CISM COURSES AND LECTURES NO. 292
INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

APPLICATIONS OF TENSOR FUNCTIONS IN SOLID MECHANICS

EDITED BY

J. P. BOEHLER

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PREFACE

The mechanical behavior of materials with oriented internal structures, produced by formation processes and manufacturing procedures (crystal arrangements, stratification, fibrosity, porosity, etc.) or induced by permanent deformation (anisotropic hardening, softening, creep internal damage growth, etc.) requires a suitable mathematical modelling. The properties of tensor valued functions of tensor variables constitute a rational basis for a consistent mathematical modelling of complex material behavior.

This book, which contains lectures presented at a CISM Advanced School, presents the principles, methods and results of applications in solid mechanics of the general laws governing tensor functions. The principles of mathematical techniques employed to derive representations of tensor functions are explained. The rules of specifying irreducible sets of tensor invariants and tensor generators for various classes of material symmetries are discussed. Representations of isotropic and anisotropic tensor functions are derived, in order to develop the general invariant forms of non-linear constitutive laws in mechanics of solids.

Within this approach, the mathematical modellization of the materials' mechanical response is explained and specific models are presented in elasticity, plasticity, hardening, internal damage and failure, for materials such as metals, composites, stratified rocks, consolidated soils and granular materials. The approach specifies a rational way to develop approximate theories and gives the necessary precision as to the number and the type of independent variables entering the mechanical laws to be used in engineering applications.

Experimental justifications as to the pertinence of the approach are given on examples of composite materials, rolled sheet-steel and stratified rocks. Information concerning proper experimental setting of tests for materials with oriented internal structures is developed.

This book is addressed to specialists in solid mechanics, both theoretical and applied, material scientists concerned with metals and composites, specialists in soil and rock mechanics and to structural engineers facing problems involving anisotropic and inelastic solids at various environments, nonlinearity and couplings.

I wish here to pay homage to the memory of my deeply missed friend Professor Antoni Sawczuk, co-coordinator of the CISM Advanced School. His untimely death did not allow him to see the final fulfilment of our shared project ; nevertheless, his help was invaluable in the preparatory phase of the Advanced School.

I would like to take this opportunity to thank CISM for having provided lecturers and participants with a chance to work together on Applications of Tensor Functions in Solid Mechanics. I am indebted to Professor Giovanni Bianchi, Secretary General of CISM, for his help with the organization of the School and to Miss Elsa Venir for her kindness and efficiency.

Jean-Paul Boehler

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APPLICATIONS OF TENSOR FUNCTIONS

IN SOLID MECHANICS

Chapter 1

PHYSICAL MOTIVATION

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1. INTRODUCTION

Theorems of representations for tensor functions are valuable for modelling non-linear constitutive laws, particularly when the mechanical response of the material depends on more than one tensor agency. It is an approach that leads to the general invariant forms of the non-linear constitutive equations and gives the number and type of the scalar variables involved.

These representations for tensor functions have proved to be even more pertinent in attempts to model the mechanical behavior of anisotropic materials, since here invariance conditions predominate and the number of independent scalar variables cannot be found by simple arguments.

In this Chapter, we present experimental evidence of anisotropic response of materials and show the complexity of phenomena observed, which indicates the need for a rational and unified formulation of anisotropic constitutive laws.

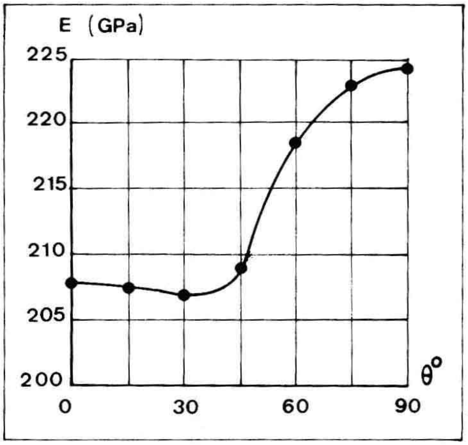

Fig.1 - Anisotropy of the elastic modulus of rolled sheet-steel (after [1]).

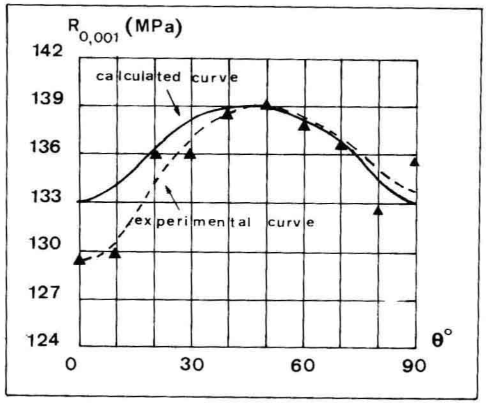

Fig.2 - Anisotropy of the elastic limit of rolled sheet-steel (after [2]).

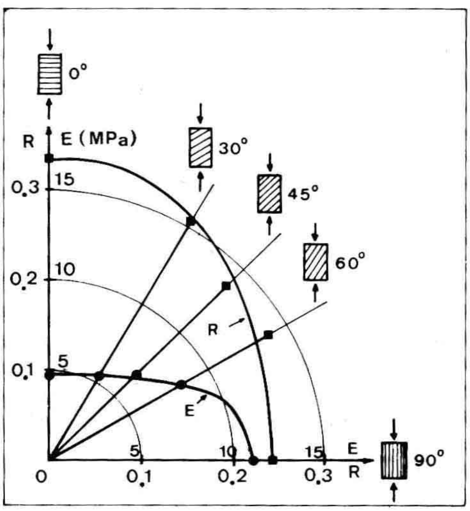

Fig.3 - Anisotropy of the elastic modulus and compressive strength of a natural clay (after [3]).

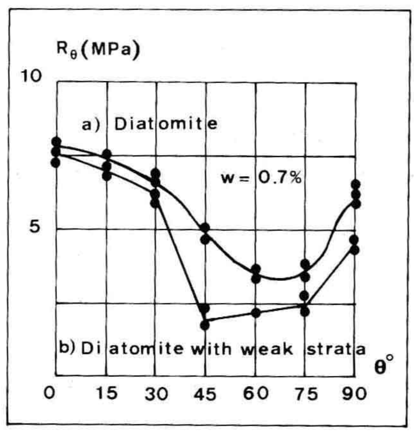

Fig.4 - Strength in simple compression for uniform and periodically non-homogeneous structure of diatomite (after [4]).

2. DIFFERENT DOMAINS OF MECHANICAL ANISOTROPY

Oriented internal structures of solids, such as oriented crystallographic axes, grains, particles, fissuration, cracks, cavities, etc..., result on the macroscopic level in a directional mechanical response to applied agencies. Different domains of the mechanical behavior can be influenced.

Fig.1 shows the variation of the elastic modulus E of rolled sheet-steel with respect to the angle θ between the direction of the tensile stress and the rolling direction (after [1]), whereas the anisotropy of the elastic limit R is presented in Fig.2 (after [2]). Anisotropy of the elastic modulus E and strength R of a consolidated clay subjected to oriented compressions is given in Fig.3 (after [3]).

According to the type of the oriented internal structure, the variation of mechanical characteristics, with respect to the orientation of the material, may be continuous or discontinuous. This is shown in Fig.4 (after [4]), where the uniaxial compressive strength of diatomite, a stratified soft rock, is traced versus the orientation θ of the specimens with respect to the normal of the strata. It is seen that the standard compressive strength possesses a minimum within the range of inclination of the strata. For a diatomite with marked layers of weaker strata, there appears a sudden drop in the strength for inclinations ranging between 30° and 45°.

In Fig.5, results regarding the strength of different consolidated clays in compression are presented (after [5, 6]) : a) range of variation for London clay ; b) Little Belt clay ; c) Vienna clay ; d) Welland clay ; e) experimental points for Grenoble clay. It is seen that the variation of strength with the orientation of the privileged direction of transverse isotropy is quite irregular. The strength either decreases, increases or passes through an extremum when the inclination of the privileged direction changes with respect to the principal stress direction. For triaxial tests on shales, Fig.6, the strength is plotted against this inclination angle for several values of the confining pressure (after [7]). Two remarks are appropriate in connection with the experimental results pre-