

Optimal Measurement Methods for Distributed Parameter System Identification

Dariusz Uciński



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About the Author

Dariusz Uciński was born in Gliwice, Poland, in 1965. He studied electrical engineering at the Technical University of Zielona Góra, Poland, from which he graduated in 1989. He received Ph.D. (1992) and D.Sc. (2000) degrees in automatic control and robotics from the Technical University of Wrocław, Poland. He is currently an associate professor at the University of Zielona Góra, Poland.

He has authored and co-authored numerous journal and conference papers. He also co-authored two textbooks in Polish: *Artificial Neural Networks — Foundations and Applications* and *Selected Numerical Methods of Solving Partial Differential Equations*. For fifteen years his major activities have been concentrated on measurement optimization for parameter estimation in distributed systems. In 2001 his habilitation thesis on this subject was granted an award from the Minister of National Education. In his career, he has been both a leader and a member of several national and international research projects. Other areas of expertise include optimum experimental design, algorithmic optimal control, robotics and cellular automata. Since 1992 he has been the scientific secretary of the editorial board of the *International Journal of Applied Mathematics and Computer Science*.

Preface

It is well understood that the choice of experimental conditions for distributed systems has a significant bearing upon the accuracy achievable in parameter-estimation experiments. Since for such systems it is impossible to observe their states over the entire spatial domain, close attention has been paid to the problem of optimally locating discrete sensors to estimate the unknown parameters as accurately as possible. Such an optimal sensor-location problem has been widely investigated by many authors since the beginning of the 1970s (for surveys, see [135, 145, 237, 298, 307]), but the existing methods are either restricted to one-dimensional spatial domains for which some theoretical results can be obtained for idealized linear models, or onerous, not only discouraging interactive use but also requiring a large investment in software development. The potential systematic approaches could be of significance, e.g., for environmental monitoring, meteorology, surveillance, hydrology and some industrial experiments, which are typical exemplary areas where we are faced with the sensor-location problem, especially owing to serious limitations on the number of costly sensors.

This was originally the main motivation to pursue the laborious research detailed in this monograph. My efforts on optimum experimental design for distributed-parameter systems began some fifteen years ago at a time when rapid advances in computing capabilities and availability held promise for significant progress in the development of a practically useful as well as theoretically sound methodology for such problems. At present, even low-cost personal computers allow us to solve routinely certain computational problems which would have been out of the question several years ago.

The aim of this monograph is to give an account of both classical and recent work on sensor placement for parameter estimation in dynamic distributed systems modelled by partial differential equations. We focus our attention on using real-valued functions of the Fisher information matrix of parameters as the performance index to be minimized with respect to the positions of point-wise sensors. Particular emphasis is placed on determining the ‘best’ way to guide scanning and moving sensors and making the solutions independent of the parameters to be identified. The bulk of the material in the corresponding chapters is taken from a collection of my original research papers. My main objective has been to produce useful results which can be easily translated into computer programmes. Apart from the excellent monograph by Werner Müller [181], which does not concern dynamic systems and has been written from a statistician’s point of view, it is the first up-to-date and comprehensive

monograph which systematizes characteristic features of the problem, analyses the existing approaches and proposes a wide range of original solutions. It brings together a large body of information on the topic, and presents it within a unified and relatively simple framework. As a result, it should provide researchers and engineers with a sound understanding of sensor-location techniques, or more generally, modern optimum experimental design, by offering a step-by-step guide to both theoretical aspects and practical design methods of sensor location, backed by many numerical examples.

The study of this subject is at the interface of several fields: optimum experimental design, partial differential equations, nonlinear programming, optimal control, stochastic processes, and numerical methods. Consequently, in order to give the reader a clear image of the proposed approach, the adopted strategy is to indicate direct arguments in relevant cases which preserve the essential features of the general situation, but avoid many technicalities.

This book is organized as follows. In Chapter 1, a brief summary of concrete applications involving the sensor-location problem is given. Some of these examples are used throughout the monograph to motivate and illustrate the demonstrated developments. A concise general review of the existing literature and a classification of methods for optimal sensor location are presented. Chapter 2 provides a detailed exposition of the measurement problem to be discussed in the remainder of the book and expounds the main complications which make this problem difficult. In Chapter 3 our main results for stationary sensors are stated and proved. Their extensions to the case of moving internal observations are reported in Chapter 4. Efficient original policies of activating scanning sensors are first proposed and examined. Then optimal design measures are treated in the context of moving sensors. A more realistic situation with nonnegligible dynamics of the vehicles conveying the sensors and various restrictions imposed on their motions is also studied therein and the whole problem is then formulated as a state-constrained optimal-control problem. Chapter 5 deals with vital extensions towards sensor location with alternative design objectives, such as prediction or model discrimination. Then Chapter 6 establishes some methods to overcome the difficulties related to the dependence of the optimal solutions on the parameters to be identified. Some indications of possible modifications which can serve to attack problems generally considered ‘hard’ are contained in Chapter 7. Chapter 8 attempts to treat in detail some case studies which are close to practical engineering problems. Finally, some concluding remarks are made in Chapter 9. The core chapters of the book are accompanied by nine appendices which collect accessory material, ranging from proofs of theoretical results to MATLAB implementations of the proposed algorithms.

The book may serve as a reference source for researchers and practitioners working in various fields (applied mathematics, electrical engineering, civil/geotechnical engineering, mechanical engineering, chemical/environmental engineering) who are interested in optimum experimental design, spatial statistics, distributed parameter systems, inverse problems, numerical anal-

ysis, optimization and applied optimal control. It may also be a textbook for graduate and postgraduate students in science and engineering disciplines (e.g., applied mathematics, engineering, physics/geology). As regards prerequisites, it is assumed that the reader is familiar with the basics of partial differential equations, vector spaces, probability and statistics. Appendices constitute an essential collection of mathematical results basic to the understanding of the material in this book.

Dariusz Uciński

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1.1 The optimum experimental design problem in context

Distributed-parameter systems (DPSs) are dynamical systems whose state depends not only on time but also on spatial coordinates. They are frequently encountered in practical engineering problems. Examples of a thermal nature are furnaces for heating metal slabs or heat exchangers; examples of a mechanical nature are large flexible antennas, aircrafts and robot arms; examples of an electrical nature are energy transmission lines.

Appropriate mathematical modelling of DPSs most often yields partial differential equations (PDEs), but descriptions by integral equations or integro-differential equations can sometimes be considered. Clearly, such models involve using very sophisticated mathematical methods, but in recompense for this effort we are in a position to describe the process more accurately and to implement more effective control strategies. Early lumping, which means approximation of a PDE by ordinary differential equations of possibly high order, may completely mask the distributed nature of the system and therefore is not always satisfactory.

For the past forty years DPSs have occupied an important place in control and systems theory. This position has grown in relevance due to the ever-expanding classes of engineering systems which are distributed in nature, and for which estimation and control are desired. DPSs, or more generally, infinite-dimensional systems are now an established area of research with a long list of journal articles, conference proceedings and several textbooks to its credit [59, 74, 76, 103, 133, 138, 140, 152, 167, 179, 200, 273, 357], so the field of potential applications could hardly be considered complete [17, 151, 317–319].

One of the basic and most important questions in DPSs is parameter estimation, which refers to the determination from observed data of unknown parameters in the system model such that the predicted response of the model is close, in some well-defined sense, to the process observations [200]. The parameter-estimation problem is also referred to as parameter identification or simply the inverse problem [118]. There are many areas of technological importance in which identification problems are of crucial significance. The importance of inverse problems in the petroleum industry, for example, is well

documented [80, 138]. One class of such problems involves determination of the porosity (the ratio of pore volume to total volume) and permeability (a parameter measuring the ease with which the fluids flow through the porous medium) of a petroleum reservoir based on field production data. Another class of inverse problems of interest in a variety of areas is to determine the elastic properties of an inhomogeneous medium from observations of reflections of waves travelling through the medium. The literature on the subject of DPS identification is considerable. Kubrusly [144] and Polis [216] have surveyed the field by systematically classifying the various techniques. A more recent book by Banks and Kunisch [16] is an attempt to present a thorough and unifying account of a broad class of identification techniques for DPS models, also see [14, 320].

In order to identify the unknown parameters (in other words, to calibrate the considered model), the system's behaviour or response is observed with the aid of some suitable collection of sensors termed the measurement or observation system. In many industrial processes the nature of state variables does not allow much flexibility as to which they can be measured. For variables which can be measured online, it is usually possible to make the measurements continuously in time. However, it is generally impossible to measure process states over the entire spatial domain. For example [211], the temperature of molten glass flowing slowly in a forehearth is described by a linear parabolic PDE, whereas the displacements occasioned by dynamic loading on a slender airframe can be described by linear second-order hyperbolic PDEs. In the former example, temperature measurements are available at selected points along the spatial domain (obtained by a pyrometer or some other device), whereas, in the latter case, strain gauge measurements at selected points on the airframe are reduced to yield the deflection data. In both cases the measurements are incomplete in the sense that the entire spatial profile is not available. Moreover, the measurements are inexact by virtue of inherent errors of measurement associated with transducing elements and also because of the measurement environment.

The inability to take distributed measurements of process states leads to the question of where to locate sensors so that the information content of the resulting signals with respect to the distributed state and PDE model be as high as possible. This is an appealing problem since in most applications these locations are not prespecified and therefore provide design parameters. The location of sensors is not necessarily dictated by physical considerations or by intuition and, therefore, some systematic approaches should still be developed in order to reduce the cost of instrumentation and to increase the efficiency of identifiers.

As already mentioned, the motivations to study the sensor-location problem stem from practical engineering issues. Optimization of air quality-monitoring networks is among the most interesting. As is well known, due to traffic emissions, residential combustion and industry emissions, air pollution has become a big social problem. One of the tasks of environmental protection