CALCULUS WITH ANALYTIC GEOMETRY

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JOHN M. H. OLMSTED

VOLUME I

CALCULUS

WITH ANALYTIC GEOMETRY

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TO CYNTHIA

PREFACE

A perennial problem facing teachers of mathematics is the determination of the proper time for introducing the student to the "delta-and-epsilon" type of mathematical rigor, sometimes referred to informally as "epsilonics." Experience in recent years has convinced many people that much of the subject matter of "epsilonics" can be effectively taught at a much earlier stage in a student's academic career than had at one time seemed possible. However, the fact remains that most students do have difficulty when first confronted with the limit concept. What is the best solution for the problem posed above? Should a considerable period of time be invested in a purely intuitive approach before the student is asked to face the harsh realities of mathematical analysis? Or is it possible to present "epsilonics" in such a fashion and with such a timing that the student can assimilate the essential ideas in a natural way and with a reasonably correct intuitive understanding?

For purposes of discussion let it be agreed that the limit concept is a complicated and difficult one for most students new to calculus. Explicitly or implicitly the formulation of a limit statement uses not only a universal quantifier and an existential quantifier but also an implication involving two inequalities (or the equivalent in terms of a neighborhood and a deleted neighborhood). One of the principal assumptions determining the character of the present book is that the most appropriate groundwork for the study of limits is a substantial amount of detailed and careful work with inequalities, implications, and quantifiers *before* these are all combined into the type of compound statement needed for limits. For this reason, and also because of the many variants that involve "neighborhoods of infinity," the treatment of limits has been placed as late as possible, in Chapter 10. This postponement of limits is made possible, in part, by the earlier discussion, in Chapter 9, of continuity (free from the complications of infinities and deleted neighborhoods), but more especially by the still earlier focusing of attention on such global concepts as the Riemann integral, in Chapters 6 and 7, and uniform continuity, in Chapter 8.

Placing the introduction of the definite integral before that of the derivative has a well-recognized historical background, but it also has a sound logical justification. When the Riemann integral is approached by means of step-functions the definition of integrability involves only two quantifiers and a single inequality, and the definition of the Riemann integral itself is simultaneously the supremum of one set and the infimum of another. The avoidance, at this stage, of implications between pairs of inequalities provides a natural arena for students to get a mathematical workout with quantifiers. Then later, when it is time to struggle with limit statements, the quantified portions have become familiar objects and are no longer a barrier.

It is possible that some readers of this preface are not disposed at first to agree with the proposition that the global concept of uniform continuity is a simpler one than the local concept of ordinary continuity. Undoubtedly, if one wishes only to talk intuitively in terms of "approach" or "nearness" with accompanying "hand-waving" gestures, uniform continuity *is* more complicated. However, it is when one expresses himself in exact terms that this concept becomes simpler. More precisely, it is nearly always simpler to find an explicit δ in terms of ϵ for a function on a set than for a function at a point. An example may help clarify this statement. For the "squaring" function $x \longrightarrow x^2$ on the compact interval $[3, 5]$ a suitable δ can be found quite simply to be $\epsilon/10$, whereas for this same function at the point 3, either δ turns out to be something like $\min(1, \epsilon/7)$ or else ϵ must be artificially limited in size. In fine, the position adopted in this book is that to concentrate on relatively simple global concepts before turning attention to point-wise continuity and limits makes good sense, and is preferable in the long run to the more traditional intuitive approach which necessitates an often painful relearning process later.

The dominant philosophy of this book emphasizes *concepts* and *structure*, with the dual objectives of developing an appreciation for a truly beautiful and well-conceived subject and exploiting the great potentials of these concepts and structures. The concept of *vector space* should serve to illustrate what is meant. Vector spaces are introduced first for function spaces, with examples such as bounded functions, step-functions, and polynomials. It then becomes natural to develop the Riemann integral as a positive linear functional on the vector space of integrable functions. Then later, many of the standard theorems of continuous functions and limits find simple expression when cast in the language of vector spaces and algebras of functions. Furthermore, these same ideas become tools when it is shown that the Riemann integral can be obtained as the limit of a sum. Later in the book, generous attention is again devoted to vector spaces and algebras, this time in the context of matrices and vectors in two and three dimensions. Eigenvalue techniques are used effectively to simplify transformations of equations of the second degree.

In order to achieve the goals enumerated above, the first two chapters contain an introduction to sets and logic. Truth sets serve to explain exactly what an implication is. The quantifier symbols $\forall$ and $\exists$ are used widely, and their role in the formulation of negations is discussed carefully and used frequently. Both the nature of a proof and the meaning of a counterexample are given considerate attention. The groundwork laid in the first two chapters forms the basis for the extensive use of both sets and logic throughout the book. It is felt that the time spent initially in forming a sound foundation for the ideas that are so essential to calculus is well worth the investment, and is amply justified by the ultimate dividends in the form of understanding and intellectual satisfaction.

Alternative formulations are sometimes included. For example, several limit statements are expressed in terms of deltas and epsilons, neighborhoods and deleted neighborhoods, and mappings. These alternatives help provide a broad and varied texture for the underlying flow of ideas, as well as emphasizing a basic unity that pervades many apparently dissimilar settings.

Much attention is given to applications. For example, Chapter 7 is devoted entirely to such applications of the Riemann integral as area, volume, work, and distance. Applications of the derivative are scattered through most of the chapters. Differential equations find their first use in Chapter 18, where they are applied to problems involving gravitation, orthogonal trajectories, radioactive decay, bacterial growth, cooling, and mixing. The final Chapter 34 returns to the subject of differential equations, concentrating on linear equations with constant coefficients and their applications. Extremal problems receive unusually thorough consideration, with full attention given to sufficiency conditions and endpoint extrema.

Area and volume are introduced axiomatically, each being a positive additive function on a ring of sets and satisfying a certain completeness axiom. For a function of a single real variable these axioms suffice for establishing the theorem that the ordinate set of a nonnegative Riemann-integrable function f on a compact interval has area equal to the integral of f.

Elliptic notation (for example, "the function $e^{2x} \sin 3x$") is discussed clearly so that the student learns to understand and use both the strict f and the elliptic notation. This is important since both notations abound in the mathematical literature of the world.

Special attention should be called to the character and arrangement of honors sections, suitable for enrichment content in honors courses. These honors sections are planned so as to permit an honors course running concurrently with a standard course to cover the standard as well as the extra material, and also to permit exploratory reading and extra assignments for interested and capable students who are not registered in a special honors class. In order that a student may be able to transfer either out of or into an honors sequence between terms it is desirable that the honors course keep pace with the standard course to which it corresponds. This is made especially feasible for courses using this book by the uniform arrangement of the honors sections, which constitute exactly twenty percent of the sections, as follows: Every chapter contains a multiple of ten sections, including sections of exercises; every section whose number ends with the digit 9 is an honors section, and every section whose number ends with the digit 0 is the accompanying honors section of exercises. All honors material is identified by the letter H.

The division of the book into two volumes is done simply to reduce the bulk of the alternative of a single tome. The place of division between Chapters 21 and 22 achieves a local minimum for cross-references between the two volumes. The use of a table of integrals is limited to the second volume.

The book is designed for courses of three semesters — or four or five quarters — meeting four or five days a week. Prerequisite is a standard high school mathematical preparation, including trigonometry and college algebra. Review material in trigonometry and such topics as mathematical induction is included.

Over 6000 exercises of all levels of difficulty are available, for practice, for challenge, and for individual exploration. These exercises form an essential part of the book, and are designed to fortify and deepen, as well as guarantee, learning. Answers to nearly all problems are given in the back portions of the two volumes. Illustrative examples are liberally provided.

The author wishes to express his appreciation for the extensive aid and numerous suggestions given by Professor R. W. Brink. He is also indebted to many others for their helpful comments, given both informally in conversation and more specifically on paper.

J. M. H. O.

Carbondale, Illinois

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