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Combinatorics '81
in honour of Beniamino Segre

Proceedings of the International Conference on
Combinatorial Geometries and their Applications
Rome, June 7-12, 1981

edited by

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PREFACE

The *International Conference on Combinatorial Geometries and their Applications* was held in Rome from 7 to 12 June, 1980, to honour the memory of Beniamino Segre (1903-1977), who stimulated a vast area of research with a series of fundamental works.

These proceedings contain 76 articles in all, devoted to recent progress in the following topics: finite geometries (arcs, caps and special varieties in a Galois space; generalized quadrangles; Benz planes; foundation of geometry), design theory, finite groups, coding theory and graph theory in its geometric and design aspects.

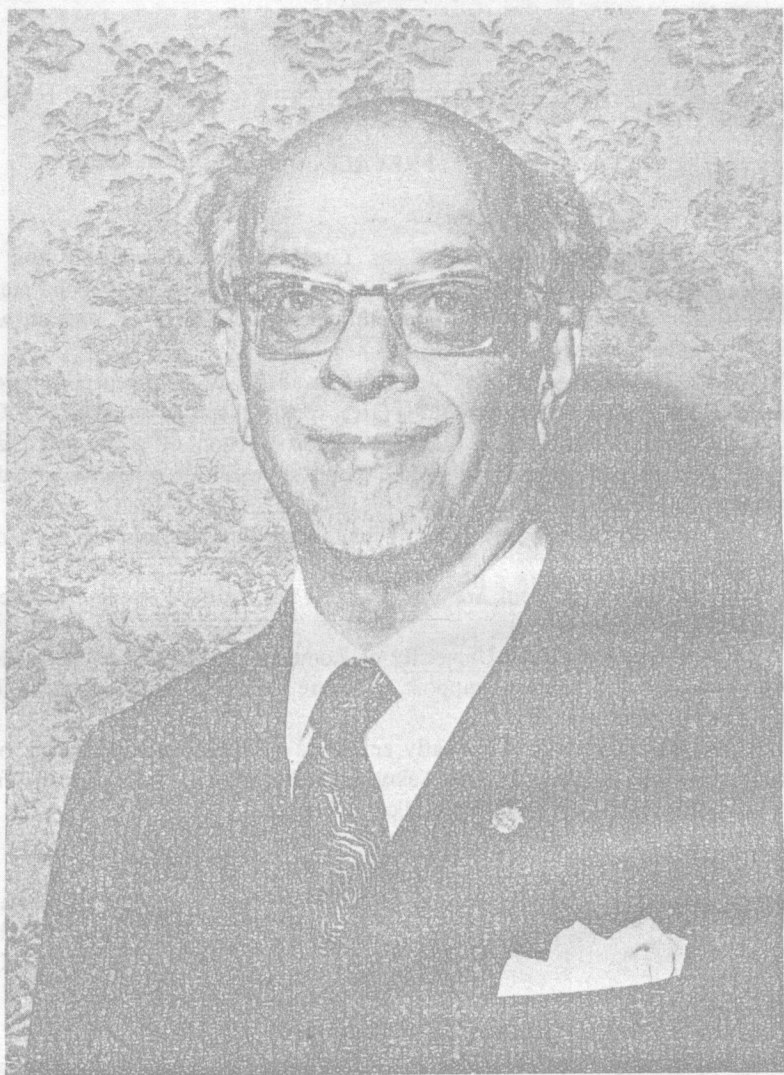
An asterisk following an author's name indicates the presenter of a multi-authored paper.

At the end of the book are a list of talks not included in these proceedings and a list of participants.

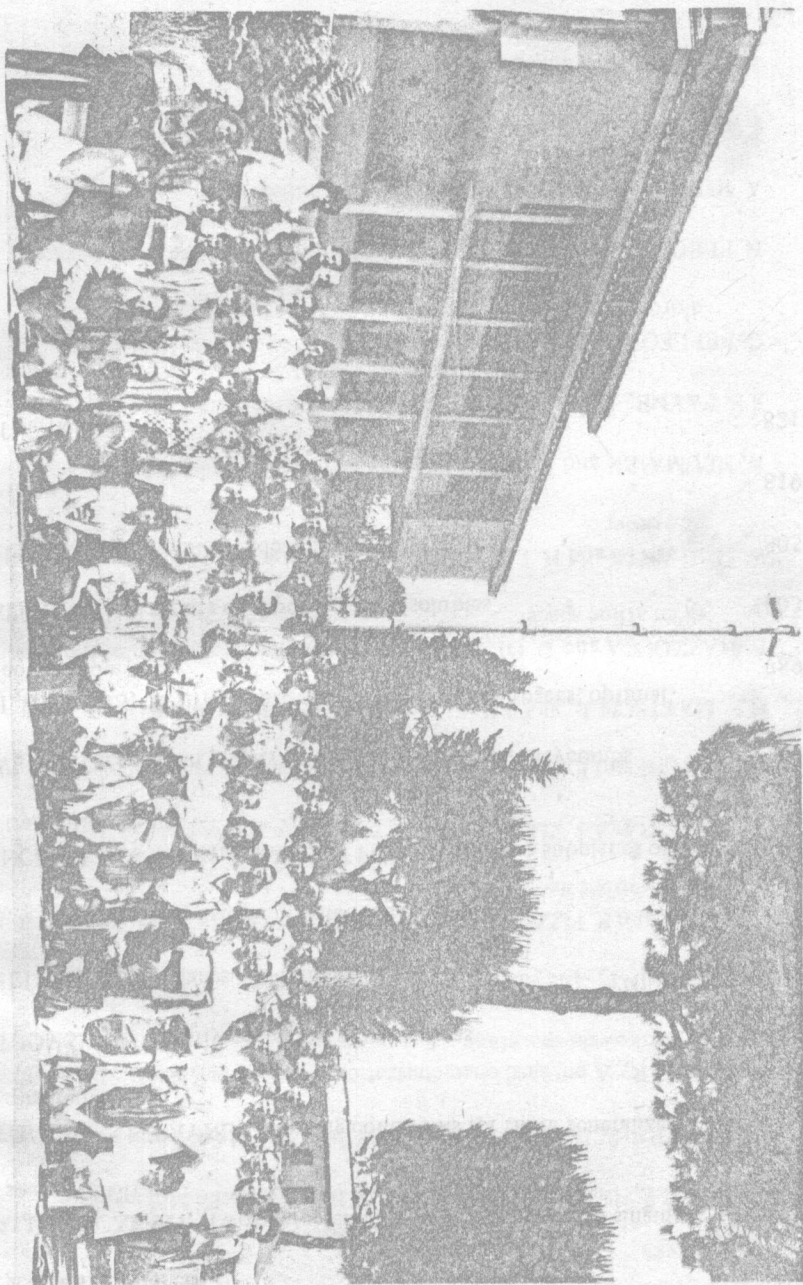
We are indebted to the University of Rome and to the "Consiglio Nazionale delle Ricerche" for financial support. We are also profoundly grateful to the referees for their assistance.

The conference was also greatly enhanced by the beautiful concert of the Maestri Giuseppe Selmi and Maria Selmi Dongellini and by the series of films on Art and Mathematics shown by Prof. Michele Emmer.

Adriano Barlotti
Pier Vittorio Ceccherini
Giuseppe Tallini



BENIAMINO SEGRE (1903-1977)



Participants of the International Conference on Combinatorial Geometries and their Applications,
Rome, 7-12 June, 1981

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ADDRESS TO PARTICIPANTS

A. Barlotti

It is really a pleasure and a honor to welcome here the large number of mathematicians who are present to take part at the "Convegno internazionale di Geometrie combinatorie e loro applicazioni".

This Conference is held in memory of the late Prof. Beniamino Segre who, since 1950, with a pioneering work opened the new field of research of the "Galois geometries" and stimulated interest in combinatorial geometry not only in Italy, but in the entire world.

Beniamino Segre was one of the most eminent mathematicians of this century. He obtained deep results in several different areas: from algebraic geometry "in the Italian trend" to algebra, analysis, combinatorics, differential geometry, number theory, topology, applied mathematics.

This Conference is restricted to the specific area of "Combinatorial Geometries and their applications", the field to which Beniamino Segre devoted a large part of his energies in the last years of his life. If the purpose of the Conference would have been only to celebrate the memory of Beniamino Segre, we would have held lectures in the many fields of his interest. But a Conference in a more restricted area is much more effective for the development of science and we had in mind the idea of the Master who was always dedicated to the progress of mathematics.

Combinatorics had a very broad développement in the last decades. In 1973, in his opening address for the "Colloquio internazionale sulle Teorie Combinatorie" held at the Accademia Nazionale dei Lincei, Beniamino Segre stressed the importance of Combinatorics with the following words: "Le Teorie Combinatorie sono un ramo assai recente di matematica, destinato a mio avviso ad avere influenza grandissima sullo sviluppo futuro di numerose scienze applicate ed altresì su quello della stessa matematica.

Il nome di tale ramo può farsi risalire a Leibniz il quale, nel 1666, pub-

blicò la sua famosa *Dissertatio de Arte Combinatoria*, in cui esprime vive speranze sulle relative possibilità di applicazioni - com'egli testualmente scrisse - "all'intera sfera della scienza". Quel ramo rimase tuttavia per così dire latente durante quasi tre secoli, il suo studio venendo ripreso in modo sistematico e con vigore crescente soltanto negli ultimi decenni e particolarmente nell'ultimo, anche sotto la spinta di varie pressanti richieste di carattere applicativo del tutto nuove, fra cui quelle derivanti dallo studio dei grafi e dei circuiti elettrici, delle geometrie finite, della statistica, della teoria dell'informazione, della cristallografia, della chimica organica, nonché dall'uso sempre più intenso e diffuso degli elaboratori elettronici.

Lo scienziato puro è così suo malgrado costretto a riconoscere che, dopo la creazione e l'evoluzione prodigiosa dell'analisi infinitesimale, della teoria degli insiemi, dell'algebra, della topologia e dei vari indirizzi astrattisti ed assiomatici, rimangono quasi completamente da esplorare vastissime regioni matematiche assai concrete, in apparenza piuttosto banali ma sovente nella sostanza ardue e profonde, attinenti spesso semplicemente al discreto o persino al finito, in relazione alle quali anche le tecniche usuali di dimostrazione ed indagine risultano carenti ed in parte da rinnovare radicalmente. Citerò soltanto in proposito due problemi ben noti, rimasti a tutt'oggi scandalisticamente insoluti, pur potendo essi sembrare agevolmente trattabili: mi riferisco a quello cartografico detto dei quattro colori, ed a quello relativo all'esistenza di geometrie finite il cui ordine non sia una potenza di un numero primo".

The first one of these two problems has now been solved. The work done toward the solution of the second problem is attested by recent papers which give the hope that at least a partial solution may be reached soon. Many other important achievements in Combinatorics have been obtained since 1973 and the program of the present Conference shows that a large number of these achievements will be considered and discussed here.

Speaking for all the participants we are honored by the presence of members of the family of Professor Segre.

In name of the organizing committee I wish to express a sincere thanks for the substantial help which made the Conference possible. So I am thanking: the Magnifico Rettore dell'Università di Roma, Professor Antonio Ruberti; the Preside della Facoltà di Scienze, Professor Giorgio Tecce; the Direttore dell'Istituto Ma-

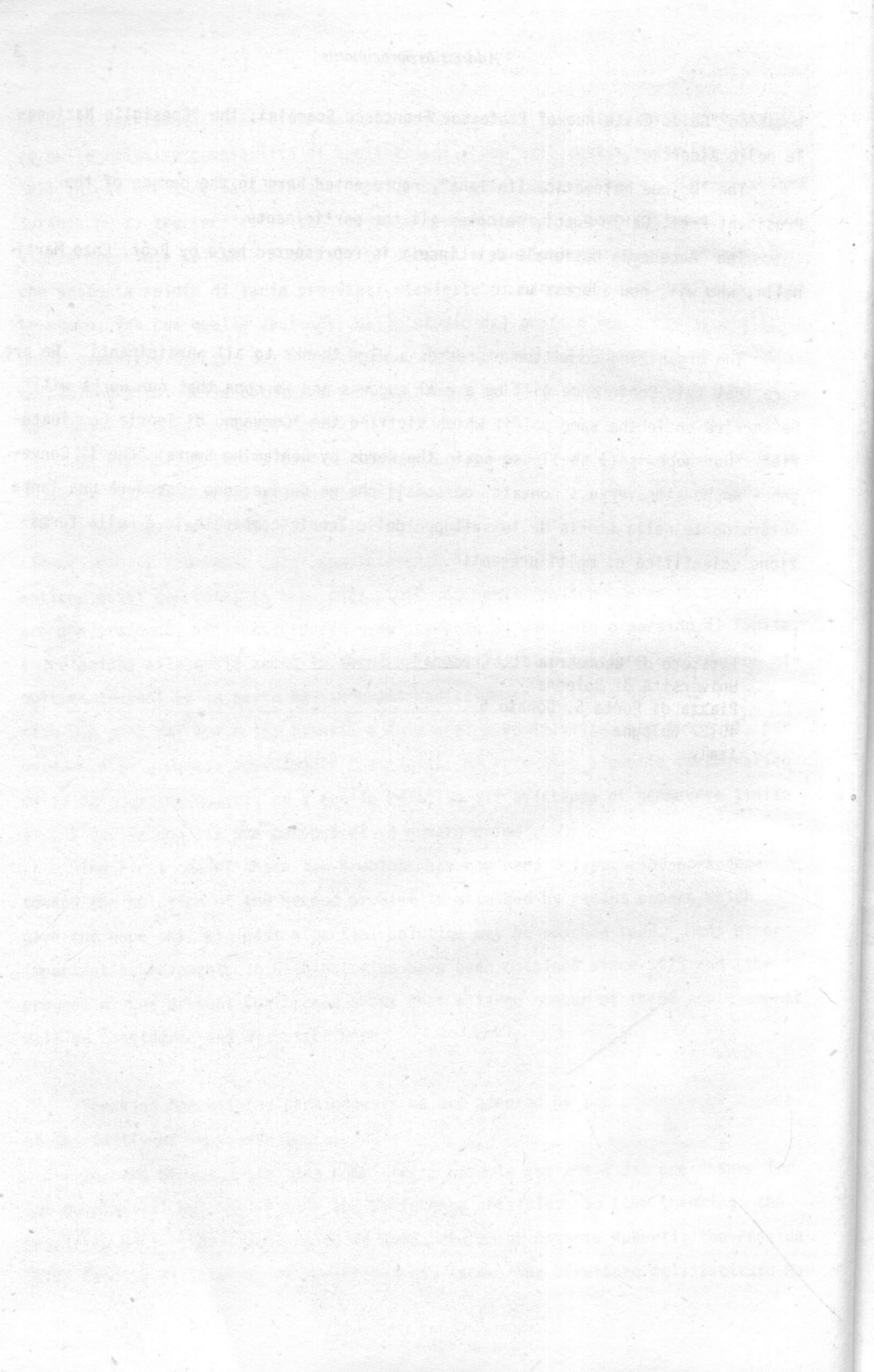
tematico "Guido Castelnuovo" Professor Francesco Scarpini; the "Consiglio Nazionale delle Ricerche".

The "Unione Matematica Italiana", represented here in the person of the President Prof. Carlo Pucci, welcomes all the participants.

The "Accademia Nazionale dei Lincei" is represented here by Prof. Enzo Martellini, who will now address us.

The organizing committee expresses a warm thanks to all participants. We are sure that this Conference will be a real success and we hope that our works will be carried on in the same spirit which vivified the "Convegno di Teorie Combinatorie". Our hope is (I shall use again the words by Beniamino Segre) "che il Convegno - anche attraverso i contatti personali che ne deriveranno - segnerà una tappa determinante nella storia dello sviluppo delle Teorie combinatorie e nella formazione scientifica di molti presenti".

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BENIAMINO SEGRE

G. Tallini

Commemorating such an eminent man as the late Professor Beniamino Segre is not an easy task. The breadth and depth of his scientific work in many branches of mathematics was remarkable: from mechanics to differential geometry, from algebraic to combinatorial geometry, from topology to algebra, his work amounted to some three hundred publications, both books and scientific papers.

As a former student of his I remember him with deep admiration and reverent devotion.

Beniamino Segre was born in Turin on February 16th 1903, and remained there for his education. He started his university studies when he was just sixteen and among his teachers there were famous mathematicians and great masters such as Giuseppe Peano, Gino Fano, Guido Fubini and Corrado Segre, a cousin of his mother's, under whose supervision he took his degree cum laude, his doctoral thesis on algebraic geometry being published in the *Atti dell'Accademia delle Scienze di Torino*. He was only twenty years old at the time and soon became assistant professor in rational mechanics, analytic, projective and descriptive geometry at Turin University. Three years later, supported by a Rockefeller scholarship, he studied in Paris with Elie Cartan and after becoming "libero docente" (the Italian title "libero docente" can be compared with the Ph.D.) in 1927, he came to Rome, appointed assistant professor of infinitesimal analysis by Francesco Severi, the latter too being among the greatest of Corrado Segre's students. Beniamino Segre stayed in Rome till 1931; during this year he became full professor and occupied the chair of analytic, projective and descriptive geometry at Bologna University. In 1932 he married Fernanda Coen, a woman of great worth and feeling who was a constant support to him throughout their life. They were extremely close and his death occurred almost exactly a year after that of his beloved wife. He remained in Bologna till 1938, when he was compelled to leave Italy because of the racial laws. The years between 1938 and 1946 were spent in London, Cambridge and Manchester.

Then he returned to Bologna where he remained till 1950 when he came to Rome as successor to Francesco Severi, holding first the chair of Algebraic Geometry and the year after the chair of Higher Geometry; at the same time he gave advanced geometry courses at the Istituto Nazionale di Alta Matematica. In 1973 he retired from teaching, remaining in the Faculty of Sciences as a retired professor and "decano".

He was appointed corresponding member of the Accademia Nazionale dei Lincei in 1947, national member in 1953, president from 1968 to 1973 and from 1976 to 1977 and vice-president from 1973 to 1975. Within the Accademia Nazionale he founded and directed the "Centro Linceo Interdisciplinare di Scienze Matematiche e loro Applicazioni" whose aim was to develop inter-relations between different branches of pure and applied mathematics. He was an excellent promoter and organizer of national and international conferences, which he used to address in powerful and striking talks. He truly believed in academies because of their aims and importance. Especially he believed in the Accademia dei Lincei, which through long years he enlivened by proclaiming its cultural and scientific functions as well as its social function, hoping that politicians and especially the rulers of the country would become aware of what the Accademia dei Lincei could do and should give to Italy and to the whole world. Indeed, he wrote "Siamo qui a disposizione del Paese, con tutto il patrimonio storico ed intellettuale che ci onora e ci onera. Ci si interPELLI, ci si consulti, in una parola ci si faccia produrre concretamente per il progresso scientifico, culturale ed anche economico-sociale del nostro popolo". ("We are here, available to our country, with all our historical and intellectual wealth, which is both an honour and a burden. Ask us questions, consult us, in a word, let us produce concrete results aiming at the scientific, cultural, social and economical progress of our people").

He was a founding member of the Unione Matematica Italiana, life member of the Soci  t   Math  matique de France, member of the American Mathematical Society and of the Tensor Society. He was on the editorial board and on the scientific committee of several important mathematical reviews, such as *Annali di Matematica Pura ed Applicata*, *Bollettino dell'Unione Matematica Italiana*, *Rendiconti del Circolo Matematico di Palermo*, *Canadian Journal of Mathematics*, *Revue de la Facult   de Sciences d'Istanbul*, *Acta Arithmetica*, and *Tensor*.

Beside being a fellow of the Accademia Nazionale dei Lincei, he was a fellow of the Accademia delle Scienze di Torino, the Accademia Nazionale dei XL, chairman