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PREFACE

In the last year or so there have been widespread rumors that group theory is finished, that there is nothing more to be done. It is not so.

While it is true that we are tantalizingly close to that pinnacle representing the classification of finite simple groups, one should remember that only by reaching the top can one properly look back and survey the neighboring territory. It was the task of the Santa Cruz conference not only to describe the tortuous route which brings us so close to the summit of classification, but also to chart out more accessible paths–ones which might someday be open to the general mathematical public.

A third concern was the elucidation of topics in related fields, and it is to one of these three areas that the papers in this volume are devoted.

Just a quick glance at the table of contents will reveal a wide variety of topics with which the modern group theorist must contend. Some of these, for example, the connections with the theory of modular functions, have very recent origins, but they leave us with the clear impression that, far from being dead, group theory has only just come of age.

Geoffrey Mason
Chicago, June 1980

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AN OUTLINE OF THE CLASSIFICATION OF FINITE SIMPLE GROUPS

DANIEL GORENSTEIN

1. Introduction. My aim here is to present a brief outline of the classification of the finite simple groups, now rapidly nearing completion. The major steps in the classification will be discussed in greater detail by many authors within these PROCEEDINGS and it is hoped that this outline will help to provide a cohesive overview of their individual articles as well as the subject of finite simple groups itself.

For expository purposes, I shall divide the outline into four parts:

(A) special classification theorems,

(B) general classification theorems,

(C) underlying techniques,

(D) the remaining open problems.

A classification theorem is considered to be *general* or *special* according as its hypothesis does or does not carry over to all subgroups and homomorphic images. This distinction is not to be taken too literally, for a special classification theorem often becomes general by a slight rewording of its hypothesis (the property of a group having dihedral Sylow 2-subgroup is not strictly speaking general, but that of a group having dihedral or cyclic Sylow 2-subgroups is). In making the division, I have been concerned primarily with providing what I felt would be the clearest picture of the global classification theorem.

Likewise the distinction between a classification theorem and an underlying technique is often blurred, for today's classification theorem becomes tomorrow's basic tool. Bender's classification of groups with a strongly embedded subgroup or Timmesfeld's root involution theorem are good illustrations of this point. However, such theorems clearly have a different flavor from Glauberman's *ZJ*- or Z^*-theorem or from the signalizer functor theorem, and I have tried to preserve this distinction in my division.

To keep the focus as sharp as possible, I shall follow a skeletal format, carefully stating the main results in each category, but limiting myself to very

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