

PROCEEDINGS OF SYMPOSIA  
IN PURE MATHEMATICS  
Volume 37

THE SANTA CRUZ CONFERENCE  
ON FINITE GROUPS

AMERICAN MATHEMATICAL SOCIETY  
PROVIDENCE, RHODE ISLAND

1980

PROCEEDINGS OF THE SYMPOSIUM IN PURE MATHEMATICS  
OF THE AMERICAN MATHEMATICAL SOCIETY

HELD AT THE UNIVERSITY OF CALIFORNIA  
SANTA CRUZ, CALIFORNIA  
JUNE 25—JULY 20, 1979

EDITED BY  
BRUCE COOPERSTEIN  
GEOFFREY MASON

Prepared by the American Mathematical Society  
with partial support from National Science Foundation grant MCS 78-24165

**Library of Congress Cataloging in Publication Data**

Santa Cruz Conference on Finite Groups, 1979.

The Santa Cruz Conference on Finite Groups.

(Proceeding of symposia in pure mathematics; v. 37)

Includes bibliographies.

1. Finite groups—Congresses. I. Cooperstein, Bruce, 1950— II. Mason, Geoffrey, 1948— III. American Mathematical Society. IV. Series.  
QA171.S26 1979 512'.2 80-26879 ISBN 0-8218-1440-0

1980 Mathematics Subject Classification. Primary 00A10, 20-02.

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## PREFACE

In the last year or so there have been widespread rumors that group theory is finished, that there is nothing more to be done. It is not so.

While it is true that we are tantalizingly close to that pinnacle representing the classification of finite simple groups, one should remember that only by reaching the top can one properly look back and survey the neighboring territory. It was the task of the Santa Cruz conference not only to describe the tortuous route which brings us so close to the summit of classification, but also to chart out more accessible paths—ones which might someday be open to the general mathematical public.

A third concern was the elucidation of topics in related fields, and it is to one of these three areas that the papers in this volume are devoted.

Just a quick glance at the table of contents will reveal a wide variety of topics with which the modern group theorist must contend. Some of these, for example, the connections with the theory of modular functions, have very recent origins, but they leave us with the clear impression that, far from being dead, group theory has only just come of age.

Geoffrey Mason  
Chicago, June 1980

## LIST OF PARTICIPANTS

NAME	AFFILIATION
Jonathan Alperin	University of Chicago
Herbert Alward	University of Oregon
Bruno Andriamanalimanana	Lehigh University
Zvi Arad	Bar-Ilan University, Israel
Michael Aschbacher	California Institute of Technology
George Avrunin	University of Massachusetts, Amherst
Eiichi Bannai	Ohio State University
Bernd Baumann	University of Bielefeld, Federal Republic of Germany
Dave Benson	Trinity College, University of Cambridge, England
Thomas Berger	University of Minnesota, Minneapolis
Harvey Blau	Northern Illinois University
Michel Broué	CNRS, University of Paris VII, France
Francis Buekenhout	Free University of Brussels, Belgium
Alan Robert Camina	University of East Anglia, England
Neville Campbell	University of California, Santa Cruz
John Cannon	University of Sydney, Australia
Andrew Chermak	University of Minnesota
David Chillag	Technion-Israel Institute of Technology, Israel
Arjeh M. Cohen	Mathematisch Centrum, Amsterdam, The Netherlands
Michael Collins	University of Oxford, England, and California Institute of Technology
John M. Conway	University of Cambridge, England
Bruce Cooperstein	University of California, Santa Cruz
Charles W. Curtis	University of Oregon
Everett C. Dade	University of Illinois, Urbana-Champaign
Stephen Davis	Ohio State University
Alberto Delgado	University of California, Berkeley
Clifton Ealy, Jr.	Northern Michigan University
Yoshimi Egawa	Ohio State University
Paul Fan	University of California, Berkeley
Walter Feit	Yale University
Pamela Ferguson	University of Miami
Larry Finkelstein	Wayne State University



Bernd Fischer	University of Bielefeld, Federal Republic of Germany
Paul Fong	University of Illinois at Chicago Circle
Richard Foote	University of Cambridge, England
Mike Fried	University of California, Irvine
Daniel Frohardt	Wayne State University
Stephen M. Gagola, Jr.	Texas A & M University
Dilip Gajendragadkar	Tata Institute of Fundamental Research, Bombay, India
Robert Gilman	Stevens Institute of Technology
George Glauberman	University of Chicago
Andrew M. Gleason	Harvard University
David Goldschmidt	University of California, Berkeley
Kensaku Gomi	University of Tokyo
Daniel Gorenstein	Rutgers University, New Brunswick
Robert Griess	University of Michigan, Ann Arbor
Fletcher Gross	University of Utah
Robert Guralnick	California Institute of Technology
Martin Guterma	Tufts University
Jonathan Hall	Michigan State University
Marshall Hall	California Institute of Technology
Koichiro Harada	Ohio State University
Morton Harris	University of Minnesota, Minneapolis
John Hayden	Bowling Green State University
Marcel Herzog	Tel-Aviv University, Israel
James Humphreys	University of Massachusetts, Amherst
David Hunt	University of New South Wales, Australia
I. Martin Isaacs	University of Wisconsin, Madison
David Jackson	University of Cambridge, England
Peter Johnson	Kansas State University
William Kantor	University of Oregon
Otto Kegel	Mathematics Institute, Freiburg, Federal Republic of Germany
Hiroshi Kimura	Hokkaido University, Japan
Laszlo Kovács	Australian National University, Australia
Burkhard Kuelshammer	University of Dortmund, Federal Republic of Germany
Peter Landrock	Aarhus University, Denmark
Jeffrey S. Leon	University of Illinois at Chicago Circle
Henry Leonard	Northern Illinois University
James Lepowsky	Rutgers University, New Brunswick
Robert Liebler	Colorado State University
Richard Lyons	Rutgers University, New Brunswick
Avinoam Mann	Hebrew University, Israel
Narendra Lal Maria	California State College, Stanislaus
Geoffrey Mason	University of California, Santa Cruz
Victor Mazurov	Institute of Mathematics, Novosibirsk, USSR

Patrick McBride	University of Michigan, Ann Arbor
Gerald McCollum	Harvard University
John McKay	Concordia University
Gerhard Michler	University of Essen, Federal Republic of Germany
Izumi Miyamoto	Yamanashi University, Japan
Paul Murphy	California Polytechnic State University
Michael Newman	Australian National University, Australia
Anni Neumann	Christian-Albrechts-University of Kiel, Federal Republic of Germany
Volker Neumann	Eberhard-Karls-University of Tübingen, Federal Republic of Germany
Jorn B. Olsson	University of Dortmund, Federal Republic of Germany
Michael O'Nan	Rutgers University, New Brunswick
David Parrott	University of Adelaide, Australia
Martin Pettet	Texas A & M University
Kok W. Phan	University of Notre Dame
Harriet Pollatsek	Mt. Holyoke College
Jerry Povse	University of California, Berkeley
Udo Preiser	University of Bielefeld, Federal Republic of Germany
Alan Prince	Heriot-Watt University, Scotland
Lluís Puig	CNRS, University of Paris VII, France
Larissa Queen	University of Cambridge, England
Mark Ronan	University of Illinois at Chicago Circle
Peter Rowley	University of Birmingham, England
Betty Salzberg	Northeastern University
Ulrich Schoenwaelder	University of Aachen, Federal Republic of Germany
William Scott	University of Utah
Gary Seitz	University of Oregon
Ernest Shult	Kansas State University
David Sibley	Pennsylvania State University, University Park
Charles Sims	Rutgers University
Justine Skalba	Kansas State University
Stephen Smith	University of Illinois at Chicago Circle
Ronald Solomon	Ohio State University
Tonny A. Springer	University of Utrecht, The Netherlands
Bhama Srinivasan	Clark University
Robert Steinberg	University of California, Los Angeles
Gernot Stroth	Ruprecht-Karl-University of Heidelberg, Federal Republic of Germany
Hiroshi Suzuki	Ohio State University
Michio Suzuki	University of Illinois, Urbana-Champaign
Sergei A. Syskin	Institute of Mathematics, Novosibirsk, USSR
Tsunj-to Tai	University of California, Berkeley

Olga Taussky-Todd	California Institute of Technology
Alvin I. Thaler	National Science Foundation
Franz G. Timmesfeld	University of Cologne, Federal Republic of Germany
Nalsey Tinberg	Southern Illinois University, Carbondale
Hsio-Fu Tuan	Peking University, People's Republic of China
David Wales	California Institute of Technology
John Walter	University of Illinois, Urbana-Champaign
Michael Ward	University of Utah
Helmut Wielandt	Eberhard-Karls-University of Tübingen, Federal Republic of Germany
Bette Warren	State University of New York, Binghamton
Robert Wilson	Rutgers University, New Brunswick
Sia K. Wong	Ohio State University
Hiromichi Yamada	University of Tokyo, Japan
Hiroyoshi Yamaki	Ohio State University
Tomoyuki Yoshida	Hokkaido University, Japan

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Andrew Chernyak

David Chinburg

Arjen M. Cohen

Michael Collins

John M. Conway

Bruce Cooperstein

Charles W. Curtis

Evelyn C. Dade

Stephen Davis

Alberto Delgado

Clifton Ealy, Jr.

Yoshimi Egawa

Paul Fapp

Walter Feit

Patricia Ferguson

Larry Finkelstein

University of Minnesota

Technion-Israel Institute of Technology, Israel

Mathematisch Centrum, Amsterdam,

The Netherlands

University of Oxford, England, and

California Institute of Technology

University of Cambridge, England

University of California, Santa Cruz

University of Oregon

University of Illinois, Urbana-Champaign

Ohio State University

University of California, Berkeley

Northern Michigan University

Ohio State University

University of California, Berkeley

Yale University

University of Miami

Wayne State University

## AN OUTLINE OF THE CLASSIFICATION OF FINITE SIMPLE GROUPS

DANIEL GORENSTEY

1. Introduction. My aim here is to present a brief outline of the classification of the finite simple groups, now reaching final completion. The major steps in the classification will be discussed. I refer to many authors within these

### PART I

### Classification theory of finite simple groups

For expository purposes, I shall divide the outline into four parts:

(A) special classification theorems,

(B) general classification theorems,

(C) underlying techniques,

(D) the remaining open problems.

A classification theorem is considered to be *general* or *special* according to its hypothesis does or does not carry over to all subgroups and homomorphic images. This distinction is not to be taken too literally, for a special classification theorem often becomes general by a slight rewording of its hypothesis (the property of a group having dihedral Sylow 2-subgroup is not strictly speaking general, but that of a group having dihedral or cyclic Sylow 2-subgroups is). In making the division, I have been concerned primarily with providing what I felt would be the clearest picture of the global classification theorem.

Like the distinction between a classification theorem and an underlying technique is often blurred, for today's classification theorem becomes tomorrow's basic tool. Bender's classification of groups with a strongly embedded subgroup or Thompson's root involution theorem are good illustrations of this point. However, such theorems clearly have a different flavor from Glauberman's  $ZJ$ - or  $Z^*$ -theorem or from the signalizer functor theorem, and I have tried to preserve this distinction in my division.

To keep the focus as sharp as possible, I shall follow a skeletal format, carefully stating the main results in each category, but limiting myself to very

# LIST OF PARTICIPANTS

John F. Adams  
David J. Alderson  
Frank W. Buehler

William C. Coker  
Rosa P. L. L. L.  
David W. L.  
John W. L.  
Michael W. L.  
Harold W. L.

Belle Warren  
Robert Wilson  
Sia K. Wong  
Hiroshi Yamada  
Hiroshi Yamada  
Tomoyuki Yoshida

California Institute of Technology  
National Science Foundation  
University of California  
Federal Republic of Germany  
Eastern Illinois University, Carbondale  
Peking University, People's Republic of China  
California Institute of Technology  
University of Illinois, Urbana-Champaign  
University of Utah  
Friedrich-Karl-University of Tübingen  
Federal Republic of Germany  
State University of New York, Binghamton  
Rutgers University, New Brunswick  
Ohio State University  
University of Tokyo, Japan  
Ohio State University  
Hokkaido University, Japan

Classification theory of finite simple groups



## AN OUTLINE OF THE CLASSIFICATION OF FINITE SIMPLE GROUPS

DANIEL GORENSTEIN

**1. Introduction.** My aim here is to present a brief outline of the classification of the finite simple groups, now rapidly nearing completion. The major steps in the classification will be discussed in greater detail by many authors within these PROCEEDINGS and it is hoped that this outline will help to provide a cohesive overview of their individual articles as well as the subject of finite simple groups itself.

For expository purposes, I shall divide the outline into four parts:

- (A) special classification theorems,
- (B) general classification theorems,
- (C) underlying techniques,
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A classification theorem is considered to be *general* or *special* according as its hypothesis does or does not carry over to all subgroups and homomorphic images. This distinction is not to be taken too literally, for a special classification theorem often becomes general by a slight rewording of its hypothesis (the property of a group having dihedral Sylow 2-subgroup is not strictly speaking general, but that of a group having dihedral or cyclic Sylow 2-subgroups is). In making the division, I have been concerned primarily with providing what I felt would be the clearest picture of the global classification theorem.

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