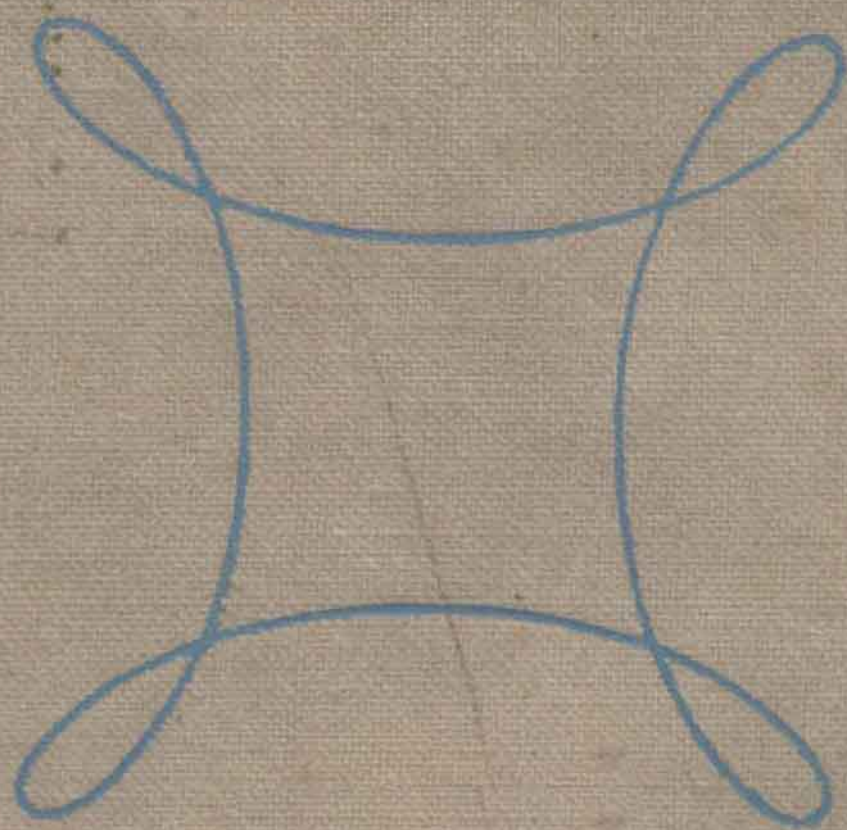


MATHEMATICAL HANDBOOK

ELEMENTARY MATHEMATICS

M. VYGODSKY



MIR PUBLISHERS MOSCOW



М. Я. ВЫГОДСКИЙ

**СПРАВОЧНИК
ПО ЭЛЕМЕНТАРНОЙ МАТЕМАТИКЕ**

**ИЗДАТЕЛЬСТВО «НАУКА»
МОСКВА**

MATHEMATICAL HANDBOOK

**ELEMENTARY
MATHEMATICS**

M. VYGODSKY

**TRANSLATED
FROM THE RUSSIAN
BY
GEORGE YANKOVSKY**

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INTRODUCTION

1. Designation of the handbook. The purpose of this book is twofold: firstly, it is designed for quick reference to mathematical concepts (to find out what a tangent is, to compute percentages, to recall formulas for the roots of a quadratic equation, etc.). All definitions, rules, formulas, and theorems are supplied with examples. Where required, hints and suggestions are given as to the use of a rule or how to avoid common mistakes, and so forth.

Secondly, the author believes that this handbook can serve as a manual for reviewing the essentials of mathematics and even as a first introductory course in its practical applications.

2. Handbook and textbook. An attempt has been made to combine the merits of both books in one text. That this has been successful is evident from numerous letters from the readers, most of whom used it as a textbook. True this handbook differs radically from an ordinary school textbook, where, especially in the senior classes, the emphasis is placed on reasoning: facts are subordinated to logic. This at any rate is how the student regards the process. In this book, the leading role is played by factual material. This does not in the least mean that the reasoning process is absent. Derivations of formulas are given, but only on occasion, as, for example, when it is necessary to stress the central idea of a given section or to overcome any doubts as to the validity of a result (say, when dealing with operations involving complex numbers). In deciding whether to keep a proof or omit it the author was guided by his own teaching experience.

3. How to use the handbook. For quick reference, use the extensive index at the back of the book. If the user has forgotten the exact name of a rule, formula, or mode of solution, he has a detailed table of contents at his disposal in the front.

We strongly advise the user to follow up any additional references he may encounter when investigating a term. Also, much useful information can be gained by reading through

the entire section containing the term or concept he is interested in.

It is wise to pay careful attention to the historical surveys contained in each division. They form an integral part of the book and contribute greatly to a deeper understanding of the subject.

The reader who desires to use this manual as a textbook should pay particular attention to the worked examples. Any proofs that are omitted in the handbook can be filled out by reference to a textbook on the subject either at the same time or later. However, it is well to bear in mind that neither handbook nor textbook alone suffices to give the reader a knowledge of the subject: he must use pencil and paper and work through the examples and problems for himself.

TABLES

1. Some Frequently Encountered Constants

Quantity	n	$\log_{10} n$	Quantity	n	$\log_{10} n$
π	3.1416	0.4971	$\sqrt[3]{1:\pi}$	0.6828	$\bar{1}.8343$
2π	6.2832	0.7982	$\sqrt[3]{\pi:6}$	0.8060	$\bar{1}.9063$
3π	9.4248	0.9743	$\sqrt[3]{3:4\pi}$	0.6204	$\bar{1}.7926$
4π	12.5664	1.0992	$\sqrt[3]{\pi^2}$	2.1450	0.3314
$4\pi:3$	4.1888	0.6221	e	2.7183	0.4343
$\pi:2$	1.5708	0.1961	e^2	7.3891	0.8686
$\pi:3$	1.0472	0.0200	$\sqrt{e}$	1.6487	0.2171
$\pi:4$	0.7854	$\bar{1}.8951$	$\sqrt[3]{e}$	1.3956	0.1448
$\pi:6$	0.5236	$\bar{1}.7190$	$1:e$	0.3679	$\bar{1}.5657$
$\pi:180$	0.0175	$\bar{2}.2419$	$1:e^2$	0.1353	$\bar{1}.1314$
$2:\pi$	0.6366	$\bar{1}.8039$	$\sqrt{1:e}$	0.6065	$\bar{1}.7829$
$180:\pi$	57.2958	1.7581	$\sqrt[3]{1:e}$	0.7165	$\bar{1}.8552$
$10800:\pi$	3437.7467	3.5363	$M=\log e$	0.4343	$\bar{1}.6378$
$648000:\pi$	206264.81	5.3144	$\frac{1}{M}=\ln 10$	2.3026	0.3622
$1:\pi$	0.3183	$\bar{1}.5029$	2!	2	
$1:2\pi$	0.1592	$\bar{1}.2018$	3!	6	
$1:3\pi$	0.1061	$\bar{1}.0257$	4!	24	
$1:4\pi$	0.0796	2.9008	5!	120	
π^2	9.8696	0.9943	6!	720	
$2\pi^2$	19.7392	1.2953	7!	5040	
$\sqrt{\pi}$	1.7725	0.2486	8!	40,320	
$\sqrt{2\pi}$	2.5066	0.3991	9!	362,880	
$\sqrt{\pi:2}$	1.2533	0.0981	10!	3,628,800	
$\sqrt{1:\pi}$	0.5642	$\bar{1}.7514$	11!	39,916,800	
$\sqrt{2:\pi}$	0.7979	$\bar{1}.9019$	12!	479,001,600	
$\sqrt{3:\pi}$	0.9772	$\bar{1}.9900$			
$\sqrt{4:\pi}$	1.1284	0.0525			
$\sqrt[3]{\pi}$	1.4646	0.1657			

2. Squares, Cubes, Square Roots, Cube Roots, Reciprocals, Circumferences, Areas of Circles, Natural Logarithms

(For three-digit numbers, use interpolation; see Sec. 64. This may result in a slight error in the last digit. On the natural logarithm, $\ln$, see Sec. 128)

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$3\sqrt{n}$	$3\sqrt{10n}$	$3\sqrt{100n}$	$\frac{1}{n}$	πn	$\frac{\pi n^2}{4}$	$\ln n$
1	1	1	1.000	3.162	1.000	2.154	4.642	1.000	3.14	0.785	0.00000
2	4	8	1.414	4.472	1.260	2.714	5.848	0.500	6.28	3.142	0.69315
3	9	27	1.732	5.477	1.442	3.107	6.694	0.333	9.42	7.069	1.09861
4	16	64	2.000	6.325	1.587	3.420	7.368	0.250	12.57	12.566	1.38629
5	25	125	2.236	7.071	1.710	3.684	7.937	0.200	15.71	19.635	1.60944
6	36	216	2.449	7.746	1.817	3.915	8.434	0.167	18.85	28.274	1.79176
7	49	343	2.646	8.367	1.913	4.121	8.879	0.143	21.99	38.484	1.94591
8	64	512	2.828	8.944	2.000	4.309	9.283	0.125	25.13	50.265	2.07944
9	81	729	3.000	9.487	2.080	4.481	9.655	0.111	28.27	63.617	2.19722
10	100	1000	3.162	10.000	2.154	4.642	10.000	0.100	31.42	78.540	2.30259
11	121	1331	3.317	10.488	2.224	4.791	10.323	0.091	34.56	95.033	2.39790
12	144	1728	3.464	10.954	2.289	4.932	10.627	0.083	37.70	113.097	2.48491
13	169	2197	3.606	11.402	2.351	5.066	10.914	0.077	40.84	132.73	2.56495
14	196	2744	3.742	11.832	2.410	5.192	11.187	0.071	43.98	153.94	2.63906
15	225	3375	3.873	12.247	2.466	5.313	11.447	0.067	47.12	176.72	2.70805
16	256	4096	4.000	12.649	2.520	5.429	11.696	0.062	50.27	201.06	2.77259
17	289	4913	4.123	13.038	2.571	5.540	11.935	0.059	53.41	226.98	2.83321
18	324	5832	4.243	13.416	2.621	5.646	12.164	0.056	56.55	254.47	2.89037
19	361	6859	4.359	13.784	2.668	5.749	12.386	0.053	59.69	283.53	2.94444
20	400	8000	4.472	14.142	2.714	5.848	12.599	0.050	62.83	314.16	2.99573

Continued

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$	πn	$\frac{\pi n^2}{4}$	$\ln n$
21	441	9261	4.583	14.491	2.759	5.944	12.806	0.048	65.97	346.36	3.04452
22	484	10,648	4.690	14.832	2.802	6.037	13.006	0.045	69.12	380.13	3.09104
23	529	12,167	4.796	15.166	2.844	6.127	13.200	0.043	72.26	415.48	3.13549
24	576	13,824	4.899	15.492	2.884	6.214	13.389	0.042	75.40	452.39	3.17805
25	625	15,625	5.000	15.811	2.924	6.300	13.572	0.040	78.54	490.87	3.21888
26	676	17,576	5.099	16.125	2.962	6.383	13.751	0.038	81.68	530.93	3.25810
27	729	19,683	5.196	16.432	3.000	6.463	13.925	0.037	84.82	572.55	3.29584
28	784	21,952	5.292	16.733	3.037	6.542	14.095	0.036	87.96	615.75	3.33220
29	841	24,389	5.385	17.029	3.072	6.619	14.260	0.034	91.11	660.52	3.36730
30	900	27,000	5.477	17.321	3.107	6.694	14.422	0.033	94.25	706.86	3.40120
31	961	29,791	5.568	17.607	3.141	6.768	14.581	0.032	97.39	754.77	3.43399
32	1024	32,768	5.657	17.889	3.175	6.840	14.736	0.031	100.53	804.25	3.46574
33	1089	35,937	5.745	18.166	3.208	6.910	14.888	0.030	103.67	855.30	3.49651
34	1156	39,304	5.831	18.439	3.240	6.980	15.037	0.029	106.81	907.92	3.52636
35	1225	42,875	5.916	18.708	3.271	7.047	15.183	0.029	109.96	962.1	3.55535
36	1296	46,656	6.000	18.974	3.302	7.114	15.326	0.028	113.10	1017.9	3.58352
37	1369	50,653	6.083	19.235	3.332	7.179	15.467	0.027	116.24	1075.2	3.61092
38	1444	54,872	6.164	19.494	3.362	7.243	15.605	0.026	119.4	1134.1	3.63759
39	1521	59,319	6.245	19.748	3.391	7.306	15.741	0.026	122.5	1194.6	3.66356
40	1600	64,000	6.325	20.000	3.420	7.368	15.874	0.025	125.7	1256.6	3.68888
41	1681	68,921	6.403	20.248	3.448	7.429	16.005	0.024	128.8	1320.2	3.71357
42	1764	74,088	6.481	20.494	3.476	7.489	16.134	0.024	131.9	1385.4	3.73767
43	1849	79,507	6.557	20.736	3.503	7.548	16.261	0.023	135.1	1452.2	3.76120
44	1936	85,184	6.633	20.976	3.530	7.606	16.386	0.023	138.2	1520.5	3.78419
45	2025	91,125	6.708	21.213	3.557	7.663	16.510	0.022	141.4	1590.4	3.80666

Continued

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$	πn	$\frac{\pi n^2}{4}$	$\ln n$
46	2116	97,336	6.782	21.448	3.583	7.719	16.631	0.022	144.5	1661.9	3.82864
47	2209	103,823	6.856	21.679	3.609	7.775	16.751	0.021	147.7	1734.9	3.85015
48	2304	110,592	6.928	21.909	3.634	7.830	16.869	0.021	150.8	1809.6	3.87120
49	2401	117,649	7.000	22.136	3.659	7.884	16.985	0.020	153.9	1885.7	3.89182
50	2500	125,000	7.071	22.361	3.684	7.937	17.100	0.020	157.1	1963.5	3.91202
51	2601	132,651	7.141	22.583	3.708	7.990	17.213	0.020	160.2	2042.8	3.93183
52	2704	140,608	7.211	22.804	3.733	8.041	17.325	0.019	163.4	2123.7	3.95124
53	2809	148,877	7.280	23.022	3.756	8.093	17.435	0.019	166.4	2206.2	3.97029
54	2916	157,464	7.348	23.238	3.780	8.143	17.544	0.018	169.6	2290.2	3.98898
55	3025	166,375	7.416	23.452	3.803	8.193	17.652	0.018	172.8	2375.8	4.00733
56	3136	175,616	7.483	23.664	3.826	8.243	17.758	0.018	175.9	2463.0	4.02535
57	3249	185,193	7.550	23.875	3.849	8.291	17.863	0.017	179.1	2551.8	4.04306
58	3364	195,112	7.616	24.083	3.871	8.340	17.967	0.017	182.2	2642.1	4.06044
59	3481	205,379	7.681	24.290	3.893	8.387	18.070	0.017	185.4	2734.0	4.07754
60	3600	216,000	7.746	24.495	3.915	8.434	18.171	0.017	188.5	2827.4	4.09434
61	3721	226,981	7.810	24.698	3.936	8.481	18.272	0.016	191.6	2922.5	4.11087
62	3844	238,328	7.874	24.900	3.958	8.527	18.371	0.016	194.8	3019.2	4.12713
63	3969	250,047	7.937	25.100	3.979	8.573	18.469	0.016	197.9	3117.2	4.14313
64	4096	262,144	8.000	25.298	4.000	8.618	18.566	0.016	201.1	3217.0	4.15888
65	4225	274,625	8.062	25.495	4.021	8.662	18.663	0.015	204.2	3318.3	4.17439
66	4356	287,496	8.124	25.690	4.041	8.707	18.758	0.015	207.3	3421.1	4.18965
67	4489	300,763	8.185	25.884	4.062	8.750	18.852	0.015	210.5	3525.6	4.20469
68	4624	314,432	8.246	26.077	4.082	8.794	18.945	0.015	213.6	3631.7	4.21951
69	4761	328,509	8.307	26.268	4.102	8.837	19.038	0.014	216.8	3739.3	4.23411
70	4900	343,000	8.367	26.458	4.121	8.879	19.129	0.014	219.9	3848.4	4.24850
71	5041	357,911	8.426	26.646	4.141	8.921	19.220	0.014	223.1	3959.2	4.26268
72	5184	373,248	8.485	26.833	4.160	8.963	19.310	0.014	226.2	4071.5	4.27667