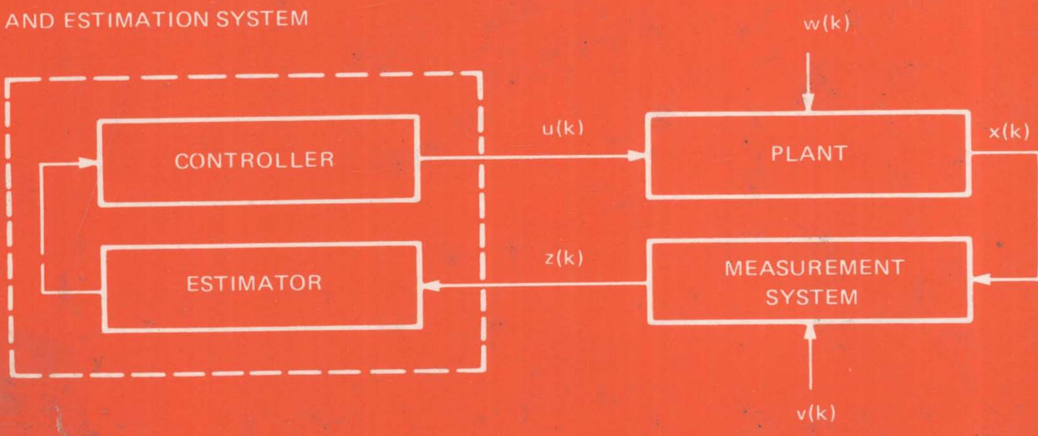
CONTROL AND SYSTEMS THEORY Volume 7

Principles of Dynamic Programming

Part II
Advanced Theory and Applications

OPTIMUM COMBINED CONTROL AND ESTIMATION SYSTEM



Principles of Dynamic Programming

Part II

Advanced Theory and Applications

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To Richard Bellman

Teacher, Inspiration, and Friend to Us Both

PREFACE

In the first part of our two-part exposition of dynamic programming methodology, the basic theme was determinism and algorithm feasibility, i.e., our attention was centered upon decision processes with no stochastic influences and we developed the basic computational procedures needed for obtaining feedback decision policies.

Taking the material of Part I as our point of departure, in this part we explore many of the extensions and generalizations inherent in the dynamic programming approach to determination of optimal policies. In one direction we make contact with classical variational theory and show how all the standard necessary conditions of the calculus of variations follow in a straightforward way from the basic functional equation of dynamic programming. In a more modern spirit, we also examine a number of questions involving optimal control processes and derive a variety of basic results for determination of optimal control laws using the principle of optimality.

Since uncertainty continually intrudes into most realistic decision processes, a rather lengthy chapter is devoted to a treatment of stochastic and adaptive processes arising in a variety of engineering and management situations. It is at this point in our narrative where the conceptual approach of dynamic programming allows us to treat truly significant applied problems, where the classical variational calculus would be powerless.

As has been well chronicled in the literature, the basic stumbling block standing in the way of a routine use of dynamic programming for most decision processes is the so-called "curse of dimensionality." This computational obstacle is attacked from two standpoints in this part. The first is via the exploitation of problem structure to reduce the original problem to more tractable form. A more or less miscellaneous collection of methods, techniques and subterfuges are presented all of which make use of particular problem structure to cut the standard dynamic programming computational requirements down to a manageable level. The second approach to lifting the "curse" involves a study of new computer architectures, particularly parallelism, and their applicability for dynamic programming calculations. It is seen that recent developments in computer hardware and software have major implications for the feasibility of employing dynamic programming in a wide variety of problems which heretofore had been beyond computational help.

Finally, to demonstrate the many uses of dynamic programming, we present a number of applications in the management and OR areas, together with two extensive case studies involving electric utility operation and planning and nuclear fuel reprocessing plants. These case studies leave no doubt as to the ability of dynamic programming methods to penetrate to the heart of complex decision-making processes involving uncertainty.

It is our hope and expectation that the material provided in Parts I and II of the treatise on dynamic programming will provide the serious system analyst with sufficient tools and confidence to tackle the variety of complex decision-making problems which the modern world continually forces upon us.

ROBERT E. LARSON
JOHN L. CASTI

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CHAPTER 1

DYNAMIC PROGRAMMING, OPTIMAL CONTROL THEORY, AND THE CALCULUS OF VARIATIONS

INTRODUCTION

A fundamental problem of modern society, and one to which considerable study has been devoted, is the control or regulation of complex, interdependent systems. These problems occur in all fields of industry and government, and they encompass systems ranging from municipal utility systems to manufacturing plants to large-scale defense systems. Although the specific systems differ greatly in the physical mechanisms of their operation, nevertheless there is a certain commonality in the control problems pertaining to all of them. The purpose of this chapter is to show how dynamic programming can be applied to the analysis of these control problems and to the effective computation of optimal control laws for a variety of systems.

Our concern here will be with what has now become known as feedback control theory, in contrast with previous mathematical analyses of control problems which centered on questions of stability under various conditions. Our analysis will focus upon situations of the type depicted in Figure 1.1. Here we have a system S characterized by its state x. This state is measured by the controlling unit C, which compares the current system performance with a predefined desired mode of operation and generates a controlling input u based upon the current deviation of the system from its desired performance level.

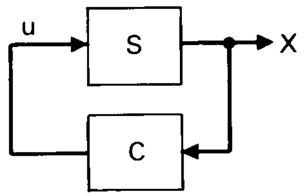

Figure 1.1

An important feature distinguishing the material of this chapter from most of our treatment in Part I, is continuous rather than discrete time. Optimal control problems have typically arisen in mechanical or engineering processes in which the response time of the system to controlling (or perturbing) influences has been quite short as compared with the duration of the process under study. Thus, it has been mathematically convenient, and physically reasonable, to develop the major results in the continuous time framework.

One way to formulate the above situation in mathematical terms is to regard the system S as being governed by the differential equation

$$\frac{dx}{dt} = g(x,u,t), \quad x(0) = c ,$$

where u is to be chosen to minimize some criterion function J which measures the cost of exerting control as well as the cost of deviation of x(t) from its desired level; the form of J is generally assumed to include both an integral term and a final-time term

$$J = \int_0^T L(x,u,t)dt + \varphi[x(T)] .$$

Many of the problems we shall consider in this chapter can be posed as questions within the classical calculus of variations.

The calculus of variations is a rather old and well-studied chapter of mathematics, having its origin in the classical problem of determining the curve of minimum time of descent between two specified points. On the other hand, dynamic programming is a relatively new field of mathematics, whose main development has been stimulated by the recent growth of computational capability and the plethora of important optimization problems which defy analysis by more traditional means. One of the main points we wish to bring out is that dynamic programming provides a very natural framework from which to study both classical and modern variational problems.

DISCRETE CONTROL PROCESSES

To introduce the concepts of this chapter, let us first consider control processes in which decisions are made at a discrete set of times $k = 0,1,2,\ldots,K$. These processes can be treated directly by the methods developed in Part I.

Let us examine the process

$$x(k+1) = g[x(k),\ u(k)]\ ,\quad x(0) = c\ ,\quad k = 0,1,\ldots,K-1\ ,$$

where $x(k)$ is an n-dimensional state vector and $u(k)$ is an n-dimensional control vector. We wish to choose the sequence $\{u(k)\}$ to minimize the criterion functional

$$J = \sum_{k=0}^{K-1} L[x(k),\ u(k)] + \varphi[x(K)]\ .$$

Let us denote the minimizing value of J by

$$I(c,a) = \min_{u(a),\ldots,u(k-1)} \sum_{k=0}^{K-1} L[x(k),u(k)] + \varphi[x(K)]\ ,$$

indicating, as before, that the minimum of J depends only upon c, the initial state, and a, the initial stage of the process.

Employing the principle of optimality and arguing as in Part I, we readily obtain the recurrence formula

$$I(c,a) = \min_{u} [L(c,u) + I(g(c,u), a+1)] , \quad 0 \le a \le K-1 ,$$

$$I(c,K) = \varphi(c) .$$

EXAMPLE

1.1 Consider the problem of minimizing

$$J[u(0),u(1),\ldots,u(N-1)] = \sum_{k=a}^{N} [x(k) - u(k)] ,$$

over all $u(k)$ subject to the conditions

$$x(k+1) = x(k) + bu(k) , \quad x(a) = c ,$$

$$0 \le u(k) \le x(k) , \quad k = a, a+1, \ldots, N ,$$

where $u(k)$, b and $x(k)$ are scalars. Derive the appropriate recurrence formula for the minimum cost function.

As the result of a decision v in state c, at stage a, the state is transformed from c to $c+bv$, and a cost $c-v$ is incurred. By definition, the optimal continuation from the new state $c+bv$ is $I(c+bv, a+1)$. Thus according to the principle of optimality

$$I(c,a) = \min_{0 \le v \le c} [(c-v) + I(c+bv, a+1)] ,$$

with

$$I(c,N) = \min_{0 \le v \le c} (c-v) = 0 .$$

CONTINUOUS CONTROL PROCESSES

In many problems of practical interest, particularly those associated with physical systems, the problem is given directly in continuous time form. Numerous problems from fields such as process control, mechanical engineering, and aerospace engineering, are specified in this manner. In these cases we wish to minimize

$$\int_a^T L[x(t),u(t),t]\,dt ,$$

over all continuous (say) u, such that

$$\frac{dx}{dt} = g(x,u,t) , \quad x(a) = c .$$

Let us examine the dynamic programming formulation of such a problem, assuming for the moment that x and u are scalar functions.

One way to proceed would be to use finite difference approximations to the integral criterion and differential equation, thereby reducing the problem to the type considered earlier. Under reasonable hypotheses on L and g, this procedure can be justified, and we can assert that the solution to the discrete problem approaches that of the continuous as the discretization step goes to zero. As far as computation of numerical results is concerned, this is the approach which must ultimately be followed, since digital computers are essentially discrete in nature. However, for the moment let us follow another route and stay entirely within the continuous domain in quest of analytical insight into the problem.

Define

$$I(c,a) = \min \int_a^T L(x,u,t)\,dt ,$$