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CHAPTER **1**

Basic Concepts and Two-Fluid Description of Plasmas

The study of the interaction of intense laser light with plasmas serves as an excellent introduction to the field of plasma physics. Both the linear and nonlinear theory of plasma waves, instabilities and wave-particle interactions are important for understanding the laser plasma coupling. Indeed, the field is a veritable testing ground for many fundamental processes. Numerous plasma effects have now been observed in laser plasma experiments, and many challenging problems remain to be understood.

Since laser plasma interactions are of interest to scientists from many different fields of expertise, little prior background in plasma physics will be assumed. Even for those with plasma experience, it can be very instructive and refreshing to begin from the basics and examine a field of applications. Two levels of description will be used – a theoretical one based on the two-fluid theory of plasmas and a numerical one based on particle simulation codes. These two descriptions both reinforce and complement one another. For example, the particle simulations allow one to both test the theory and develop some understanding of the nonlinear effects.

1.1 BASIC PLASMA CONCEPTS

Let's begin. A plasma is basically just a system of N charges which are coupled to one another via their self-consistent electric and magnetic fields. Consider then following the evolution of these N charges. Even neglecting magnetic fields and electromagnetic waves, we must in principle solve 6N coupled equations:

$$\begin{aligned} m_i \ddot{\mathbf{r}}_i &= q_i \mathbf{E}(\mathbf{r}_i) \\ \mathbf{E}(\mathbf{r}_i) &= \sum_j \frac{q_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} (\mathbf{r}_i - \mathbf{r}_j) . \end{aligned}$$

Here m_i, q_i and $\mathbf{r}_i$ are the mass, charge and position of the i^{th} particle, and $\mathbf{E}$ is the electrostatic field. This is clearly an unpromising approach if a nontrivial number of charges is considered.

Fortunately a very great simplification is possible if we focus our attention on collisionless plasma behavior. We can decompose the electric field into two fields ($\mathbf{E}_1$ and $\mathbf{E}_2$) which have distinct spatial scales. The field $\mathbf{E}_1$ has spatial variations on a scale length much less than the so-called electron Debye length, which is the length over which the field of an individual charge is shielded out by the response of the surrounding charges. $\mathbf{E}_1$ represents the rapidly fluctuating microfield due to multiple and random encounters (collisions) among the discrete charges. In contrast, $\mathbf{E}_2$ represents the field due to deviations from charge neutrality over space scales greater than or comparable to the Debye length. This field gives rise to "collective" or coherent motion of the charges.

We thus have a natural separation into collisional and collective behavior. Not surprisingly, the collisional behavior becomes negligible when the number of electrons in a sphere with a radius equal to the electron Debye length becomes very large. To motivate this, let us carry out a simple calculation of electron scattering by ions. As illustrated in Fig. 1.1, we consider an electron with velocity v, mass m and charge e streaming past an ion with charge Ze. The distance of closest approach is b. The electron undergoes a change in velocity Δv which is approximately

$$\Delta v = \frac{Z e^2}{m b^2} \left(\frac{2b}{v} \right) ,$$

which is just the maximum electrostatic force times the interaction time ($\sim 2b/v$). If we assume many randomly spaced ions, $\langle \Delta v \rangle = 0$, where the brackets denote an average. However, there is a change in the mean square velocity. This average rate of change is given by $(\Delta v)^2$ times the rate of encounters, which is $n_i \sigma v$. Here n_i is the ion density and σ is the cross-section of impact. Summing over all encounters gives

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle (\Delta v)^2 \rangle = \int 2\pi b \, \mathrm{d}b \, n_i \, v \, (\Delta v)^2 .$$

If we substitute for Δv and integrate over impact parameters, we obtain

$$\langle (\Delta v)^2 \rangle = \frac{8\pi \, n_i \, Z^2 e^4 \ \ln \Lambda}{m^2 v} \, t ,$$

where Λ is the ratio of the maximum and minimum impact parameters (b_{max} and b_{min}). The maximum impact parameter is approximately the electron Debye length, since other electrons in the plasma shield out the Coulomb potential over this distance. The minimum impact parameter is the larger of either the classical distance of closest approach ($b_{\mathrm{min}} \approx Ze^2/mv^2$) or the DeBroglie wavelength of the electron ($b_{\mathrm{min}} \approx \hbar/mv$), where $\hbar$ is Planck's constant. Using the first, the distance of closest approach, we have $\Lambda \approx 9N_D/Z$, where N_D is the number of electrons in a Debye sphere. In particular $N_D = \frac{4}{3}\pi \, n_e \lambda_{\mathrm{De}}^3$, where n_e is the electron density and λ_{De} is the electron Debye length. This important length will be derived later in this chapter.

It is convenient to define a ninety-degree deflection time (t_{90°) by the condition that the root-mean-square change in velocity becomes as large as the velocity. Hence

$$t_{90^\circ} = \frac{m^2 v^3}{8\pi \, n_i \, Z^2 e^4 \ \ln \Lambda} .$$

Averaging over a Maxwellian distribution of velocities then provides us with a convenient measure of the mean rate ($\nu_{90^\circ} \equiv 1/t_{90^\circ}$) at which electron-ion collisions scatter electrons through a large angle:

$$\nu_{90^\circ} = \frac{8\pi \, n_i \, Z^2 e^4 \ \ln \Lambda}{6.4 \, m^2 v_e^3} . \tag{1.1}$$

Here $v_e = \sqrt{\theta_e/m}$ is the electron thermal velocity and θ_e is the electron

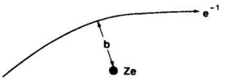

Figure 1.1 An electron is deflected as it streams past an ion.

temperature. We note that

$$\frac{\nu_{90^\circ}}{\omega_{pe}} \simeq \frac{Z \ln \Lambda}{10} \frac{1}{N_D},$$

where ω_{pe} is the electron plasma frequency, which we will see is a frequency characteristic of collective electron motion.

The important point we wish to make is now apparent. The fine scale, collisional interactions can be neglected to zeroth order in the parameter $1/N_D$. If we express the electron density in cm^{-3} and the electron temperature in eV, then $N_D = 1.7 \times 10^9 (\theta_e^3/n_e)^{1/2}$. N_D can be very large even in a rather dense plasma, provided the electron temperature is high. For example, if $n_e = 10^{21}$ cm^{-3} and $\theta_e = 1$ keV, $N_D \approx 1700$. In the collisionless limit ($N_D \to \infty$), the fine scale fluctuating microfields associated with discrete charges are completely negligible. The plasma behavior can then be investigated by solving for the motion of the charges in the smoothed or coarse-grained fields which arise from the collective motion of large numbers of charges.

We will develop two parallel levels of description for the collective behavior. One level is analytical. Starting from the Vlasov equation, we will derive moment (fluid-like) equations for the electrons and ions by averaging over the velocities of the charges. This so-called two-fluid description will then be used extensively to describe a wide variety of laser plasma interactions. The second level of description is numerical: the use of particle simulations. These simulations are a powerful tool for investigating nonlinear effects and kinetic effects (effects which depend on the details of the velocity distribution of the particles).

1.2 THE VLASOV EQUATION

The natural starting point for describing the evolution of a collisionless plasma is the Vlasov equation. We first introduce the phase space distribution function $f_j(\mathbf{x}, \mathbf{v}, t)$. This is simply the function which characterizes the location of the particles of species j in phase space $(\mathbf{x}, \mathbf{v})$ as a function of time. Knowing the laws of motion, we can readily derive an equation for $f_j(\mathbf{x}, \mathbf{v}, t)$. Since particles are assumed to be neither created nor destroyed as they move from one location in phase space to another (no ionization or recombination), $f_j(\mathbf{x}, \mathbf{v}, t)$ must obey the continuity equation:

$$\frac{\partial f_j}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (\dot{\mathbf{x}} f_j) + \frac{\partial}{\partial \mathbf{v}} \cdot (\dot{\mathbf{v}} f_j) = 0 \,. \tag{1.2}$$

From the laws of motions, we have

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \frac{q_j}{m_j} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) , \end{aligned} \tag{1.3}$$

where q_j and m_j are the charge and mass of the j^{th} species and $\mathbf{E}$ and $\mathbf{B}$ are the coarse-grained fields associated with the collective behavior. Noting that $\mathbf{x}$ and $\mathbf{v}$ are independent variables and substituting Eq. (1.3) into Eq. (1.2), we arrive at the Vlasov equation:

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \frac{\partial f_j}{\partial \mathbf{x}} \cdot + \frac{q_j}{m_j} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f_j}{\partial \mathbf{v}} = 0 \,. \tag{1.4}$$

This equation simply says that $f_j(\mathbf{x}(t), \mathbf{v}(t), t)$ is a constant; i.e., the phase space density is conserved following a dynamical trajectory. Such an equation applies to each charge species in the plasma.

The Vlasov equation, augmented with Maxwell's equations, is a complete description of collisionless plasma behavior. In practice, we need a more tractable description which can be obtained by averaging over the velocities of the individual particles. By taking different velocity moments of the Vlasov equation, we can derive equations for the evolution in space and time of the density, mean velocity, and pressure of each species. As we will see, each moment brings in the next higher moment, generating an infinite set of moment equations. However, we can fortunately truncate the series of equations by introducing assumptions about the heat flow.

1.3 THE MOMENT EQUATIONS

Let us now derive the moment equations and motivate their truncation. First, we note that the density (n_j), mean velocity ($\mathbf{u}_j$), and pressure tensor ($\underset{\approx}{\mathbf{P}}_j$) are determined by averaging the various moments of the phase space distribution function over velocities:

$$n_j = \int f_j(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} \tag{1.5}$$

$$n_j\, \mathbf{u}_j = \int \mathbf{v}\, f_j(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} \tag{1.6}$$

$$\underset{\approx}{\mathbf{P}}_j = m_j \int (\mathbf{v} - \mathbf{u}_j)(\mathbf{v} - \mathbf{u}_j)\, f_j(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}\,. \tag{1.7}$$

In deriving the moment equations, we will suppress the subscript j, since it is clear that these equations will apply to each charge species. Averaging the Vlasov equation over velocity gives

$$\int d\mathbf{v} \left[\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f}{\partial \mathbf{v}} \right] = 0\,. \tag{1.8}$$

The first two terms in Eq. (1.8) give

$$\int d\mathbf{v}\, \frac{\partial f}{\partial t} = \frac{\partial n}{\partial t}$$

$$\int d\mathbf{v}\, \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} = \int d\mathbf{v} \sum_i v_i \frac{\partial f}{\partial x_i}$$

$$= \frac{\partial}{\partial \mathbf{x}} \cdot (n\, \mathbf{u})\,.$$

The third term in Eq. (1.8) vanishes, as can be seen by integrating by parts and noting that $f \to 0$ as $|\mathbf{v}| \to \infty$. Hence the first moment of the Vlasov equation gives the continuity equation for the particle density:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (n\mathbf{u}) = 0\,. \tag{1.9}$$

The next moment of the Vlasov equation is

$$\int d\mathbf{v}\, \mathbf{v} \left[\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f}{\partial \mathbf{v}} \right] = 0\,. \tag{1.10}$$

The first term in Eq. (1.10) is straightforward:

$$\int d\mathbf{v}\,\mathbf{v}\,\frac{\partial f}{\partial t} = \frac{\partial}{\partial t} n\,\mathbf{u}\,.$$

The second term gives

$$\int d\mathbf{v}\,\mathbf{v}\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\cdot\int d\mathbf{v}\,\mathbf{v}\mathbf{v}\,f = \frac{\partial}{\partial \mathbf{x}}\cdot\left(\frac{\underset{\approx}{\mathbf{P}}}{m} + n\,\mathbf{u}\mathbf{u}\right).$$

This result is readily obtained by rewriting the integral as

$$\int d\mathbf{v}\,(\mathbf{v} - \mathbf{u} + \mathbf{u})(\mathbf{v} - \mathbf{u} + \mathbf{u})\,f = \frac{\underset{\approx}{\mathbf{P}}}{m} + n\,\mathbf{u}\,\mathbf{u}\,,$$

since $\int (\mathbf{v} - \mathbf{u})\,f\,d\mathbf{v} = 0$. Evaluation of the last term in Eq. (1.10) yields

$$\int d\mathbf{v}\,\mathbf{v}\,\frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial f}{\partial \mathbf{v}} = -\frac{n\,q}{m}\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c}\right),$$

where we have integrated by parts. Collecting the above terms, we obtain the equation of motion for the charged fluid:

$$\frac{\partial}{\partial t}(n\,\mathbf{u}) + \frac{\partial}{\partial \mathbf{x}}\cdot(n\,\mathbf{u}\mathbf{u}) = \frac{nq}{m}\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c}\right) - \frac{\partial}{\partial \mathbf{x}}\cdot\frac{\underset{\approx}{\mathbf{P}}}{m}. \qquad (1.11)$$

It is convenient to rewrite the first two terms of Eq. (1.11) using the continuity equation and to assume that the pressure is isotropic, i.e., $\underset{\approx}{\mathbf{P}} = \underset{\approx}{\mathbf{I}}\,p$ where $\underset{\approx}{\mathbf{I}}$ is the unit dyad. Then

$$n\,\frac{\partial \mathbf{u}}{\partial t} + n\,\mathbf{u}\cdot\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \frac{nq}{m}\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c}\right) - \frac{1}{m}\frac{\partial p}{\partial \mathbf{x}}\,. \qquad (1.12)$$

Observe that each moment brings in the next higher one. The continuity equation for the density involves the mean velocity; the force equation for the velocity brings in the pressure. The next moment will give us an equation for the pressure (energy density) which involves the heat flow. Continuing, we would end up with an infinite set of coupled equations, hardly a practical description.

Fortunately, we can truncate the moment equations by making various assumptions about the heat flow, which gives us a so-called equation of state. The simplest assumption is that the heat flow is so rapid that the temperature of the charged fluid is a constant. In this case, we have the isothermal equation of state: $p = n\theta$, where the temperature θ is a constant. This equation of state, plus the continuity and force equations for the fluid, and Maxwell's equations form a closed description.

The isothermal equation of state is appropriate when $\omega/k \ll v_t$, where ω and k are the frequency and wave number characteristic of the physical process being considered and v_t is the thermal velocity of the particles. In the opposite limit ($\omega/k \gg v_t$) we can simply neglect the heat flow. This assumption leads to an adiabatic equation of state, which we will now derive.

To obtain an equation for the pressure, we multiply the Vlasov equation by the kinetic energy and average over velocity:

$$\int \frac{mv^2}{2}\,\mathrm{d}\mathbf{v}\left[\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial f}{\partial \mathbf{v}}\right] = 0\,. \quad (1.13)$$

At this point, let us specialize to one-dimension to simplify the algebra. The first term can be written as

$$\frac{m}{2}\frac{\partial}{\partial t}\int f\,(v - u + u)^2\,\mathrm{d}v = \frac{1}{2}\frac{\partial}{\partial t}(p + nmu^2)\,.$$

The next term in Eq. (1.13) gives

$$\frac{m}{2}\frac{\partial}{\partial x}\int f\,(v - u + u)^3\,\mathrm{d}v = \frac{\partial Q}{\partial x} + \frac{3}{2}\frac{\partial}{\partial x}(up) + \frac{m}{2}\frac{\partial}{\partial x}(nu^3)\,,$$

where $Q \equiv (m/2)\int(v-u)^3 f\,\mathrm{d}v$. The final term in Eq. (1.13) is simply

$$\frac{q}{2}\int v^2 E\,\frac{\partial f}{\partial v}\,\mathrm{d}v = -\,nquE\,.$$

Collecting terms, we obtain

$$\frac{1}{2}\frac{\partial}{\partial t}(p + nmu^2) + \frac{3}{2}\frac{\partial}{\partial x}(up) + \frac{1}{2}\frac{\partial}{\partial x}(nmu^3) + \frac{\partial Q}{\partial x} = qnuE\,. \quad (1.14)$$

A great deal of simplification results from the use of the lower moment equations. In particular,

$$\frac{\partial}{\partial t}\left(\frac{n m u^2}{2}\right) = \frac{m u^2}{2}\frac{\partial n}{\partial t} + n m u \frac{\partial u}{\partial t} .$$

Using Eqs. (1.9) and (1.12), substituting into Eq. (1.14), and cancelling terms gives

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} + 2\frac{\partial Q}{\partial x} = 0 . \tag{1.15}$$

To obtain the adiabatic equation of state, we neglect the heat flow. This assumes that $\partial Q/\partial x$ is much less than the other terms in Eq. (1.15). For example, demanding that $\partial Q/\partial x \ll \partial p/\partial t$ gives $\omega p \gg k Q$, where ω and k are a frequency and wavenumber characteristic of the process being considered. Clearly $Q < Q_{\max} \sim n\theta v_t$, where v_t is the thermal velocity. Hence, to neglect heat flow it is sufficient to assume that $\omega/k \gg v_t$.

With this assumption, Eq. (1.15) reduces to

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} = 0 . \tag{1.16}$$

The continuity equation allows us to express $\partial u/\partial x$ as

$$\frac{\partial u}{\partial x} = -\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\ln n . \tag{1.17}$$

Substituting Eq. (1.17) into Eq. (1.16) gives

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\ln p - \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\ln n^3 = 0 ,$$

or

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\frac{p}{n^3} = 0 . \tag{1.18}$$

This equation shows that, following the plasma flow, $p/n^3 =$ constant, which is the adiabatic equation of state for motion with one degree of freedom. This equation of state is readily generalized to $p/n^\gamma =$ constant, where $\gamma = (2+N)/N$ and N is the number of degrees of freedom.