Mathematics Encyclopedia

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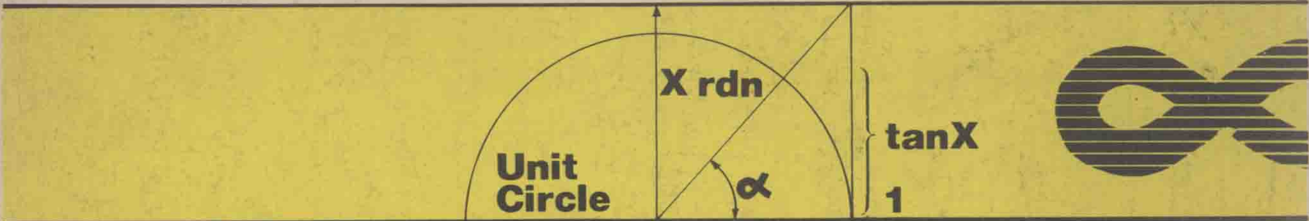
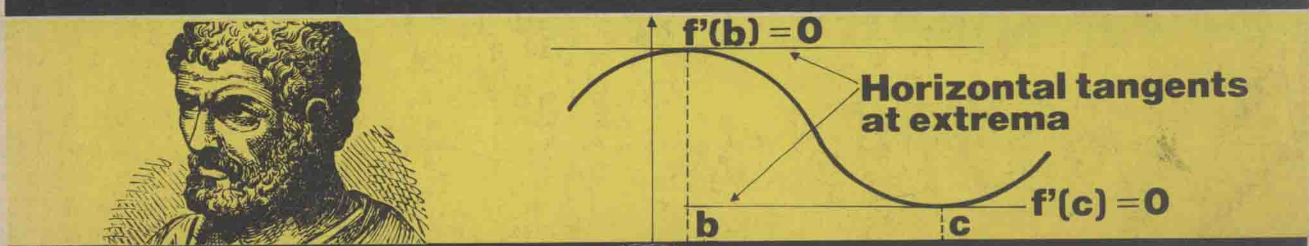
Max S. Shapiro
Executive Editor

A Made Simple Book



Algebraic Curve
Bisector
Central Limit Theorem

X-Axis
Y-Axis
Zone of a Sphere



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Introduction

Intended for practical use in school, home and business, *Mathematics Encyclopedia* contains all the mathematical topics covered by academic curricula through the college level. Sections on Arithmetic, Algebra, Geometry, Trigonometry, Analytic Geometry, Calculus, the "New Mathematics," and Statistics, to name only a few of the major topics, aid both the student in his coursework and the layman interested in teaching himself this crucial science. The entries include lucid explanations, problems with step-by-step solutions as well as hundreds of clear, functional drawings that aid immeasurably in understanding the text and in solving the problems. All of which represents a real advance beyond ordinary encyclopedic method.

Another important feature of this encyclopedia is the Special Reference Section. Here you will find the important tables used in Mathematics plus a wealth of additional reference information: a complete review of mathematical formulas; an A to Z listing of Weights and Measures arranged both alphabetically and by quantity in ascending order of size; Conversion Factors; Simple and Compound Interest Tables; Logarithms; Powers, Roots and Reciprocals; and much more.

The authors and editors of *Mathematics Encyclopedia* have endeavored to create a useful tool for their readers, one that will satisfy their need for an easy-to-use, comprehensive and up-to-date mathematical reference work. To this end, this volume has been thoroughly cross-referenced. Words appearing in small capital letters indicate that there is an entry under that heading. In addition the reader is referred to other entries by a *see* or *see also* reference. And finally a Pronunciation Guide immediately follows this page.

Pronunciation Guide

Pronunciation keys appear within parentheses following the last element of the entry heading. Example: **DESCARTES, RENE** (day-kahrt′ ru-nay′) . . . Keys are provided for selected entries, and when two or more entries have the same pronunciation, a key is provided only for one heading. In order to avoid using intricate symbol systems which are less familiar, a system of phonetic keys was devised using only the standard letters of the alphabet. Only primary stress is given. Combinations of silent letters are used where necessary to make the pronunciation clear. Below are the phonetic keys used, accompanied by a brief indication of how they are intended to be pronounced.

Vowels

ay	p*ay*
a	p*a*t
ahr	*ar*k
ah	f*a*ther, Mari*a*, p*o*t, h*o*t
e	p*e*t
ehr	*care, air*
ee	b*e*, *re*al
i	*ih* p*i*t
ahy	*pie, buy*
o	(or) oh no, toe
aw	p*aw*, f*o*r
oi	n*oi*se, b*oy*s
au	*ou*t, ab*ou*t
oo	b*oo*t, n*ew*
uh	t*oo*k, sh*ou*ld
u	c*u*t, *a*bout, it*e*m, nat*io*n
ur	*ur*ge, s*er*ge, f*ir*m

Consonants

s	*c*ellar
sh	*sh*ell (unvoiced)
k	*c*ome
ch	*ch*ur*ch*
g	*gag*
j	*j*udge
zh	plea*s*ure (voiced)
th	*th*in, *th*en (both voiced and unvoiced)

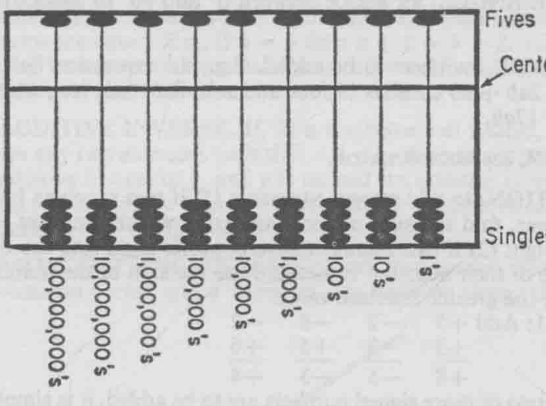
Foreign Sounds

ah	Fr. *a*mi
uh	Fr. f*eu*, Ger. sch*ö*n
Kh	Ger. i*ch*, Scot. lo*ch*
ur	Fr. s*oe*ur
n	Fr. bo*n*, Port. sao (saun)
oo	Fr. t*u*, Ger. *ü*ber

Contents

A

ABACUS (ab′-u-kus), an ancient device used for computation; predecessor of the modern calculating machine. The abacus is still widely used, especially in China and Japan. The Japanese abacus, called a *soroban*, has five beads on each of nine vertical place rods. One bead on each rod is isolated above a center bar on the abacus. Each rod has PLACE VALUE in the DECIMAL NUMBER SYSTEM. Each bead below the center bar counts as a single



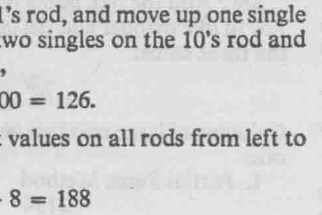
unit of the place, and each bead above the bar counts as five times the place value. Thus, one bead on the ten's rod below the bar is ten while the bead above on the same rod is 50. Beads "count" only as they are moved toward or away from the center bar.

The speed and accuracy of computation by abacus was effectively demonstrated in 1946, when a soroban was pitted against a modern electric desk calculator. The abacus won in addition subtraction, division, and in a combination of these, while the desk caluclator won only in multiplication.

Sample of addition on the abacus:

Add 62 to 126.

a. Set the 62 by moving up two singles on the 1's rod, one single on the 10's rod, and then move down the five on the 10's rod. Thus,



$$2 + 10 + 50 = 62.$$

b. Continue by setting the 126.
Move down the five bead on the 1's rod, and move up one single on the same rod; then move up two singles on the 10's rod and one single on the 100's rod. Thus,

$$6 + 20 + 100 = 126.$$

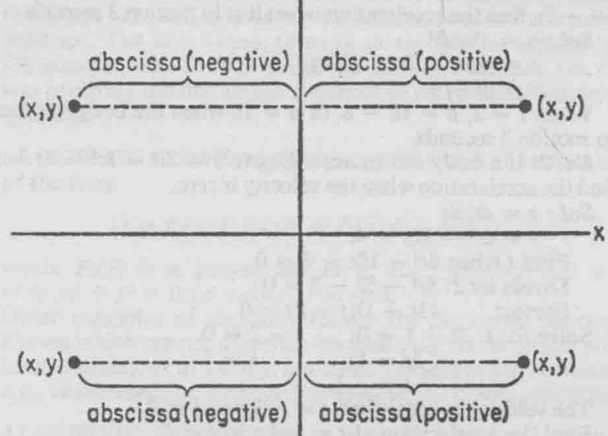
c. Now to get total, read correct values on all rods from left to right at the center bar:

$$100 + 80 + 8 = 188$$

the sum of 62 and 126.

ABSCISSA (ab-sis′-sa), the horizontal or *x*-distance of any point from the *y*-axis in a system of rectangular coordinates (*see* COORDINATES, RECTANGULAR). Its sign is positive to the right of the *y*-axis and negative to the left of the *y*-axis. The coordinates of a point are given as an ORDERED PAIR (x, y). The first member of the ordered pair is the abscissa.

Ex. 1: The abscissa of the point (1, 7) is 1.
Ex. 2: The abscissa of the point $(-3, 2)$ is -3.

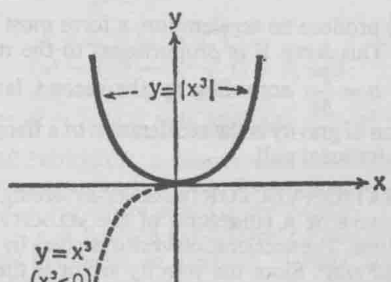


ABSOLUTE CONVERGENCE, a SERIES, $\Sigma|a_n|$, formed by taking the ABSOLUTE VALUES of terms of a series Σa_n may or may not converge even if Σa_n converges. When $\Sigma|a_n|$ does converge, the original series is said to converge absolutely. E.g., the series $1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} \ldots$ is absolutely convergent, since the series $1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} \ldots$ is convergent. A series which converges absolutely also converges in the ordinary sense. A series which converges but does not converge absolutely is said to converge conditionally (*see* CONDITIONAL CONVERGENCE).

ABSOLUTE VALUE, the value of a signed number without regard to its sign. For any number x the "absolute value of x" is denoted by the symbol $|x|$.

$|x| = x$, if x is a positive number or zero and
$|x| = -x$, if x is a negative number.

Ex. 1: a. $|2| = 2$ b. $|0| = 0$ c. $|-3| = 3$
Ex. 2: Graph the function $y = |x^3|$.



Note that where x^3 is positive or zero, the graph of $y = |x^3|$ is the same as the graph of $y = x^3$. However, $|x^3|$ is positive where x^3 is negative. Thus, the graph of $y = |x^3|$ is the mirror image of the graph of $y = x^3$ (dotted curve in the figure for $x < 0$. Since x^3 is negative when $x < 0$, the graph of $y = |x^3|$ is the solid curve shown in the figure.

ACCELERATION (ak-sel-er-ay′-shun), the derivative (*see* DERIVATIVE OF A FUNCTION), or the rate of change of the VELOCITY, v, of a body with respect to time, t. Acceleration is given by $a = dv/dt$. Since the velocity is the derivative of the position, s, of a moving body at time t, the acceleration may also be given as the second derivative of s with respect to t: $a = d^2s/dt^2$.

Ex. 1: If a body moves according to the law of motion $s = t^3 - 4t^2 - 3t$, find the acceleration when it is in motion 3 seconds.

Sol.: $a = d^2s/dt^2$

$a = d(3t^2 - 8t - 3)/dt$

$a = 6t - 8.$

When $t = 3$, $a = 18 - 8$, or $a = 10$ when the body has been in motion 3 seconds.

Ex. 2: If a body moves according to $s = 2t^3 - 5t^2 - 4t - 3$, find the acceleration when the velocity is zero.

Sol.: $v = dt/ds$

$v = 6t^2 - 10t - 4.$

Find t when $6t^2 - 10t - 4 = 0.$

Divide by 2: $3t^2 - 5t - 2 = 0$

Factor: $(3t + 1)(t - 2) = 0$

Solve for t: $3t + 1 = 0$; $t - 2 = 0$

$3t = 1$; $t = 2$

$t = -\frac{1}{3}$.

The velocity is zero when $t = -\frac{1}{3}$ and $t = 2$.

Find the acceleration at $t = -\frac{1}{3}$ and $t = 2$:

$a = dv/dt = d(6t^2 - 10t - 4)/dt = 12t - 10.$

When $t = -\frac{1}{3}$, $a = 12(-\frac{1}{3}) - 10 = -14.$

When $t = 2$, $a = 12(2) - 10 = 14.$

The acceleration is ± 14 when the velocity is zero.

Since velocity is a VECTOR quantity, acceleration is also a vector. Acceleration may change either the speed of an object, its direction, or both. Negative acceleration, that is, when the speed is decreased, is sometimes called deceleration. A body moving in a circle at constant speed is being acted on by a constant acceleration at right angles to the direction of its motion. For motion along a straight line, the change in velocity is equal to the acceleration a multiplied by the time t, so that the final velocity v is given by the original velocity v_0 plus the change in velocity, or

$$v = v_0 + at$$

For motion in a circle of radius r at a constant speed v, the acceleration is *centripetal*, that is directed toward the center, and is given by the equation

$$a_c = \frac{v^2}{r}$$

In order to produce an acceleration, a force must be exerted on an object. This force F is proportional to the mass m of the object, or $a = \frac{F}{M}$, according to the second law of motion. Acceleration of gravity is the acceleration of a freely falling body due to gravitational pull.

ACCELERATION VECTOR (ak-sel-er-ay′-shun), the derivative (*see* DERIVATIVE OF A FUNCTION) of the VELOCITY VECTOR with respect to time. The acceleration vector is given by $\mathbf{a} = d\mathbf{v}/dt = \mathbf{i}\, d^2x/dt^2 + \mathbf{j} d^2y/dt^2$. Since the velocity vector is the derivative of the POSITION VECTOR, the acceleration vector is the second derivative of the position vector.

The acceleration vector may also be expressed in terms of its tangential (*see* TANGENT VECTOR) and normal (*see* NORMAL VECTOR) components. Expressing $\mathbf{v}$ in terms of the tangent vector, $\mathbf{T}$, $\mathbf{v} = \mathbf{T}ds/dt$. The derivative of this with respect to t gives $\mathbf{a} = d\mathbf{v}/dt = \mathbf{T}d^2s/dt^2 + \frac{ds}{dt}\frac{d\mathbf{T}}{dt}$. By the chain rule, $\frac{d\mathbf{T}}{dt} = \frac{d\mathbf{T}}{ds}\frac{ds}{dt}$. Since the normal vector $\mathbf{N}_K = d\mathbf{T}/ds$, $\mathbf{a} = \mathbf{T}d^2/dt^2 + \frac{ds}{dt}\mathbf{N}_K\frac{ds}{dt} = \mathbf{T}d^2s/dt^2 + \mathbf{N}_K(ds/dt)^2$.

ACCUMULATION POINT (u-keum-eu-lay′-shun), a point x is an accumulation point of a SET A when every NEIGHBORHOOD of x contains at least one point of A distinct from x.

ACCUMULATOR (u-keum′-eu-lay-tor), an arithmetic part of a computing machine that adds to the stored number each new number that it receives. An accumulator is also called an *adder*.

ACRE, *see* TABLE NO. 4, 5.

ACUTE ANGLE, an ANGLE between 0° and 90° in ABSOLUTE VALUE.

ADDENDS, numbers to be added. E.g., the expression $8ab + 6ab + 2ab + ab$ consists of four addends, $8ab$, $6ab$, etc., whose sum is $17ab$.

ADDER, *see* ACCUMULATOR.

ADDITION, to add SIGNED NUMBERS: (1) if two numbers have like signs, find the sum of their ABSOLUTE VALUES and use the same sign; (2) if two numbers have opposite signs find the difference of their absolute values and use the sign of the number having the greater absolute value.

Ex. 1: Add

+5	−2	−8	−2
+3	−3	+5	+6
+8	−5	−3	+4

When two or more signed numbers are to be added, it is simpler to first add all POSITIVE NUMBERS, next add all NEGATIVE NUMBERS, and then combine the two sums.

Ex. 1: Add $(-8) + (+5) + (-2) + (+6)$.

$$\begin{aligned}(-8) + (+5) + (-2) + (+6) &= +(5 + 6) - (8 + 2)\\ &= 11 - 10\\ &= 1.\end{aligned}$$

To add POLYNOMIALS, it is convenient to place similar terms (*see* TERMS, SIMILAR) in columns and add their NUMERICAL COEFFICIENTS. Add each column independently of the other columns, observing rules for adding signed numbers.

Ex. 1: Add $4x^2 + 3xy + ab$ and $3x^2 + 7xy - 6ab$.

$$\begin{array}{r} 4x^2 + 3xy + ab \\ 3x^2 + 7xy + 6ab \\ \hline 7x^2 + 10xy - 5ab \end{array}$$

The numerical coefficients of like terms may also be added mentally and the sums of the unlike terms with their proper signs are then written as a simplified polynomial.

Ex. 2: Combine $6a^2 - 3ab - 2a^2 - b^2 - 2ab + b^2 + 3a^2 - 2b^2$.

Sol.: Add the like terms $6a^2 - 2a^2 + 3a^2$, the sum of which is $7a^2$. In like manner add the ab terms and the b^2 terms. Then write the three sums:

$$7a^2 - 5ab - 2b^2.$$

Columns of large numbers may be added by the following methods:

1. Partial Sums Method

2134	
4742	
7358	
+ 3972	
16	Sum of units
19	Sum of tens
20	Sum of hundreds
16	Sum of thousands
18206	

2. Carrying Method

2134	The sum of the unit's column is 16.
4742	Write 6 in the unit's place of the sum and carry the
7358	1 ten to the ten's column. Then the sum of the ten's
+ 3972	column is 20. Write 0 in the ten's column to serve as
18206	a place holder and carry the 2 hundreds to the hun-

dred's column. The sum of the hundred's column is 22. Write 2 in the hundred's column and carry the 2 thousands to the thousand's column. Grouping numbers that equal 10 speeds any addition operation. Add from top to bottom, check by adding up.

The first method is helpful if long columns of large numbers are to be added because checking is simpler. *See also* PLACE VALUE.

ADDITION AXIOM, an AXIOM of EQUALITY (or INEQUALITY): If equal quantities are added to the same or equal quantities the sums are equal. E.g., if $a = b$ then $a + 2 = b + 2$. The axiom may be expressed symbolically as follows: If $a = b$ and $c = d$, then $a + c = b + d$.

ADDITIVE INVERSE. If, in a mathematical system, x and y are any two elements such that $x + y = 0$, then x is termed the additive inverse of y, and y is termed the additive inverse of x: e.g., in arithmetic 5 and -5 are additive inverses of each other because $5 + (-5) = 0$.

ADJACENT ANGLES (ad-jay′-sent), two angles that have a common vertex and a common side between them. In Fig. 1,

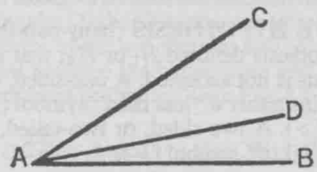


Fig. 1

$\angle BAD$ and $\angle DAC$ are adjacent angles; the common vertex is A, and AD is the common side between them.

If two adjacent angles are formed by PERPENDICULAR lines, the angles are equal, and each contains 90°. In Fig. 2, if $CD \perp AB$, then $\angle ADC = \angle CDB$.

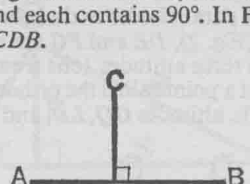


Fig. 2

AGE PROBLEM, in mathematics.

Ex.: A man is 7 times as old as his son. In two years the father will be only 5 times as old as his son. What is the age of each?

Sol.: Let

x = son's age now
$7x$ = father's age now
$x + 2$ = son's age in 2 years
$7x + 2$ = father's age in 2 years.

Then
$7x + 2 = 5(x + 2)$
$7x + 2 = 5x + 10$
$7x - 5x = 10 - 2$
$2x = 8$
$x = 4$, years in son's age now
$7x = 28$, years in father's age now.

AHMES PAPYRUS (ahms pa′-pi-rus), also **Rhind Papyrus,** the oldest surviving work of ancient Egyptian mathematics from ab. 1600 B.C., and now in the British Museum. It was compiled by Ahmes, a scribe, from an earlier treatise dating ab. 2300 B.C.

ALGEBRA (al′-ji-brah), traditionally and still most commonly, the branch of mathematics in which operations of ARITHMETIC are generalized by the use of letters to represent quantities, as in FORMULAS and EQUATIONS.

The name "algebra" was taken from one of the works of Mohammed ibn-Musa al-Khowarizmi which was written in Bagdad ab. 825 A.D. The title was *Al-jabr w'al muqabalah.* The word *al-jabr* is Arabic while *muqabalah* is Persian and it is thought that each referred to an equation. This work was so important and had such influence upon European mathematicians that the name "algebra" was adopted, with various spellings.

The earliest known treatise which could be called algebraic is the AHMES PAPYRUS, now in the British museum and written ab. 1600 B.C. The only Greek to write extensively on algebra was Diophantus, who lived in about the mid-third century A.D. and was probably the first to use a symbol to represent an unknown quantity.

ALGEBRAIC CURVE (al-ji-bray -ik), the graph of an equation of the form

$$O = P_0(x) + P_1(x)y + P_2(x)y^2 + Pn(x)y^{n'},$$

where $P_i(x)$ is a polynomial in x. E.g., $x^2 + xy^2 - 1 = 0$; $x^3 + xy + y^3 = 0$; $(x + 5)(y - 4) = 0$.

Other examples of algebraic curves are the CONIC SECTIONS. Curves which are not algebraic because they do not graph polynomial equations in x and y, are called TRANSCENDENTAL CURVES, e.g., TRIGONOMETRIC CURVES. For sample, *see* CURVE SKETCHING.

ALGEBRAIC EXPRESSION, a quantity made up of letters, NUMERALS, and other ALGEBRAIC SYMBOLS. The parts of an expression that are connected by plus and minus signs are the *terms* of an expression. The sign immediately preceding the term is the sign of that term. Thus, $3x^2yz^2$, $2a$, and x/y are expressions of one term each. The expression $3x - 2y$ has two terms, $3x$ and $-2y$. In the expression $2a^2b - 3ab/2c + 2a(b + c)$, the three terms are $2a^2b$, $-3ab/2c$, and $2a(b + c)$. If there is no sign preceding the term it is understood to be plus.

If an algebraic expression has one term, it is called a *monomial;* if it has two terms, it is called a *binomial*; if it has three terms, it is called a *trinomial.* An algebraic expression of more than one term is called a POLYNOMIAL.

ALGEBRAIC FUNCTION, any FUNCTION whose dependent value (*see* VARIABLE) is an ALGEBRAIC EXPRESSION. If y is a function of x such that values of y corresponding to every x can be found from $f_0(x)y^n + f_1(x)y^{n-1} + \ldots + f_{n-1}(x)y + f_n(x) = 0$, where $f_0(x), f_1(x), \ldots, f_n(x)$ are polynomials in x, then y is an algebraic function of x. E.g., $y = \sqrt{\frac{x^2+1}{x^2-1}}$ is an algebraic function of x since $y^2 = \frac{x^2+1}{x^2-1}$,

$$y^2(x^2 - 1) = x^2 + 1,$$
$$(x^2 - 1)y^2 - (x^2 + 1) = 0, \text{ and}$$
$$(x^2 - 1)y^2 + (-x^2 - 1) = 0.$$

Here, $f_0(x) = x^2 - 1$, $f_1(x) = 0$, and $f_2(x) = -x^2 - 1$.

ALGEBRAIC NUMBER, a NUMBER that satisfies a polynomial equation with integral coefficients. Since a RATIONAL NUMBER is defined by $x = p/q$ where p and q are integers, x satisfies the equation $qx - p = 0$ which has integral coefficients. Therefore, every rational number is an algebraic number, and every nonalgebraic number (TRANSCENDENTAL NUMBER) must be irrational. However, there are IRRATIONAL NUMBERS that are algebraic numbers: $\sqrt{2}$ satisfies $x^2 - 2 = 0$, a polynomial equation with integral coefficients. Not all numbers have been classified as either algebraic or transcendental. E.g., it is not yet known whether π^π is algebraic or transcendental.

ALGEBRAIC SYMBOLS, as in TABLE NO. 2, conventional signs which enable us to abbreviate mathematical expressions which otherwise would have to be made at length in words.

Ex. 1: If n represents a certain number, express the following algebraically:

1. The PRODUCT of 5 and the number.	$5n$
2. The number divided by 3.	$n/3$
3. 7 divided by 2 times the number.	$7/2n$
4. 5 times the sum of the number and 6.	$5(n+6)$
5. The SQUARE ROOT of 2 less than the number.	$\sqrt{n-2}$
6. The number increased by 3.	$n+3$

Ex. 2: Represent the following:

1. The difference of x and y.	$x-y$
2. The sum of 8 and b subtracted from a.	$a-(8+b)$
3. The product of 2, a, and b.	$2ab$
4. The sum of x and y divided by the product of x and y.	$(x+y)/xy$
5. The QUOTIENT when b divides a.	a/b

ALGEBRA OF PROPOSITIONS, a two-valued BOOLEAN ALGEBRA of logical relationships among PROPOSITIONS. Also called the **propositional calculus** or **symbolic logic,** it does for traditional logic what analytic geometry did for traditional geometry.

In this algebra the small letters, $p, q, r, \ldots$ generally stand for propositions. The algebra's unity and ZERO symbols, 1 and 0, stand for the truth values, true and false, respectively. Other standard symbols are:

$\sim$ for "not" or negation (*see* NEGATION, LOGICAL).

$\wedge$ or $\cdot$ (dot) for "and" or CONJUNCTION.

$\vee$ or $+$ for "or" or DISJUNCTION.

$\rightarrow$ for "if. . . , then . . ." or the conditional (*see* CONDITIONAL STATEMENT).

$\leftrightarrow$ for "if and only if . . ." or the bi-conditional (*see* BI-CONDITIONAL, LOGICAL).

Thus, $\sim p$ is read "not p;" $p \wedge q$, "p and q;" $p \vee q$, "p or q;" $p \rightarrow q$, "if p, then q;" $p \leftrightarrow q$, "p if and only if q."

ALGOL (al'-gol). In computer terminology, an acronym for **Algorithmic Language,** one of several major key automatic coding systems for computers. *ALGOL* is really a language, developed for the purpose of translating higher level instructions into the specific codes of the computer. *See* COMPUTER.

ALGORITHM (al'-guh-rith-um), any method that is used to perform a calculation. The term is applied more commonly to those methods of calculation that require repetition of some process, as in DIVISION, or finding the GREATEST COMMON DIVISOR by EUCLID'S ALGORITHM.

ALIQUOT PART (al-i-kwot'), any factor of an integer except the integer itself. For example, 2, 3, 5, 6, 15 are called aliquot parts of 30.

ALMAGEST (al'-muh-jest), title which the Arabians gave to a translation of *Megiste Syntaxis*, a treatise on astronomy by PTOLEMY ab 150 A.D. The word means "the greatest." This work formed a basis for the later development of trigonometry.

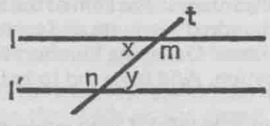
ALPHA (al-fah), the Greek letter symbolized by α. *See* TABLE NO. 1.

ALPHA ERROR, in STATISTICS, the error that results when a NULL HYPOTHESIS is erroneously rejected. The size of this error is usually denoted by α and chosen to be .005, .01, .05, or .10. An alpha error is also called a Type I error.

ALPHA LEVEL, in STATISTICS, the SIGNIFICANCE LEVEL of a test; the size of the ALPHA ERROR. If the alpha level of a test is .05, it means that 5 times out of 100, on the average, the NULL HYPOTHESIS would be rejected even though it is true.

ALTERNATE-INTERIOR ANGLES, when lines are cut by a TRANSVERSAL, the pairs of angles between the lines and on opposite sides of the transversal.

In the figure, alternate-interior angles are $\angle x$ and $\angle y$ and $\angle m$ and $\angle n$. If two PARALLEL LINES are cut by a transversal, the alternate-interior angles are equal. In terms of the figure, if $l \| l'$, then $\angle x = \angle y$ and $\angle m = \angle n$.



If two lines are cut by a transversal and the alternate-interior angles are equal, the lines are parallel. Thus, if $\angle x = \angle y$, or if $\angle m = \angle n$, then $l \| l'$.

ALTERNATING SERIES, a SERIES in which every other term is negative.

Ex.: $\sum_{n=1}^{\infty} (-1)^{n+1}/n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is an alternating series.

An alternating series is called a CONVERGENT SERIES if $a_n \rightarrow 0$ as $n \rightarrow \infty$ in such a way that $|a_{n+1}| \leq |a_n|$. *See also* CONDITIONAL CONVERGENCE.

ALTERNATION, the process by which a given PROPORTION is changed to another by interchanging the means, so that the first term is to the third term as the second is to the fourth.

Ex.: $a/b = c/d$, by alternation, $a/c = b/d$.

ALTERNATIVE HYPOTHESIS (hahy-pah-thuh-seez), in statistics, the hypothesis denoted H_1 or H_A, that is accepted if the NULL HYPOTHESIS is not accepted. A one-sided, or one-tailed, alternative contains either a "less than" symbol ($<$) or a "greater than" symbol ($>$). A two-sided, or two-tailed, alternative contains a "not equal to" symbol ($\neq$).

ALTITUDE OF A GEOMETRIC FIGURE, the PERPENDICULAR from any vertex to the opposite side, or to the opposite side extended. In PARALLELOGRAM $ABCD$ (Fig. 1), DE, GH, and CF are altitudes to the base AB.

The altitude of a TRAPEZOID is the perpendicular from any point on one of the parallel sides to the other parallel side. In the trapezoid $ABCD$ (Fig. 2), DE and FG are altitudes to base AB.

A TRIANGLE has three altitudes (one from each vertex) which are CONCURRENT at a point called the orthocenter. In OBTUSE TRIANGLE LGF (Fig. 3), altitudes GQ, LD, and FJ intersect at S. S is

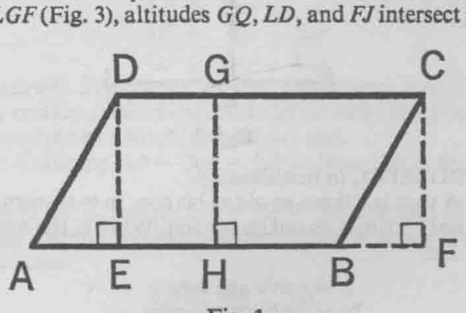


Fig. 1

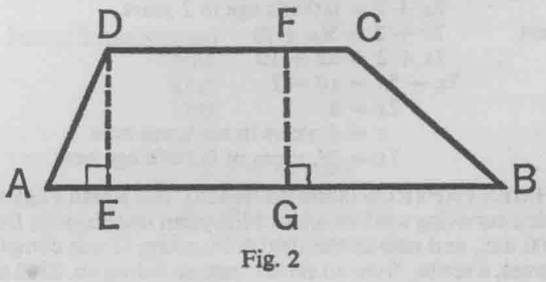


Fig. 2

the orthocenter of triangle LGF. The altitude of an EQUILATERAL TRIANGLE may be found by the formula $h = \frac{s}{2}\sqrt{3}$, in which s represents any one of the equal sides and h represents the altitude to that side.

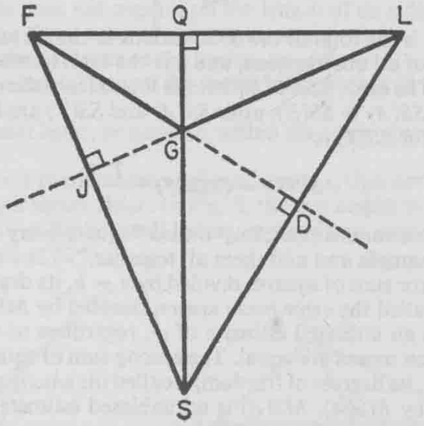


Fig. 3

In a RIGHT TRIANGLE, either leg is an altitude and the third side is the BASE. When an altitude is drawn to the base of a right triangle, (1) the two triangles formed are similar and each is similar to the original triangle; (2) the altitude is the MEAN PROPORTIONAL between the segments of the hypotenuse; and (3) either leg is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg. (See Fig. 4.)

1. $\triangle ACD \sim \triangle CDB$; $\triangle ACD \sim \triangle ACB$; $\triangle CDB \sim \triangle ACB$.
2. $\frac{r}{h} = \frac{h}{s}$.
3. $\frac{c}{b} = \frac{b}{r}$; $\frac{c}{a} = \frac{a}{s}$.

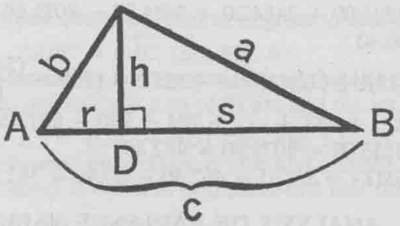


Fig. 4

AMBIGUOUS CASE (am-big′-eu-us), in the solution of triangles by TRIGONOMETRY, the case in which two sides and the angle opposite one of them are given. It is so called because one, two, or no triangles may be constructed depending on the relations of the given parts.

If the given parts are a, b, and A, the ALTITUDE h from C can be found from the equation $h/b = \sin A$; $h = b \sin A$. If A is acute and $a < b$ and if $a > b \sin A$, or if $a > h$, there will be two triangles, ABC and $AB'C$. (See Fig. 1.)

If A is acute, $a < b$, and $a = b \sin A$, or h, there is one triangle, a right triangle. (See Fig. 2.)

If A is acute, $a < b$, and $a = b \sin A$, or h, there can be no triangle and no solution. (See Fig. 3.)

If A is a right angle, there can be one solution if $a > b$, and no solution if $a = b$ or if $a < b$. If A is obtuse, there can be one solution if $a > b$, and no solution if $a = b$ or if $a < b$.

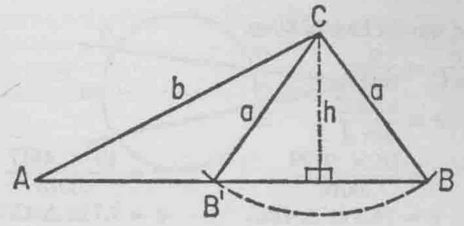


Fig. 1

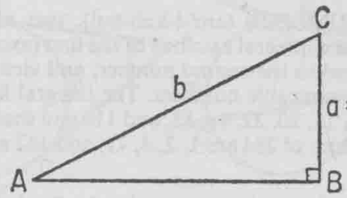


Fig. 2

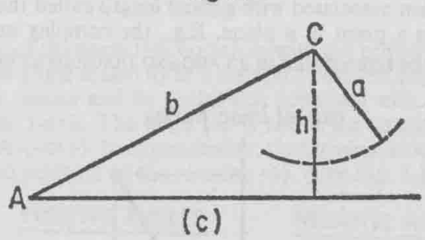


Fig. 3

Ex. 1: Solve triangle ABC, given $A = 54°$, $b = 16$, and $a = 9$.

$$h = b \sin A$$
$$h = 16 \times .8090 = 12.944.$$

Therefore, there can be no solution, because the altitude is greater than a.

Ex. 2: Solve triangle ABC, given $A = 37°$, $b = 15$, and $a = 10$.

$$h = b \sin A$$
$$h = 15 \times .6018 = 9.027.$$

Since $a > b \sin A$, two triangles may be constructed.

By the Law of Sines:

$$\frac{\sin B}{\sin A} = \frac{b}{a} \text{ and } \sin B = \frac{b \sin A}{a}.$$

Since angle B is determined by its sine, it may have two values which are supplementary; therefore either of the values may be taken. (See Fig. 4.)

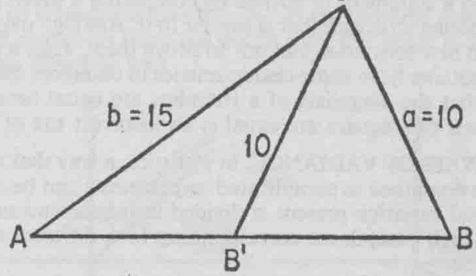


Fig. 4

$$\sin B = \frac{b \sin A}{a} = .9027$$

$B = 64° 30'$ or $115° 30'$ to the nearest minute.

$C = 180° + (A + B)$

$C = 78° 30'$ or $27° 30'$.

To find c, use the Law of Sines.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

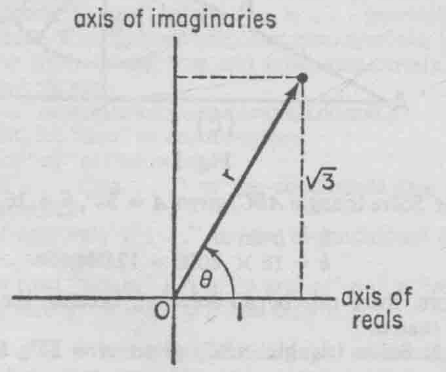
$$c = \frac{10 \times .9799}{.6018} \qquad c = \frac{10 \times .4617}{.6018}$$

$$c = 16.3 \text{ in } \triangle ABC. \qquad c = 7.7 \text{ in } \triangle AB'C.$$

See also SINES, LAW OF.

AMICABLE NUMBERS (am'-i-kuh-bul), two numbers such that the sum of the integral FACTORS of the first (except the number itself) is equal to the second number, and vice versa. Thus, 220 and 284 are amicable numbers. The integral factors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110 and their sum is 284. The integral factors of 284 are 1, 2, 4, 71, and 142 and their sum is 220.

AMPLITUDE, of a COMPLEX NUMBER, the fixed positive angle which, when associated with a fixed length called the MODULUS, determines a point in a plane. E.g., the complex number $1 + \sqrt{3}i$, may be represented in an ARGAND DIAGRAM as in the figure.



Then θ is the angle whose TANGENT is $\sqrt{3}/1$ (arctan $\sqrt{3}/1$) or 30°. The amplitude of $1 + \sqrt{3}i$ is 30°. In general, the amplitude of the complex number $a + bi$ is arctan b/a.

Of a TRIGONOMETRIC CURVE, the greatest numerical ORDINATE of the curve of a graph of a TRIGONOMETRIC FUNCTION. In $y = \sin x$, the amplitude of the curve is 1; in $y = \frac{1}{2} \sin x$ it is $\frac{1}{2}$. In acoustics, intensity of a tone is determined by the amplitude and pitch is determined by the period.

ANALOG COMPUTER, *see* COMPUTER.

ANALOGY (u-nal'-uh-jee), in mathematics, a form of reasoning whereby a statement is formed by comparing a given structure with another structure that is similar to it. Analogy may be used to form new THEOREMS, but not to prove them. E.g., a RHOMBUS and a SQUARE have some characteristics in common, but to conclude that the diagonals of a rhombus are equal because the diagonals of a square are equal is an incorrect use of analogy.

ANALYSIS OF VARIANCE, in statistics, a way that the information contained in complicated experiments can be analyzed. The total variation present is divided into independent components, each component corresponding to a different source of variation.

The simplest situation to which the analysis of variance technique can be applied is the *one-way analysis of variance*. There are k independent samples of sizes $n_1, n_2, \ldots, n_k$ from k normal populations, each with variance σ^2. The NULL HYPOTHESIS is that the means of the k populations are equal, H_0: $\mu_1 = \mu_2 = \cdots = \mu_k$. The ALTERNATIVE HYPOTHESIS is that they are not all equal. The total variation present, called the *total sum of squares*, is composed of two separate pieces: the *among samples sum of squares*, denoted by $SS(A)$; and the *within samples sum of squares* (or *error sum of squares*), denoted by $SS(E)$.

The formula for $SS(A)$ is

$$SS(A) = \left(\sum_{i=1}^{k} \frac{T_{i.}^2}{n_i} \right) - \frac{T_{..}^2}{n},$$

where $T_{i.}$ is the total of the observations in the ith sample, $T_{..}$ is the total of all observations, and n is the total number of observations. The error sum of squares is found from the relationship $SS(T) = SS(A) + SS(E)$, after $SS(A)$ and $SS(T)$ are found. The formula for $SS(T)$ is

$$SS(T) = (\Sigma\Sigma x_{ij}^2) - \frac{T_{..}^2}{n}.$$

The double summation $\Sigma\Sigma x_{ij}^2$ means "square every observation in every sample and add them all together."

The error sum of squares divided by $n - k$, its degrees of freedom, is called the *error mean square*, denoted by $MS(E)$. $MS(E)$ furnishes an unbiased estimate of σ^2, regardless of whether the population means are equal. The among sum of squares divided by $k - 1$, its degrees of freedom, is called the *among mean square*, denoted by $MS(A)$. $MS(A)$ is an unbiased estimate of σ^2 if the populations are equal. If the population means are not equal, the among mean square is expected to be larger than the error mean square. If it is significantly larger, the hypothesis of equal population means is rejected. The test-statistic is $F = \frac{MS(A)}{MS(E)}$. The null hypothesis of equal means is rejected if F is equal to or greater than the upper α-point of the F-distribution with $k - 1$ and $n - k$ degrees of freedom.

Ex.:

	Sample 1		*Sample 2*		*Sample 3*
	17		20		27
	22		15		28
	14		18		19
	23		31		28
	29		27		30
Totals	105	+	111	+	132 = 348

$$SS(A) = \frac{(105)^2}{5} + \frac{(111)^2}{5} + \frac{(132)^2}{5} - \frac{(348)^2}{15}$$

$$= 2205.00 + 2464.20 + 3484.80 - 8073.60$$

$$= 80.40$$

$$SS(T) = (17)^2 + (22)^2 + \cdots + (28)^2 + (30)^2 - \frac{(348)^2}{15}$$

$$= 289 + 484 + \cdots + 784 + 900 - 8073.60$$

$$= 8536.00 - 8073.60 = 462.40.$$

$$SS(E) = SS(T) - SS(A) = 462.40 - 80.40 = 382.00.$$

ANALYSIS OF VARIANCE TABLE

Source	*Degrees of Freedom d.f.*	*Sample Sum of Squares S.S.*	*Mean Square M.S.*	*F*
Among (*A*)	2	80.40	40.20	1.26 (not significant)
Within (*E*)	12	382.00	31.83	
Total	14	462.40	—	

ANALYSIS OF VARIANCE TABLE, a table that displays, for an analysis of variance, the sources of variation, along with the degrees of freedom, *d.f.*, the sum of squares, *S.S.*, and the mean square, *M.S.*, for each source of variation. *See* ANALYSIS OF VARIANCE.

ANALYTIC FUNCTION, a complex-valued function of a complex variable which is differentiable. *See also* CAUCHY-RIEMANN EQUATIONS.

ANALYTIC GEOMETRY, *see* GEOMETRY, ANALYTIC.

ANCHOR RING, *see* TORUS.

ANGLE, a geometric figure formed by two RAYS (or half-lines) drawn from the same POINT. The symbol for angle is ∠. In Fig. 1, rays *BA* and *BC* meet in point *B* forming $\angle ABC$. The rays are called the sides of the angle and the point is its VERTEX. The size of an angle does not depend on the length of its sides.

There are three ways of naming an angle:

1. By the capital letter at the vertex, as $\angle B$.
2. By three capital letters, the vertex letter always in the middle, as $\angle ABC$.
3. By a small letter, or number, within the angle near the vertex, as $\angle x$.

If there are two or more angles at a vertex, they are never read by the single vertex letter. In Fig. 2, the two angles at *D* are read $\angle ADC$ and $\angle BDC$. Small letters, or numbers, may be used, as $\angle 1$ and $\angle 2$ at *C*.

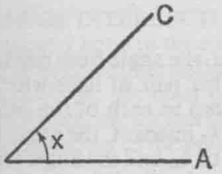


Fig. 1

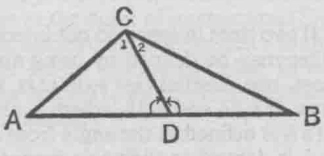


Fig. 2

In a circle, the following theorems are used to measure angles:

A CENTRAL ANGLE is equal in degrees to its ARC. $\angle AOB \triangleq \widehat{AB}$. (See Fig. 3.)

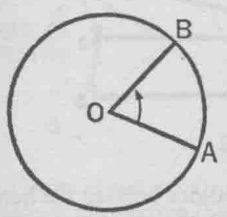


Fig. 3

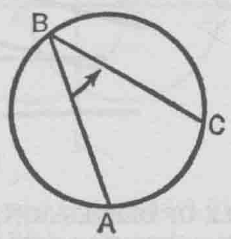


Fig. 4

An INSCRIBED ANGLE is equal in degrees to one half its intercepted arc. $\angle ABC \triangleq \frac{1}{2}\widehat{AC}$. (See Fig. 4.)

An angle formed by two CHORDS intersecting in a circle is equal in degrees to one half the sum of its arc and the arc of its VERTICAL ANGLE. $\angle x \triangleq \frac{1}{2}(\widehat{AC} + \widehat{BD})$. (See Fig. 5.)

An angle formed by a TANGENT LINE and a chord drawn to the point of tangency is equal in degrees to one half the arc it intercepts. $\angle ABC \triangleq \frac{1}{2}\widehat{AB}$. (See Fig. 6.)

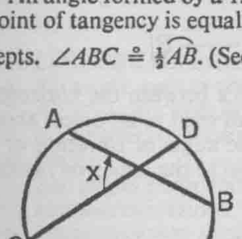


Fig. 5

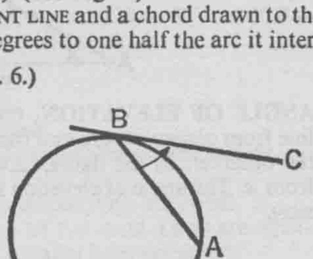


Fig. 6

An angle formed by two secant lines, two tangent lines, or a SECANT LINE and a tangent line meeting outside the circle is equal in degrees to one half the difference of the intercepted arcs. $\angle A = \frac{1}{2}(\widehat{m} - \widehat{n})$. (See Fig. 7.)

One of the rays that form the angle is called the initial side of the angle. In trigonometry the initial side (*see* SIDE OF AN ANGLE)

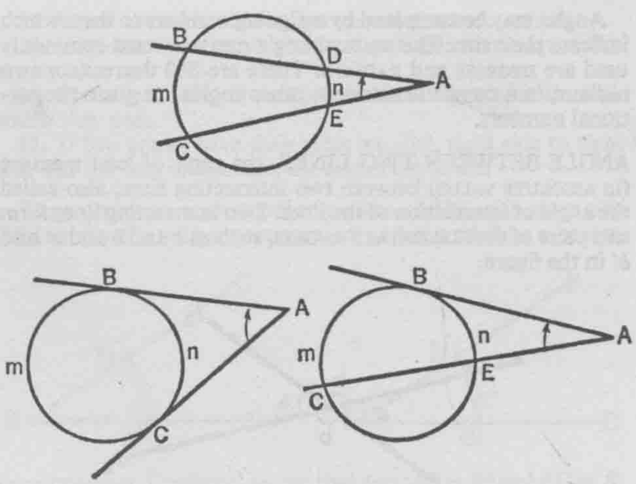


Fig. 7

is the side of the angle that coincides with the positive side of the *x*-axis. An angle is said to be a standard position angle if its vertex is the ORIGIN and its initial side coincides with the positive side of the *x*-axis. The other ray is called the terminal side (*see* SIDE OF AN ANGLE). In trigonometry, the terminal side of an angle is the final position of the rotating ray. (See Fig. 8.) The size of

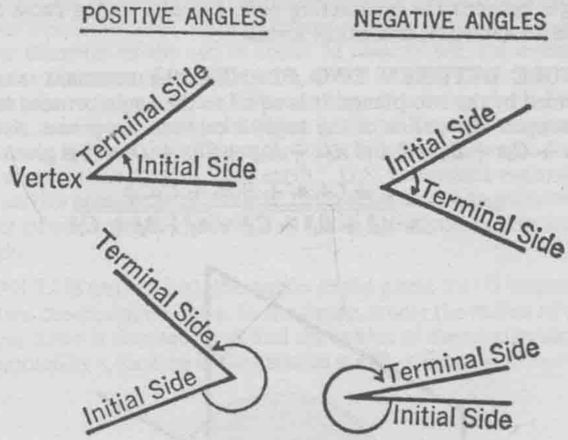


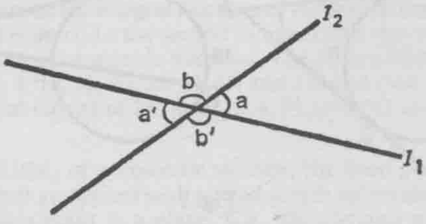
Fig. 8

an angle is determined by its terminal side. An angle is considered to be of the QUADRANT in which its terminal side lies. The rotating ray may make more than one revolution. If the terminal side has been rotated counterclockwise from the initial side, the angle is said to be positive (See Fig. 9a.) If the terminal side has been rotated clockwise from the initial side, the angle is said to be negative. (See Fig. 9b.)

Fig. 9

Angles may be compared by assigning numbers to them which indicate their size. The units of angle measure most commonly used are DEGREES and RADIANS. There are 360 degrees, or two radians, in a complete rotation; other angles are given proportional numbers.

ANGLE BETWEEN TWO LINES, the angle of least measure (in ABSOLUTE VALUE) between two intersecting lines; also called the angle of intersection of the lines. Two intersecting lines form two pairs of SUPPLEMENTARY ANGLES, such as a and b and a' and b' in the figure.

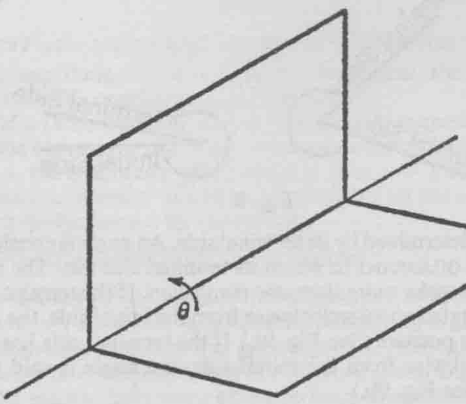


The angle between the two lines is a or a'. Since they are VERTICAL ANGLES, $a = a'$ (and $b = b'$).

If two lines in space do not intersect an angle between them may be defined by using another pair of lines which do intersect, one parallel (*see* PARALLEL LINES) to each of the original lines. The angle between the nonintersecting pair is then defined as the angle between the intersecting pair. *See also* ANGLE FROM ONE LINE TO ANOTHER; DIRECTION ANGLE.

ANGLE BETWEEN TWO PLANES, the DIHEDRAL ANGLE formed by the two planes. It is equal to the angle between their NORMALS. The cosine of the angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by:

$$\cos\theta = \frac{\pm(A_1A_2 + B_1B_2 + C_1C_2)}{\sqrt{A_1^2 + B_1^2 + C_1^2}\,\sqrt{A_2^2 + B_2^2 + C_2^2}}$$



Angle Between Two Planes

If A_1, B_1, C_1 and A_2, B_2, C_2 are proportional, the phases are parallel.

Ex.: Find the cosine of the acute angle between the two planes $2x - 2y + 8z - 6 = 0$ and $x + 3y + 2z - 6 = 0$.

Sol.: $$\cos\theta = \frac{\pm(2 - 6 + 16)}{\sqrt{4 + 4 + 64}\,\sqrt{1 + 9 + 4}} = \frac{\pm 12}{\sqrt{72}\,\sqrt{14}}$$

or approximately .9.

See also ANGLE BETWEEN TWO LINES.

ANGLE FROM ONE LINE TO ANOTHER, the angle of least positive measure from one line to the other. If a pair of lines l_1 and l_2 intersect at point P, then the angle from l_1 to l_2 is the smallest angle formed by rotating l_2 counterclockwise from l_1. There will be two such angles, such as a and a' in Fig. 1, but they are VERTICAL ANGLES and are therefore equal. In Fig. 1, the angle from l_1 to l_2 is a, while the angle from l_2, to l_1 is b.

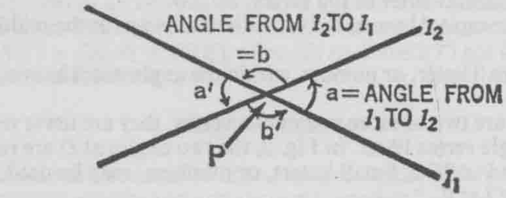


Fig. 1

If two lines in space do not intersect, the angle from one to the other may be defined by using another pair of lines which do meet, one parallel (*see* PARALLEL LINES) to each of the original lines. If $k_1 \| l_1$ and $k_2 \| l_2$, where k_1 and k_2 intersect, the angle from l_1 to l_2 is defined as the angle from k_1 to k_2, and the angle from l_2 to l_1 is defined as the angle from k_2 to k_1. (See Fig. 2.) *See also* ANGLE BETWEEN TWO LINES.

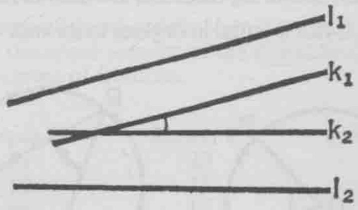
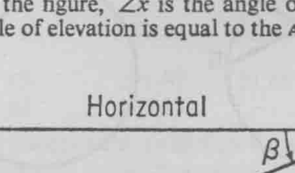
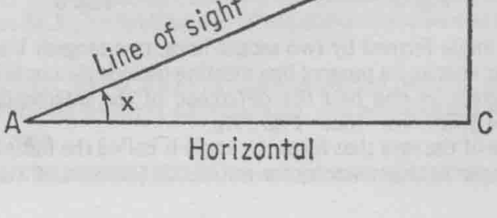


Fig. 2

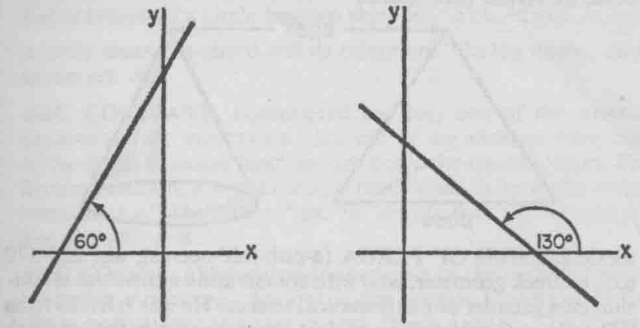
ANGLE OF DEPRESSION, the ANGLE between the horizontal line from observer's eye and the line of sight to an object below observer. In the figure, $\angle y$ is the angle of depression of A from C, where AC represents the line of sight from observer at C to object A, and CD represents the horizontal.



ANGLE OF ELEVATION, the ANGLE between the horizontal line from observer's eye and the line of sight to an object above the observer. In the figure, $\angle x$ is the angle of elevation of C from A. The angle of elevation is equal to the ANGLE OF DEPRESSION.



ANGLE OF INCLINATION, the positive angle between 0° and 180° that a line makes with the x-axis. (See figure.)



angle of inclination = 60° angle of inclination = 130°

ANGLE OF INTERSECTION OF CURVES, the angle between the TANGENT LINES to the curves at the point of intersection (Fig. 1a). The angle of intersection of circles can be computed without using calculus, but most other curves require calculus. In the case of circles, use the fact that a tangent to a circle is perpendicular to the RADIUS to the point of contact.

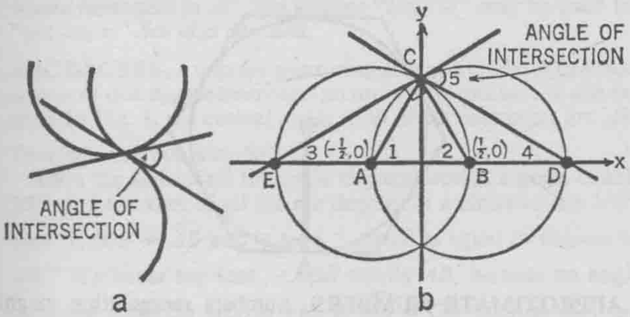


Fig. 1

Ex.: Circles of radius 1 are centered at $(-\frac{1}{2}, 0)$ and $(\frac{1}{2}, 0)$. Find their angle of intersection (See Fig. 1b.).

Sol.:

Method 1: Use the fact that the slopes of perpendicular lines (*see* SLOPE OF A LINE) are negative reciprocals; compute slopes and then find the angle in TABLE NO. 22.

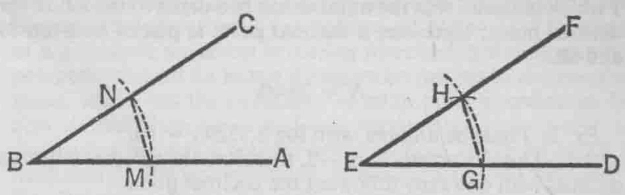
Method 2: Observe that the cosine of angle 1 is $\frac{1}{2}$; therefore $\angle 1$ is 60°. Triangle ACD is a right triangle with $\angle 1$ in it, so $\angle 4$ is $90° - 60° = 30°$. Similarly, $\angle 3 = 60°$. Then $\angle ECD$ is 120°, and $\angle 5$ is 60°.

Method 3: $\triangle ABC$ is equilateral, so $\angle ACB = 60°$. $\angle ACB = \angle 5$. *See also* ANGLE BETWEEN TWO LINES.

ANGLES, EQUAL, in geometry:

1. All RIGHT ANGLES are equal.
2. All STRAIGHT ANGLES are equal.
3. BASE ANGLES of an ISOSCELES TRIANGLE and ISOSCELES TRAPEZOID are equal.
4. The angles of an EQUILATERAL TRIANGLE are equal.
5. ALTERNATE-INTERIOR ANGLES OF PARALLEL LINES are equal.
6. CORRESPONDING ANGLES of parallel lines are equal.
7. VERTICAL ANGLES are equal.
8. Complements of the same angle (*see* COMPLEMENTARY ANGLES), or of equal angles, are equal.
9. Supplements of the same angle, (*see* SUPPLEMENTARY ANGLES) or of equal angles, are equal.
10. Corr. angles of CONGRUENT POLYGONS are equal.
11. Corr. angles of SIMILAR POLYGONS are equal.
12. If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.
13. Opp. angles of a PARALLELOGRAM are equal.
14. Equal ARCS of a CIRCLE have equal CENTRAL ANGLES.
15. INSCRIBED ANGLES which intercept the same or equal arcs are equal.
16. The DIAGONALS of a RHOMBUS bisect the angles through which they pass.
17. If two angles have their sides parallel, right side to right side and left side to left side, the angles are equal.

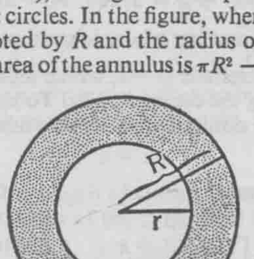
To construct an angle equal to a given $\angle ABC$ with given side ED (see figure), with any convenient radius and B as a center,



use a compass. Construct an arc that cuts AB at M and BC at N. With the same radius and E as a center, construct the arc that cuts ED at G. With a radius equal to the segment joining M and N and with G as the center, draw an arc that cuts the first arc at H. Draw EF passing through point H. $\angle DEF = \angle ABC$. Since radii of the same or of equal circles are equal, $BN = EH$, $BM = EG$, and $NM = HG$. $\triangle BMN \cong \triangle EGH$ (S.S.S.), and $\angle E = \angle B$.

ANGULAR DIAMETER, the diameter of an object from an observer's point of view, without reference to its distance. The angular diameter of the sun is about 32 min. of arc, the average angular diameter of the full moon, about 31 min. Of course, the actual diameters are much different, the sun's being about 865,000 mi. and the moon's only about 2,160 mi. The moon appears almost as large as the sun because it is much closer. As seen from the moon at the time of "full earth," U.S. astronauts evaluated the earth's angular area, the area of an object from an observer's point of view, about 13 times that of the full moon as seen from earth.

ANNULUS (an'-eu-lus), the region in the plane that is bounded by two CONCENTRIC circles. In the figure, where the radius of the larger circle is denoted by R and the radius of the smaller circle is denoted by r, the area of the annulus is $\pi R^2 - \pi r^2$ or $\pi(R^2 - r^2)$.



ANTECEDENT, the first term, or the NUMERATOR, of a RATIO. In the ratio x/y, x is the antecedent. *See also* CONSEQUENT.

ANTIDERIVATIVE OF A FUNCTION, the INDEFINITE INTEGRAL of that function, also called **Primitive Function.** For a function $f(x)$, the antiderivative is a function $g(x)$, such that $g'(x) = f(x)$. Hence, $g(x) = \int f(x)dx + c$.

Ex. 1: Find the antiderivative of $x^2 + 1$.

Sol.: Antiderivate $= g(x) = \int f(x)dx + c$

$$g(x) = \int f(x^2 + 1)dx + c$$

The antiderivative $= \dfrac{x^3}{3} + x + c$. (*See* TABLE NO. 14.)

Ex. 2: Find the antiderivative of $f(x) = 1/x^2 + x, x > 0$.

Sol.: $g(x) = \int (1/x^2 + x) + c$

$$g(x) = -1/x + x^2/2 + c.$$

ANTILOGARITHM, the number corresponding to a LOGARITHM. Finding an antilogarithm (abbreviated *antilog*) is the inverse of finding the logarithm of a number. If the mantissa is not in a Table of Logarithms, (TABLE NO. 22) the number is found by INTERPOLATION.

Ex. 1: Find the number whose logarithm is 1.59770.

Sol.: Let $N =$ the number, then $\log N = 1.59770$.

The mantissa .59770 is found in the Table of Logarithms in the 0 column opposite 396 in the number column. The characteristic is 1 which indicates that the number has two digits to the left of the decimal point; therefore, a decimal point is placed between 39 and 60.

$$N = 39.60.$$

Ex. 2: Find the number with log $8.35296 - 10$.

Sol.: The characteristic is -2, therefore the number will be a decimal with one zero following the decimal point.

Let $\log N =$ the number, then $\log N = 8.35276 - 10$.

$$N = .02253.$$

Ex. 3: Find the number whose logarithm is 4.87749.

Sol.: Let $N =$ the number, then $\log N = 4.87749$.

$$N = 75420.$$

Ex. 4: Find the number whose logarithm is 2.57464.

Sol.: The characteristic 2 indicates a number having three digits to the left of the decimal point. The mantissa .57464 is not in the table but lies between 57461 and 57473. We know the number is between 37550 and 37560. The last digit is determined by interpolation.

$$\begin{array}{ll} \textit{Number} & \textit{Mantissa} \\ 10\left[\begin{array}{l} 37560 \\ x\left[\begin{array}{l} 37550 + x \\ 37550 \end{array}\right. \end{array}\right. & \left.\begin{array}{l} 57473 \\ \left.\begin{array}{l} 57464 \\ 57461 \end{array}\right]3 \end{array}\right]12 \end{array}$$

$$\frac{x}{10} = \frac{3}{12};\ 12x = 30;\ x = 2\tfrac{6}{12} \text{ or approximately } 3.$$

$$37550 + x = 37553;\ N = 375.53.$$

Ex. 5: Find the number whose logarithm is 9.893.46–10.

Sol.: The characteristic is -1, which indicates a decimal with no zeros following the decimal point. To simplify the interpolation, decimals are omitted and only the numbers and mantissas are used.

$$\begin{array}{ll} \textit{Number} & \textit{Mantissa} \\ 10\left[\begin{array}{l} 78250 \\ x\left[\begin{array}{l} 78240 + x \\ 78240 \end{array}\right. \end{array}\right. & \left.\begin{array}{l} 89348 \\ \left.\begin{array}{l} 89346 \\ 89343 \end{array}\right]3 \end{array}\right]5 \end{array}$$

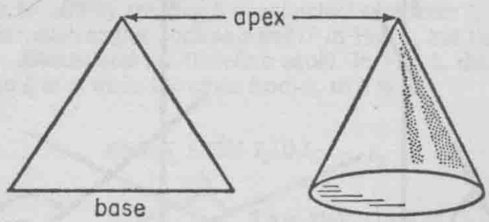
$$\frac{x}{10} = \frac{3}{5};\ 5x = 30;\ x = 6.$$

$$78240 + x = 78246;\ N = .78246.$$

See LOGARITHM, CHARACTERISTIC OF A; LOGARITHM, MANTISSA OF A.

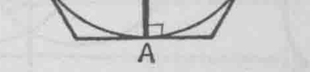
ANY, in mathematics, the word used to indicate the most general case of a term or figure. "Any point on a line" means "whatever point one may choose to pick without exception" and hence "every point." If *any* point on the perpendicular bisector of a line segment is proved equidistant from the ends of the segment, then *every* point in the perpendicular bisector is also so proven. Since ISOSCELES, RIGHT and EQUILATERAL TRIANGLES have properties unique to each, "any" triangle should be represented by a SCALENE TRIANGLE so that conclusions reached will hold true for *all* triangles.

APEX, of a plane or solid geometric figure, the point that is the greatest distance from a given base line or plane. The apex of a triangle is the vertex opposite the side designated as the baes; of a cone, its vertex. (See figure.)



APOLLONIUS OF PERGA (a-puh-loh′-nee-us), ab. 255–170 B.C., a Greek geometer, who with EUCLID and ARCHIMEDES is considered a founder of mathematical science. He was referred to as "The Great Geometer" in ancient times, primarily for his work in CONIC SECTIONS. It was Apollonius who first used the terms PARABOLA, HYPERBOLA, and ELLIPSE to name the conics.

APOTHEM OF A REGULAR POLYGON (ap′-oh-them), the perpendicular from the center of the polygon to a side. The apothem is the RADIUS of the CIRCLE inscribed in the polygon. In the figure, OA is the apothem of the polygon and is also the radius of inscribed circle O.



APPROXIMATE NUMBERS, numbers representing magnitudes which cannot be expressed exactly to a given number of decimal places. E.g., the square root of 2, which to five decimal places is 1.41421, is an approximate number. Before adding or subtracting approximate numbers round off the numbers to the same number of decimal places so that the sum, or difference, will have the same unit of measurement. E.g., given that $\sqrt{2} = 1.41421$ and $\sqrt{3} = 1.732$, to add $\sqrt{2}$ and $\sqrt{3}$, use $1.414 + 1.732$. Thus, the sum of $\sqrt{2} + \sqrt{3} = 3.146$.

The product of two approximate numbers should not have more SIGNIFICANT figures than either the multiplicand or the multiplier, nor should the quotient of two approximate numbers contain more significant figures than either the dividend or the divisor. Thus, to find the product of $\sqrt{2}$ and $\sqrt{3}$, multiply 1.414×1.732 which gives 2.449048, and round off the product to three decimal places. The product is 2.449. If computations involve both exact and approximate numbers, the result should have the same unit of measurement as the approximate number. *See also* ROUNDING OFF A NUMBER.

APPROXIMATION BY DIFFERENTIALS, *see* DIFFERENTIAL.

ARC (ahrk), the part of a CIRCLE between any two points on the