{\underline{\theta}}$	Estimate of $\underline{\theta}$
$\underline{\lambda}(i)$	Lagrange multiplier of dimension $n \times 1$
$\hat{\rho}_j$	Estimate of the lag $j$ autocorrelation coefficient of measurement residuals
$p(\underline{v} \underline{\theta})$	Probability density function of the innovations
$\Phi$	State transition matrix
$\underline{\Phi}$	State transition parameter vector of dimension $n$

$\underline{\varphi}'$	Reduced state transition parameter vector of dimension $n-1$
$\omega(i)$	Random input for Box and Jenkins model in the example

## I. INTRODUCTION

It is assumed in this study that the ultimate objective of modeling a dynamic system is to predict or control the output of the system by observing or manipulating the inputs. In concrete terms the model is a digital computer program, which, when supplied the measurements of the past and present input and output, computes the predicted future output of the system. The random nature of the problem is considered in developing the model, but the model itself is a completely deterministic system. System characterization and system identification are the principal aspects of modeling. System characterization is concerned with defining a class of mathematical models and system identification with the determination of the specific model belonging to this preselected class which best fits the observations.

The class of models examined in this study are linear stationary multistage processes. The usefulness and convenience of linear models are well known and many techniques have been proposed to fit linear models to input/output data ([1] to [7], for example). In fact, because of the many publications in this area, the main contributions of this study are listed below:

(a) The class of Kalman filter models developed by Mehra [7] for free linear systems is extended to forced linear systems and to the specific problem of fitting models to input/output data.

(b) A recursive form for the gradient of the likelihood function is derived which greatly reduces the computer memory

requirements.

(c) The numerical problems resulting from a singularity in the gradient of the likelihood function for the Kalman filter representation are eliminated by rescaling the likelihood function.

(d) The direct application of the results obtained in this study to a class of nonlinear system representations is shown.

## II. LINEAR MODELS

The following is a list of comments and assumptions which describe the class of models examined in this study:

(a)  $N$  simultaneous measurements of a scalar input sequence and scalar output sequence of an isolated system are made at uniformly spaced instants of time and are denoted  $\{u(i): i = 1, \dots, N\}$  and  $\{z(i): i = 1, \dots, N\}$ , respectively.

(b) The measurements are assumed to be error-free.

(c) The measurements are assumed to be generated by a multistage time-invariant linear process of order  $n$  which is driven by both the measured inputs and unmeasured inputs.

(d) The unmeasured inputs are assumed to be mutually independent random variables which are identically distributed and independent of the measured input and output.

(e) The prediction ability of the model may be degraded for any of the following reasons:

1. Actual random input disturbances may be present.
2. The system may not be a linear, time-invariant multistage process, as assumed in Paragraph (c).
3. There may be measurement errors.
4. There may be errors in identifying the parameters