LASERS AND THEIR APPLICATIONS IN PHYSICAL RESEARCH

Edited by N. G. Basov

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Lasers and Their Applications in Physical Research

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PREFACE

In this volume an approach is developed for describing the dynamic processes in lasers based on a spectral representation of the polarization of the material and the radiation field. Reviews are given of results on heterojunction lasers based on an entire series of new semiconductor solution systems and of results on the problem of controlling the spectral composition, directionality, and polarization of the output from heterolasers. Different materials are studied for use as active media in high-power pulsed Raman lasers. The state of theoretical and experimental work on laser ranging of the moon is discussed. It is shown that the basic characteristics of the earth–moon system can be determined from one to three orders of magnitude more accurately by using laser ranging measurements than by using other optical methods.

This anthology is intended for physicists doing research on and working with lasers.

CONTENTS

FLUCTUATING INTENSITY REGIMES IN LASERS AND MASERS

V. A. Dement'ev, T. N. Zubarev, and A. N. Oraevskii

The conditions for spiking in lasers (and masers) are studied. A method is developed for analyzing the equations describing laser processes which is a variant of the Fourier method in combination with the small-parameter method applied to this particular problem. This method makes it possible to consider in a unified way the stationary states of lasers, their stability, and transition processes. The following laser models are analyzed: standing wave and traveling wave lasers, lasers with a point active medium, with an inhomogeneous luminescence line, and with a Q-dispersive cavity. The effect of pumping instability and the cavity parameters on the operating regime of a laser is studied. Stochastic methods are used to study random lasing and a laser with noise pumping. These cases have made it possible to formulate criteria for the different spiking regimes in various types of lasers and masers. An attempt is made to compare some of the conclusions of this theory with available experimental data.

INTRODUCTION

Research on the dynamics of lasers and masers has been going on for more than ten years. During this period much experimental and theoretical work has been done on fluctuating intensity regimes (spiking) in molecular masers and various types of lasers [1-9]. It is found experimentally that under certain conditions a spikeless lasing regime is replaced by chaotic or regular spiking. In some cases spiking is regarded as a favorable factor while in others it is undesirable. The spikes have different characteristics. There are the so-called free lasing spikes (usually lasting milliseconds), the envelope of which is modulated at a low frequency (of the order of a kilohertz); self Q-switching spikes (lasting nanoseconds in lasers), which are a sequence of giant pulses with a repetition rate of the order of the effective lifetime of the upper working level; and ultrashort pulses (lasting picoseconds in lasers) with a repetition rate equal to the transit time of a photon through the cavity. The conditions for disruption of spikeless operation of a laser depend very subtly on a whole group of parameters. Detailed experimental studies have been made of the effect of such factors as the mode composition of cavities [10-12], the amount of pumping [4, 10, 13-16], and the stability of the laser parameters [17]. Definite progress has been made in the theoretical description of laser operation. Undamped oscillations in the density of photons in inverted systems have been obtained in [3, 18-21, 161]. Nonetheless, up to now (1) there is no single opinion on the nature of spiking; (2) no approach has been proposed that can uniquely yield all the experimentally observed regimes; and (3) the theoretical models are often very far from experiment, so it is difficult to compare the theoretical results with experiment. In this article we summarize some of our results on formulating a theory which satisfies these requirements.

Up to now the theory of lasers has involved three approaches [22]. The first approach consists of work on the quantum theory of the laser [23-26] in which the active medium and the electromagnetic field are considered from the standpoint of quantum theory. This is the microscopic level of investigation and, as a result, the quantum statistics of laser radiation has come to be understood and profound analogies have been discovered between laser instabilities and physically well-known phase transitions and critical phenomena near a state of thermodynamic equilibrium. The second approach combines work on the macroscopic level [22] and information theory to describe processes in lasers. The third approach involves a semimicroscopic treatment with a quantum-mechanical analysis of the active medium and a classical examination of the field [3, 27]. The quasiclassical equations are sufficiently general for studying the dynamics of lasers and are the basis of this paper as well as most work on the theory of lasers. This is why a number of problems in laser kinetics lie beyond the scope of this article. For example, we do not discuss the operating regimes of lasers with a transverse inhomogeneity in the field [28, 29], multicomponent media [30-32], multichannel lasing [33], the effect of the dependence of losses and the refractive index of materials on the field strength [3, 34], the effect of the relationship between the pumping and lasing channels [35] on the operation of lasers, and so on.

The active medium of a laser is essentially an ensemble of quantum objects with two working levels† which interact with one another through an electromagnetic field. In the absence of energy sinks and sources the probability of a two-level molecule situated in an electromagnetic field being in one of the levels oscillates at a frequency equal to the ratio of the product of the moment of the transition (dipole or magnetic) times the field strength to Planck's constant [36]. In the microwave range the working transition is usually a magnetic dipole transition and the frequency of the oscillations in the inverted population has an obvious physical meaning: It is the nutation frequency of the magnetic moment of the molecule about the direction of the magnetic field. In the optical range the working levels are part of an electric dipole transition which has no simple classical analog, but this frequency is still called the optical nutation frequency.

The operation of a laser cannot be understood without including the processes of dissipation and pumping. Dissipative processes in the laser are described by three phenomenological relaxation constants. The lifetime of the upper working level of the molecule, or the longitudinal relaxation time of the material, determines the rate of change of the population inversion in the material. The width of the luminescence line, which is inversely proportional to the transverse relaxation time of the material,‡ defines the duration of a coherent wavetrain in spontaneous emission of the molecule. The spectral width of a mode, which is inversely proportional to the lifetime of a photon in the cavity, characterizes the rate of damping of the field in the cavity. The effect of pumping is described by the change from the stationary value of the inversion in the absence of a field. When, due to pumping, more energy enters the system than is lost, the laser is self-excited. After the pump is turned on, the threshold population inversion is established after a time of the order of the lifetime of the laser level. The spectra of the modes in the laser output are formed after a further delay equal to the photon lifetime in the cavity. If the product of the transit time of the photons across the cavity and the optical nutation frequency is of the order of unity, then the electromagnetic wave strongly modulates the absorption coefficient or refractive index of the material, the modes may be coupled in phase (as when they are synchronized by an external force), and the laser emits ultrashort pulses.

† In the following we shall speak of a two-level molecule for brevity.

‡ The terms "longitudinal" and "transverse" relaxation times of the material, as well as the term "nutation frequency," have been taken from microwave terminology.

The envelope of the ultrashort pulses is formed due to the development of deviations in the partial intensities of the modes from their stationary values. These intensity deviations undergo self-consistent relaxation (at customary pumping intensities) oscillations together with the deviations in the inverted population and polarization of the material. Oscillations at the relaxation frequency arise in the same way as in the predator–prey (Volterra's) problem. The relaxation oscillations in the output intensity may be damped or may grow. In the latter case the stationary envelope of the ultrashort pulses is a sequence of giant pulses which repeat at a frequency equal to the lifetime of the laser level since the stationary regime is determined by gross energy factors. Thus, relaxation and nutation oscillations in the inversion are potential sources of instabilities in the spiking regime of a laser as well. In its most developed form the spike structure of the laser output is a sequence of ultrashort pulses with an envelope made up of giant pulses. In the opposite limiting case the laser operates without spiking. To realize these cases a number of sufficient conditions must be satisfied which differ according to the type of laser, and the resulting lasing regime may be a very poor copy of the picture sketched above.

Laser instabilities depend on a whole series of factors of which the following are most important:

(a) The ratio of the optical nutation frequency to the relaxation constants of the material and field. If this frequency is much greater than the relaxation constants, then the inverted population mainly undergoes nutation oscillations, while in the opposite limiting case the inversion oscillates at the frequency of the relaxation oscillations.

(b) The relations among the relaxation constants of the material. If the linewidth is much greater (less) than the spectral width of a mode, then the polarization (field) lags behind the field (polarization). The lifetime of the upper working level of a laser, except in the case of molecular and spin lasers, is much greater than the other characteristic times, and this often makes it possible to divide spike formation into pumping and emission stages.

(c) The duty cycle of the cavity. The lasing kinetics depend to a great extent on the ratio of the dimensions of the active medium to the wavelength. If the dimensions of the medium are much less than the wavelength, then its dimensions can be neglected and the laser can be treated as having a point active medium.

(d) The degree of spatial inhomogeneity in the field, which is characterized by the relations among the intensities of waves traveling in opposite directions.

(e) Spatial dispersion in the Q factor of the resonator cavity. This explains the different losses of modes with different indices.

(f) The nature of line broadening. The luminescence line may be homogeneously and inhomogeneously broadened. A homogeneous line has a Lorentzian shape while an inhomogeneous line may be described by a Gaussian function or by discrete distributions.

We now present a classification of the instabilities and give a brief review of the contents of this article. First of all, the instabilities may be divided into two large classes. The appearance of instabilities of the first class is determined by the energy balance in the laser or maser, and the condition for instability may have the form of a condition for self-excitation of the laser including saturation. This kind of instability is aperiodic or oscillatory with a frequency equal to the difference between the modal frequencies of the cavity and a growth rate of the order of the width of the mode (in lasers). An instability of this type describes the excitation of the spectrum of axial modes in the kinetic theory of lasers (oscillatory instability) or the destruction of a packet of modes (mode capture or trapping instability) in three- and single-mode lasers (cf. Section 5, paragraph 5.2).

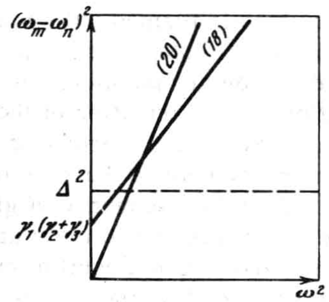

Fig. 1. The dependence of the frequency of nutation (18) and relaxation (20) oscillations in the inversion on the pumping $\gamma_3 > \gamma_1 + \gamma_2$. If $\gamma_3 < \gamma_1 + \gamma_2$ the branches (18) and (20) do not intersect. The dashed line corresponds to a frequency Δ for oscillations in the field, inversely proportional to the transit time of a photon through the cavity.

As an instability of the second class develops, the inversion oscillates at the relaxation or nutation frequency while the growth rate is determined by various factors and is of the order of or less than the Einstein coefficient for spontaneous emission. Various models have been studied.

A. A laser or maser with homogeneous parameters, that is, one using traveling waves with a homogeneous luminescence line and a dispersion-free cavity completely filled with the active medium (Sections 2-4).

1. Single-mode laser or maser (Section 3). The frequencies of the relaxation and nutation oscillations in the inversion depend in different ways on the laser parameters. It may happen that for certain parameter values these frequencies are of comparable magnitude (cf. Fig. 1). Then one speaks of an intersection of the relaxation and nutation branches of the oscillations. The strong interaction between the different forms of oscillation which may occur when the branches intersect is the reason for instability in the spikeless regime in masers, semiconductor lasers, and molecular lasers if the spectral width of a mode is of the order of or much greater than the width of the luminescence line. The mechanism of the instability is as follows. The deviations in the inverted population and other dynamic variables from their values in the spikeless regime primarily excite relaxation (nutation) oscillations in the inversion at small (large) pumping levels in the sense of factor (a). Since the frequencies of the relaxation and nutation oscillations are close, the primary excited oscillation is amplified by interacting with the weakly excited oscillation. The developing instability brings the laser into a self-Q-switched spiking regime at the optical nutation frequency at high pumping levels. Weak and strong regimes of spike excitation are possible.

2. A multimode traveling wave laser or maser (Section 4). The instability in the spikeless regime is a consequence of either the Cerenkov effect or an autoparametric resonance. The Cerenkov instability occurs when the phase velocity of an electromagnetic wave is greater than the propagation velocity of light in a medium (cf. Figs. 2 and 7). With an autoparametric resonance, heating between modes at a frequency equal to twice the optical nutation frequency pumps oscillations at the nutation frequency (cf. Figs. 4 and 7). Whereas with the Cerenkov instability a laser can "weakly" go into a spiking regime at the nutation frequency, the auto-

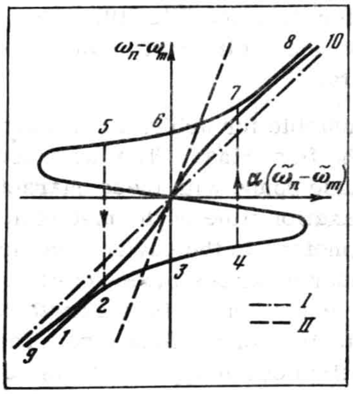

Fig. 2. The dependence of $\omega_n - \omega_m$ on $\tilde{\omega}_n - \tilde{\omega}_m$ for various values of pumping with $\gamma_1 \ll \gamma_3$: 1–8) $\omega_1^2 > 2\gamma_1^2$; 9, 10) $\omega_1^2 < 2\gamma_1^2$. (I) the asymptote $\omega_n - \omega_m = \alpha(\tilde{\omega}_n - \tilde{\omega}_m)$; (2-4-7-5) hysteresis which occurs on tuning the cavity; (II) the straight line $\omega_n - \omega_m = \tilde{\omega}_n - \tilde{\omega}_m$.

parametric resonance yields only a nonlinear instability, that is, a "hard" spike excitation regime at the nutation frequency. The growth rate is inversely proportional to the effective photon lifetime in the cavity with a proportionality coefficient of the order of the square of the ratio of the nutation frequency to the linewidth.

B. Lasers and masers with inhomogeneous parameters (Section 5).

I. Point model (Paragraph 5.1).

1. A single-mode laser with a point active medium behaves as a single-mode traveling wave laser.

2. A two-mode laser with a linewidth much greater than the width of a mode. Mode capture is unstable when the difference between the mode frequencies is of the order of the widths of the modes (cf. Fig. 8) and the stability boundary is "hazardous," that is, for smaller differences between the mode frequencies a nonlinear instability may occur and the laser may undergo a strong transition into a self-Q-switched spiking regime.

3. Three-mode and single-mode laser with a linewidth much greater than the spectral width of a mode. Mode capture is stable for arbitrary differences in the mode frequencies. Apparently only a strong transition to a self-Q-switched spiking regime with mode locking is possible (cf. Fig. 8).

II. Spatial inhomogeneity of the field in the cavity (Paragraph 5.2).

1. Single-mode laser. In the neighborhood of an antinode (node) of a standing wave the field strength is greater (less) than the field strength in a traveling wave laser. Thus, field oscillations at the relaxation frequency are damped more strongly (weakly) near an antinode (node) of a standing wave than in a traveling wave laser. Near the lasing threshold these effects compensate one another on the average. As the pumping level is increased the standing wave begins to burn out the inversion near the nodes by means of multiphoton processes, the hole in the spatial distribution of the population inversion becomes larger, and the damping rate of the spikes becomes greater than in a traveling wave laser.

2. Two-mode laser. Mode capture sets in after a time of the order of the damping decrement of the spikes in a single-mode laser. The spectrum of the modes which are set to oscillating due to spatial burnout of the inversion is formed with a growth rate proportional to the spectral width of the mode, and in a two-mode laser the coefficient of proportionality is of the order of the square of the ratio of the frequency difference between the modes to the spectral width of the modes. Thus, the second process is more effective than the first when the frequency difference between the modes is greater than the frequency of the relaxation oscilla-

tions, and mode capture is unstable at such frequency differences (cf. Fig. 10). The stability boundary is hazardous and the spikes at the relaxation frequency either are damped or grow. In the latter case a self-Q-switched spiking regime results.

3. Three- and multimode laser. Mode capture is unstable for arbitrary frequency differences between the modes because of an instability of the first class. The laser operating regime is described by a complex integral manifold in phase space with these characteristic time scales (in order of appearance): the transverse relaxation time of the material, the transit time of photons across the cavity, the lifetime of photons in the cavity, and the longitudinal relaxation time of the material. In this regime a laser produces mode-locked self-Q-switched spikes. The partial intensities of the modes are established with a growth rate of the order of the spectral width of the mode because of an instability of the first type. The envelope (a giant pulse) is formed in a time of the order of the lifetime of the upper working level as an instability develops at the relaxation frequency. The phases of the modes are synchronized with a growth rate of the same form as in a multimode traveling wave laser, that is, mode synchronization occurs due to optical nutation. It follows from obvious physical considerations that, beginning with a sufficiently large number of synchronized modes, the development of ultrashort pulses should take place in the same way in traveling wave and standing wave lasers; thus, as far as the physics of the phenomenon of synchronization (locking) is concerned, we can repeat all that was said above in Paragraph A.2. From this standpoint the effect of a bleachable filter in mode locking is to isolate the strongest fluctuating emission outburst of the laser which then initiates these dynamic synchronization mechanisms.

III. Spatial dispersion in the Q-factor (Paragraph 5.2).

The spectrum of the laser modes is established during competition among the position dependences of the gain and absorption coefficients. Burning a gap in the spatial distribution of the inversion widens the lasing spectrum, and increased loss of modes as the mode frequencies are shifted further away from the center of the luminescence line narrows the spectrum of the working modes. The first effect exceeds the second for frequency differences between the modes of the order of the square root of the difference in the squares of their widths, and an oscillatory instability at the relaxation frequency appears. The development of the instability brings the laser into a stationary spiking regime of the same type as in Paragraph B.II.3.

IV. Inhomogeneous luminescence line (Paragraph 5.3).

1. Single-mode laser. The field burns a hole of width equal to the homogeneous line broadening in the inhomogeneous luminescence line. If the relaxation frequency for the luminescence centers involved in lasing is equal to the width of this hole, then an oscillatory lasing instability develops at the relaxation. Bifurcations in a spikeless lasing regime at the stability boundary are studied for a laser with a mode width much greater than the line width. The stability boundary may be hazardous or safe and the stationary spiking regime may consist of spikes with a width and period of the order of the transverse and longitudinal relaxation times of the material, respectively.

2. Multimode laser. A new feature is observed in the amplitude characteristic of a two-mode laser in which the transverse relaxation time of the material is much less than the longitudinal relaxation time while the latter is much less than the photon lifetime in the cavity. It is known that if the frequency difference between the modes is less than the homogeneous broadening of the line, then the partial intensities of the modes are reduced by roughly a factor of 2 because of overlapping of the holes burned by the modes in the luminescence line (the so-called Lamb dip). If the frequency difference between the modes is reduced further, to the point that the product of the frequency difference and the longitudinal lifetime of the material is of order unity, then the population inversion follows the field and oscillates with a large

amplitude. The resulting combination frequencies lie outside the amplifying bandwidth of the cavity and are absorbed. Because of this the partial intensities of the modes are further reduced by a factor of 1.5. Thus, against the background of the Lamb dip there is a narrower dip with a width inversely proportional to the longitudinal lifetime of the material.

C. Nonautonomous systems (Section 6).

1. Single-mode laser (Paragraphs 6.1 and 6.3). A parametric instability with modulation of the cavity losses develops when the ratio of the frequency of the external force to the relaxation frequency is a rational number. The amplitude–frequency characteristics of the harmonic (i.e., at the frequency of the external influence) and subharmonic (with a frequency equal to half the lowest modulation frequency) intensity oscillations are determined. When the loss modulation amplitude is greater than the ratio of the damping increment of the relaxation spikes in an autonomous single-mode laser to the spectral width of the mode, then the resonance becomes nonlinear, the maximum of the amplitude–frequency characteristic is shifted toward lower frequencies, and hysteresis phenomena may occur. Modulating the pumping yields these effects at powers such that the optical nutation frequency is much less than the linewidth. In the inverse limit the amplitude–frequency characteristic has a resonance at the optical nutation frequency when the pump is modulated. When the losses are modulated this effect does not occur since at higher pumping levels the nutation frequency is the characteristic frequency of the fluctuations in the inversion; thus, if an external force at this frequency acts on the field (as during modulation of the losses) then, as with a roll dampener on a ship, the inverted population oscillates with a large amplitude while the field hardly oscillates.

2. Multimode laser (Paragraph 6.2). Modulating the losses yields the same type of phenomena as in a single-mode laser. When the refractive index of the material is modulated, we obtain a phase modulation of the output with a maximum in the partial amplitude–frequency characteristic near frequencies of the order of the slow relaxation frequency in a multimode laser. (This frequency is roughly equal to the relaxation frequency of a single-mode laser divided by the square root of the number of excited modes.)

D. Stochastic effects (Section 7).

1. Single-mode laser (Paragraphs 7.1, 7.2, and 7.3). If the external influence modulates the laser parameters over a wide spectrum of frequencies and to such a depth that the system can change its stationary states within a resonance and go from one resonance to another, then the output becomes highly irregular and close to random. For pumping with white noise the noise intensity required for randomization within a resonance is directly proportional to the square of the ratio of the lifetime of a photon in the cavity to the effective lifetime of the upper laser level. The stochastic instability is the most hazardous kind of instability for the operation of a laser. The analysis of the conditions for the stochastic instability relies on the kinetic equations for the radiant energy density. A study of the stationary solutions of these equations shows that the laser output has an almost-periodic turbulence spectrum, i.e., a linear spectrum. If the spectral width of a mode is much less than the linewidth, then a stationary random regime with a continuous spectrum corresponding to developed turbulence in the laser output is possible. The laser emits random spikes with a frequency and width of the order of the relaxation frequency while in the case of almost-periodic turbulence the spikes are grouped in packets whose repetition rate equals the difference between the frequencies in the spectrum. An examination of the stability of the spikeless regime and an analysis of the transition processes shows that the turbulence may develop strongly or weakly. If the cavity frequency equals the frequency of the line, the spikeless regime is stable but strong excitation of a developed stationary turbulence is possible. As the cavity frequency is displaced from the frequency of the line, the spectrum of the stationary turbulence is depleted and the number of stationary states is reduced. However, the role of the nonstationary motions is enhanced at

the same time that the threshold for excitation of turbulent emission is reduced. If the shift in the cavity frequency from the line frequency exceeds the product of the nutation frequency and the square root of the maximum possible number of spectral components in the output of a laser whose mode frequency equals the line frequency (Fig. 2), then the spikeless regime is unstable and the resulting turbulence is nonstationary.

2. Multimode laser (Paragraphs 7.3 and 7.4). If the mode spectrum has a single characteristic scale (that is, only axial modes operate), then random relaxation spikes occur in the laser output when the product of the slow relaxation frequency and the lifetime of the upper laser level exceeds unity. If a laser generates a group of transverse modes near the frequency of an axial mode such that the emission is coherent within that group while the emission from groups belonging to different axial modes is incoherent, then the laser may emit partially ordered trains of spikes. Strong excitation of the spiking regime occurs when the slow relaxation frequency is of the order of the distance between the frequencies of the transverse modes. A random self-Q-switched spiking regime appears. Because each group of transverse modes oscillates independently, a giant pulse splits randomly into smaller pulses, whose number is proportional to the number of axial modes generated.

As we proceed to prove these statements, we note that the theory of instabilities may be constructed deductively. As a rule, in order to explain the nature of the instabilities, a different approach is preferred. At first the instabilities in the simplest model of a laser are analyzed. This model is then refined as more new factors are included. The figures also serve this purpose and most of them are qualitative.

1. Derivation and Preliminary Analysis of the Equations

To model the processes taking place in a laser or maser we shall consider an active medium made up of two-level molecules located in a resonator cavity. The interaction of the electromagnetic field with the matter obeys the following system of equations [1, 2]:

$$\dot{\nu} = \gamma_1 (1 - \nu) + 2i\Omega e (\rho^* - \rho), \quad (1)$$

$$\dot{\rho} = -(i\omega_0 + \gamma_2)\rho - i\Omega e\nu, \quad (2)$$

$$\ddot{e} + 2\gamma_3\dot{e} - c^2\nabla^2 e + 2\frac{\Omega}{\omega_0}(\ddot{\rho} + \ddot{\rho}^*) = 0, \quad (3)$$

where $e = (2\pi n_0 \hbar\omega_0)^{-1/2} E$ (E is the electric field strength†); $\rho = \rho_{12}$ and $\nu = \rho_{22} - \rho_{11}$, where ρ_{ij} (i, j = 1, 2) is the density matrix of the two-level molecule in the energy representations; that is, the real and imaginary parts of ρ describe the polarization and displacement current of the medium, respectively, and ν gives the inverted population of the medium; ω_0 is the frequency of the transition between the working levels of the molecule; $\Omega = (2\pi n_0 \omega_0/\hbar)^{1/2} D$ (D is the absolute value of the dipole moment of the transition and n_0 is the density of inverted molecules in the absence of a field in the stationary state); γ_1^{-1} is the effective lifetime of the upper working level of the molecule or the longitudinal relaxation time of the medium; γ_2 is the luminescence linewidth, inversely proportional to the transverse relaxation time of the medium; γ_3^{-1} is the photon lifetime in the cavity and is inversely proportional to the spectral width of the mode. Table 1 shows the relaxation constants for various types of lasers and masers.

† For a maser the electric field and transition dipole moment in Eqs. (1)-(3) are replaced by the magnetic field strength and the transition magnetic dipole respectively. To be specific, in the following we shall speak of an electric dipole transition.

TABLE 1

Type of laser (maser)	γ_1, sec^{-1}	γ_2, sec^{-1}	δ, sec^{-1}	γ_3, sec^{-1}	References
Ruby laser, 300°K	10^3	10^{11}	—	10^7—10^9	[2, 37]
Same, 4-77°K	10^3	10^7—10^9	$2 \cdot 10^{10}$	10^7—10^9	[38]
Dysprosium laser $CaF_2 : Dy^{2+}$, 27° K	10^2—10^5	10^9	—	10^8	[39, 40]
Ruby maser	10^2	10^8	—	10^6—10^8	[2, 41]
Semiconductor laser	10^9	10^{12}	—	10^9—10^{12}	[42]
Molecular maser (laser)	10^4	10^4	—	10^6	[1]
He–Ne laser	10^8	10^8—10^9	10^{10}	10^7	[43]
Atmospheric pressure CO_2 laser	10^3	10^{10}	10^{10}	10^8	[44–46]
Dye lasers	10^8—10^9	10^{12}	—	10^8—10^9	[47]

Note: δ is the inhomogeneous line broadening (see Section 5).

Since we are usually interested in the electric field in the cavity, it is natural to eliminate the elements of the density matrix from Eqs. (1)-(3). This may be done in a general way, but the weak nonlinearity of the equations and the existence of dispersion make it possible to limit ourselves to the approximation

$$\ddot{\rho} + \ddot{\rho}^* \simeq - \omega_0^2 (\rho + \rho^*) \tag{4}$$

upon substituting Eq. (2) into Eq. (3). After this we find the following equation for the field from Eqs. (1)-(3):

$$\frac{AB}{4\omega_0^2} e = - \Omega^2 \left\{1 + \int e\hat{e} \exp\left[- \gamma_1 (t - \tau)\right] d\tau\right\} e, \tag{5}$$

which is an integrodifferential equation with partial derivatives and a retarding kernel $\exp[-\gamma_1(t - \tau)]$ and

$$\begin{aligned} A &= - c^2\nabla^2 + \frac{\partial^2}{\partial t^2} + 2\gamma_3 \frac{\partial}{\partial t}, \\ B &= \omega_0^2 + \frac{\partial^2}{\partial t^2} + 2\gamma_2 \frac{\partial}{\partial t}, \\ \hat{e} &= \frac{A}{\omega_0^2}\left(\frac{\partial}{\partial t} + \gamma_2\right) e. \end{aligned} \tag{6}$$

Recurrence is a feature of the autooscillatory motions in which all the dynamic characteristics of the system are roughly repeated with a quasi-period depending on the time and accuracy of the repetition [48]. The most general class of recurrent motions which have been studied analytically up to now are the almost-periodic oscillations for which the quasi-period is independent of time but depends on the accuracy of the repetition. If the period is also independent of the accuracy of repetition, then we obtain a periodic oscillation which is a special case of an almost-periodic oscillation. In general a periodic motion is a superposition of a countable number of sinusoidal oscillations with frequencies that are multiples (of a single fundamental), while an almost-periodic motion decomposes into a Fourier series with incommensurable frequencies. The simplest example of an almost-periodic function is, therefore, the beating of two harmonic oscillations with close frequencies. In [49] the basic properties of periodic functions are generalized to almost-periodic functions. In particular, it is proved that arithmetic operations, differentiation, and integration (in the last case the Fourier expansion does not have to contain a free term) are defined in the class of almost-periodic functions.

These simple properties of almost-periodic functions are the basis of their extensive use in various problems in the theory of oscillations. Thus, we seek the stationary states of a laser as solutions of Eqs. (5) in the form of trigonometric series with real frequencies:

$$e = \sum_p e_p \exp(-i\omega_p t), \qquad \omega_{-p} = \overset{*}{\omega}_p, \qquad e_{-p} = \overset{*}{e}_p, \tag{7}$$

where the components e_p are expanded in a series over the eigenfunctions (modes) of the cavity Φ_λ:

$$e_p = \sum_\lambda e_p^\lambda \Phi_\lambda, \qquad -c^2\nabla^2\Phi_\lambda = \tilde{\omega}_\lambda^2 \Phi_\lambda. \tag{8}$$

In general the electromagnetic field has an infinite number of degrees of freedom and is described by partial differential equations. From a mathematical standpoint the stability of the solutions of partial differential equations is a complicated and still mostly unsolved problem. The major techniques and assumptions of stability theory were formulated by Lyapunov for systems with a finite number of degrees of freedom and obeying ordinary differential equations [50]. In recent years it has been shown that they can be generalized to equations with an infinite number of degrees of freedom if the variation equations for the motions whose stability is being studied have a bounded spectrum, that is, the solutions of the variation equations that have the exponential form exp (γt) are such that $|\gamma| < C$, where C is a constant [51]. The wave equation (3) contains the unbounded operator $c^2\nabla^2$ whose spectrum $\tilde{\omega}_\lambda^2$ [see Eq. (8)] extends to infinity. Nevertheless, it is clear that an instability cannot develop on modes whose frequencies differ from the center of the emission line of the material by much more than the width of the line. Intensity fluctuations in such modes are damped rapidly. Because of this rapid process the system lies in the neighborhood of an integral manifold (a manifold of the trajectories) of finite dimensionality. Therefore, the classical methods of studying stability [50] are applicable to the dynamics of lasers for dispersive media, that is, media with a finite linewidth, and in analyzing the stability of the stationary states of a laser we shall seek solutions of the equations in the form of Eqs. (7) and (8) with complex frequencies ω_p.

We shall study transition processes in lasers using the trigonometric expansions (7) with complex frequencies ω_p which vary slowly in time. This variant of the averaging method is known in the literature as the quasiclassical approximation or the WKB method [36; 52]. Thus, the stationary states of a laser and their stability and transitions are analyzed in a unified way with the aid of solutions of the form (7) and (8) with real and complex, constant and slowly time-varying frequencies ω_p. Expansions of this form are used to solve nonlinear problems in dynamic and stochastic theories [53]. This method has many variants [52–54], including Poincaré's small parameter method, the averaging method, the harmonic balance principle, the stroboscopic method, the WKB method, and so on. In the form given above it is well suited to solving various nonlinear problems in the theory of lasers and makes it possible to explain the instabilities as well as to study the effects of the mode composition of the cavity, the inhomogeneity of the laser parameters, and randomness on the operation of the laser.

We shall derive some general, mainly qualitative results which are valid for the entire class of stationary laser states. A stationary state of the field is characterized by a spectrum of output (generated) frequencies ω_p and a spectrum of oscillating (generating) modes $\tilde{\omega}_\lambda$. In general these are different concepts. In fact, if the frequency difference between the modes is less than the spectral width of a mode γ_3, then such modes may be captured and form a packet of modes which oscillate at a single frequency while different packets, generally speaking, overlap one another. In the space and time Fourier expansion (7) and (8) the quantity $\sum_\lambda |e_p^\lambda|^2$ characterizes the partial intensity of a line in the frequency spectrum while $\sum_p |e_p^\lambda|^2$ gives the partial intensity of a mode in the spectrum of stationary state modes. To characterize the spectra we introduce the moments of the distribution of partial intensities. The first moment determines the location of the center of the spectrum and the second gives the width of the spectrum. It appears that in a stationary laser state there is a simple relationship between the center of the frequency spectrum $\overline{\omega} = \sum_{p,\lambda} \omega_p |e_p^\lambda|^2 [\sum_{p,\lambda} |e_p^\lambda|^2]^{-1}$ and the center of the mode spectrum $\widetilde{\omega} = \sum_{p,\lambda} \widetilde{\omega}_\lambda |e_p^\lambda|^2 [\sum_{p,\lambda} |e_p^\lambda|^2]^{-1}$. To derive this relationship we substitute Eq. (7) in Eq. (5). We obtain the following equation† :

$$\frac{A_p B_p}{4\omega_0^2} e_p = -\Omega^2 \left\{ e_p + \sum_{\omega_k+\omega_l+\omega_m=\omega_p} \frac{e_k \hat{e}_l e_m}{\gamma_1 - i(\omega_l + \omega_m)} \right\}, \tag{9}$$

where A_p, B_p, and $\hat{e}_p$ are given by Eq. (6) with $i\omega_p$ substituted in place of $\partial/\partial t$. We multiply Eq. (9) by e_{-p}, sum it over $p \geq 0$, integrate it over space, and separate the imaginary part. The fourfold sum (over $p \geq 0$, k, l, m) in the latter equality contains every term together with its complex conjugate because the width of the frequency spectrum of the field is much less than $\overline{\omega}$ ($|\omega_m - \omega_n| \ll \overline{\omega}$) and $\gamma_1 \ll \overline{\omega}$, so it goes to zero. We thus obtain the desired dispersion relation

$$\overline{\omega} = \alpha\widetilde{\omega} + \beta\omega_0, \tag{10}$$

where $\alpha = \gamma_2/(\gamma_2 + \gamma_3)$ and $\beta = 1 - \alpha$. This expression means that the average nonlinear frequency pulling and pushing balance one another, and only so-called linear frequency pulling remains. In particular, for a single-mode system with a cavity eigenfrequency $\widetilde{\omega}_n$ Eq. (10) becomes the well-known formula [27, 55]

$$\omega_n = \alpha\widetilde{\omega}_n + \beta\omega_0.$$

The nonlinear terms in Eq. (9) may be resolved into "diagonal" ($\omega_l + \omega_m = 0$) and "nondiagonal" ($\omega_l + \omega_m \neq 0$) terms. The "nondiagonal" terms are due to oscillations in the population inversion with time, and their ratio to the "diagonal" terms is of order $\gamma_1 |\gamma_1 - i(\omega_l + \omega_m)|^{-1}$. We shall call the approximation in which the "nondiagonal" terms are rejected the "diagonal" approximation. In the "diagonal" approximation, in particular, all the results for equilibrium points of the balance or kinetic equations are obtained. This approximation yields a crude picture of the operation of a laser whose frequency spectrum contains no anomalously close frequencies [56], i.e.,

$$|\omega_l - \omega_m| \sim \gamma_1. \tag{11}$$

If close frequencies (11) are contained in the frequency spectrum, then in this frequency interval the population inversion follows the field quasistatically and the retarding kernel $\exp[-\gamma_1(t - \tau)]$ of Eq. (5) becomes of order unity; thus, the nonlinear terms have an especially strong effect on the behavior of the system.

A more subtle effect of the oscillations in the inverted population that is described by the "nondiagonal" terms in Eq. (9) is observed even in studies of the stability of a single frequency regime in Eq. (7) with $e_p = 0$ when $p \neq \pm n$. This kind of regime is also called harmonic [3] or monochromatic [57]. In accordance with the method described above, to study the stability of

† If we do not make assumption (4), then the factor ω_0^{-2} in the terms $(A_p B_p)/4\omega_0^2$ and $\hat{e}_p$ goes into ω_p^{-2}. In addition, to avoid misunderstanding, we note that the sum in Eq. (9) is over both positive and negative frequencies.