# SUPERSPACE AND SUPERGRAVITY

## PROCEEDINGS OF THE NUFFIELD WORKSHOP, CAMBRIDGE

June 16 - July 12, 1980

edited by

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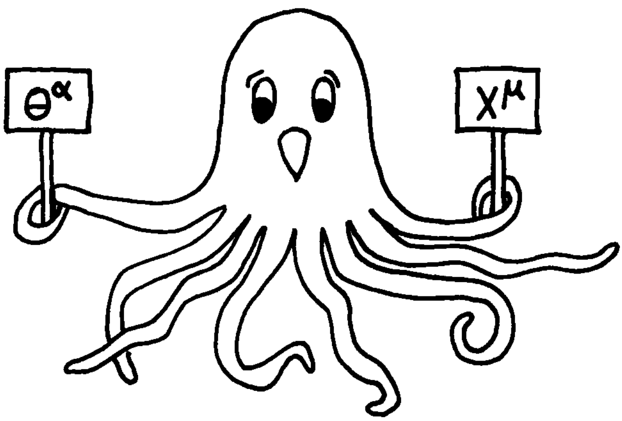


"ACHTBEIN THE OCTOPUS"

Achtbein's eight legs refer to the eight Superspace coordinates.

## PREFACE

The Workshop on Supergravity was held in the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge from 16 June to 12 July, 1980. It featured a variety of topics, and was generally held to be an instructive way to spend four weeks. The outcome included a number of articles, a blackboard (displayed on the cover), and, of course, these proceedings. We would like to thank the Nuffield Foundation for financing the Workshop, and Judy Fella without whose efforts nothing would have happened. We would also like to thank Jacques Richer for late night help with a computerized typing system.

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## INTRODUCTION

The Newtonian theory of gravity was very successful in predicting planetary and stellar orbits, but because it implied that gravitational effects propagated with infinite velocity, it was incompatible with the local validity of special relativity. This difficulty was overcome in 1916 with the formulation of General Relativity in which the gravitational field was represented by the metric of curved spacetime. Since that time, the predictions of General Relativity have been found to be in excellent agreement with observation. However, the theory is incomplete in at least two ways:

1. It does not relate gravity to interactions and to matter fields that occur in physical theories.

2. It is a purely classical theory, whereas all other fields seem to be quantized.

Einstein himself was unhappy about the first shortcoming, and spent most of the rest of his life looking for a Unified Field Theory. He failed, partly because not much was known at the time about the other interactions except electromagnetism, and partly because he ignored the second limitation of General Relativity listed above. The necessity to quantize the gravitational field has become more urgent because it has been shown that classical General Relativity inevitably predicts singularities of spacetime.

In recent years there has been considerable success in unifying the other interactions apart from gravity. The Salam-Weinberg model combined electromagnetism with the weak nuclear forces while a number of Grand Unified Theories (GUTs) have been proposed to unite these two interactions with the strong nuclear forces. In the GUTs, the unification takes place at an energy of the order of $10^{15}$ GEV, which is quite near the Planck energy of $10^{19}$ GEV at which quantum gravitational effects might be expected to become important. It therefore seems natural to try to extend these theories to include gravity. At the present, the best hope of doing this seems to lie in the various supergravity theories. In these, the graviton, the spin 2 particle that carries the gravitational interaction, is related to particles of lower integer and half integer spin by so called "supersymmetry" transformations. This has the great merit of not only unifying the "interactions", represented by integer spin fields, but also of ending the old dichotomy between them and the "matter", represented by half-integer spin fields.

Another great advantage of supergravity theories is that many of the divergences that plague quantum field theory seem to cancel between the integer and half integer spin particles. Pure gravity on its own is known to be finite at the one loop level in topologically trivial situations. However, it is not finite when coupled to matter, or in topologically nontrivial configurations, and there are indications that it may not be finite at the two-loop level in any situation. On the other hand, it has been shown that the extended supergravity theories are all finite at the one and two levels in topologically trivial situations, and that the "N=8" theory is finite at one and two loops in all situations (see the article by M. J. Duff in this volume). It has also been shown that the analogous N=4 supersymmetric Yang-Mills theory is finite to three loops. These results suggest that the N=8 theory may turn out to be finite to all orders (although there is a possible counterterm at seven loops, see the article by P. Howe and U. Lindström in this volume). It is to be hoped that this is indeed the case, because it can be shown that theories including gravity are either finite or non-renormalizable: if there are divergences, one would have to add an infinite sequence of counterterms with a corresponding infinite number of undetermined renormalization parameters.

Supergravity is a very recent and rapidly developing subject. It is therefore difficult to get a clear view of the field and to see what progress has been made. We hope that this volume will do something to improve the situation. It is based on the Workshop on Supergravity that was held in Cambridge from 16 June to 12 July, 1980, and was supported by the Nuffield Foundation. The volume is divided into six parts. Part I contains two sets of introductory lectures on supergravity. Those by P. van Nieuwenhuizen concentrate on the component approach in ordinary spacetime. This is the form in which supergravity was first formulated, and in which it has the most direct physical interpretation in terms of fields of different spin. However, the supersymmetry is more apparent in the superspace formulation which is described for the case of unextended supergravity in the second set of lectures by M. Roček. We hope that these two sets of lectures provide an introduction to supergravity which is comprehensible to readers with some knowledge of General Relativity and quantum field theory, and which will enable them to follow the remaining more specialized articles.

Part II contains five articles on the quantization and regularization of supergravity and supersymmetric gauge theories. The first article, by M. T. Grisaru, describes the background superfield method which enables the perturbation expansion to be written in a very compact and manifestly supercovariant form of supergraphs. These supergraphs have a number of gauge invariances which in general require the introduction of several generations of ghosts as described in the article by W. Siegel. They also require a manifestly supersymmetric regularization technique. The next two articles describe two such schemes. The first, by W. Siegel, P. K. Townsend and P. van Nieuwenhuizen, deals with dimensional regularization by dimensional reduction. This seems to work well but has unresolved problems about anomalies. The second, by A. A. Slavnov, uses additional higher derivative terms with gauge invariant Pauli-Villars regularization of the remaining one loop divergences. It is more complicated to apply, and has so far been worked out only for supersymmetric Yang-Mills theory, but not for supergravity. The final article in this part, by C. Teitelboim, deals with the Hamiltonian quantization of supergravity.

Part III contains nine articles dealing with extended supergravity and related topics. In simple supergravity, described in the lectures in part I, there is only one supersymmetry generator which is a Majorana spinor whereas in extended supergravity there are N such generators, with $N \leq 8$. The Lagrangians and supersymmetry transformations are known, but only in component form and without the auxiliary fields needed to close the algebra of transformations. The first two articles by E. Sokatchev and S. J. Gates, Jr., describe the progress that has been made towards superspace formulations of the N=2 theory. It seems we do not yet have an elegant formulation of the theory. The next article, by B. de Wit and co-workers, uses extended supersymmetric versions of the conformal group to clarify the structure of the component formulation of N=2 supergravity with auxiliary fields. The approach seems promising, although there are difficulties which appear to limit it to $N \leq 4$. The article by S. MacDowell attempts to formulate all the extended theories by giving covariant constraints in superspace. It unfortunately gives an implicit form of the theories which is difficult to use. In the article by E. Cremmer, the extended theories are formulated in their component form (without auxiliary fields) in five dimensions. This is a useful starting point for obtaining symmetry breaking by dimensional reduction to four dimensions, and for attempting to derive auxiliary fields by a Legendre transformation as described in the following article by M. Sohnius, K. S. Stelle and P. C. West. This deals primarily with supersymmetric Yang-Mills theories. There are problems with constraints which arise because of gauge invariance. The authors try to solve these by introducing an infinite number of Lagrange multiplier fields which effectively are the coefficients of a Taylor expansion in a fifth dimension. A systematic attempt, from the point of view of group theory, to relate theories in lower and higher dimensions is presented in the next article by B. Julia. Related group theoretical considerations follow in the contribution of J. Thierry-Mieg and B. Morel, and the final article in this part, by J. G. Taylor, finds linearized representations of extended supersymmetry using superspin projection operators.

Part IV deals with the maximally extended N=8 supergravity theory, which is the best candidate for a complete unified theory without divergences. In the article by M. J. Duff, it is shown that the one loop term is finite even in topologically nontrivial situations if one takes a proper account of the antisymmetric tensors that arise in the derivation of the theory by dimensional reduction from eleven dimensions. These antisymmetric tensors can also give rise to a cosmological term, as described in the article by A. Aurilia, H. Nicolai and P. K. Townsend. There is a possible counter term for the N=8 theory at seven loops, as is shown in the article by P. Howe and U. Lindström, but it is not known whether this divergence actually appears or whether it is absent because of some as yet unknown symmetry. The N=8 theory at it stands cannot describe the known physical fields, since its 28 spin one fields cannot include the SU(3) x SU(2) x U(1) gauge symmetry that seems to be present in particle physics. A possible solution to this problem is presented in the article by B. Zumino, where it is suggested that the elementary fields of the theory (apart from the graviton) are not observed directly, but that they form a supermultiplet of bound states invariant under an SU(8) gauge group. Most of these bound states acquire very large masses on the order of $10^{19}$ GEV and are not observed, but a subset of them form a SU(5) Grand Unified Theory with three generations of quarks and leptons. This seems to be what is observed. It is a very promising idea to reconcile supergravity to the observed particle spectrum. It is however, at the moment, only a scenario because nothing is known about the mechanisms that would confine these bound states. The final article in this part, by J. F. Adams, describes the special mathematical features of Spin(8), triality, $F_4$ and all that.

Part V contains two articles on Kähler manifolds. D. Z. Freedman uses the properties of Kähler spaces to prove results about the divergences of supersymmetric nonlinear sigma-models. C. N. Pope uses the existence of gauge covariantly constant spinors in Kähler-Einstein backgrounds to find relations between the eigenvalue spectra of different spin wave operators. These relations are responsible for the remarkable cancellations between integer and half-integer spins in supersymmetric theories.

Part VI contains five articles which do not fit under any of the previous headings. The first, by M. Sohnius, describes techniques useful for simplifying constraints in superspace, for example, of the sort used by S. MacDowell above. The article by M. Perry deals with the instabilities that arise because the gravitational part of the action is not positive definite, as for example, in a black hole background field. The next article, by U. Lindström, A. Karlhede and M. Roček, gives the explicit relation between the superspace and component formulations of N=1 supergravity. M. Kaku describes a scheme for quantising gravity and supergravity on a lattice, and C. Aragone presents failed attempts to find consistent interacting theories for fields with spins higher than two.

We feel that the articles and the lectures described above give a general introduction to the field, and indicate most of the current areas of research. Other surveys and research articles can be found in Recent Developments in Gravitation Cargèse 1978, eds. M. Lévy and S. Deser, Plenum, in Supergravity, eds. P. van Nieuwenhuizen and D. Z. Freedman, North-Holland, and in a forthcoming issue of Physics Reports C by P. van Nieuwenhuizen. At the present time, we think that the outstanding problems are:

1. The construction of a complete formulation of extended supergravity with auxiliary fields, preferably in superspace.

2. The development of general arguments to determine whether or not the higher loops in the N=8 supergravity theory are finite. This might follow immediately from the solution of the first problem.

3. The reconciliation of supergravity with the observed phenomenology of particle physics. This might be along the lines suggested in the article by B. Zumino.

It is impossible to predict the timescales, but given the current interest in the field, we can hope that all three problems will be solved in the next few years.

S. W. Hawking

M. Roček

# I INTRODUCTION TO SUPERGRAVITY