COLLEGE MATHEMATICS

Through Applications

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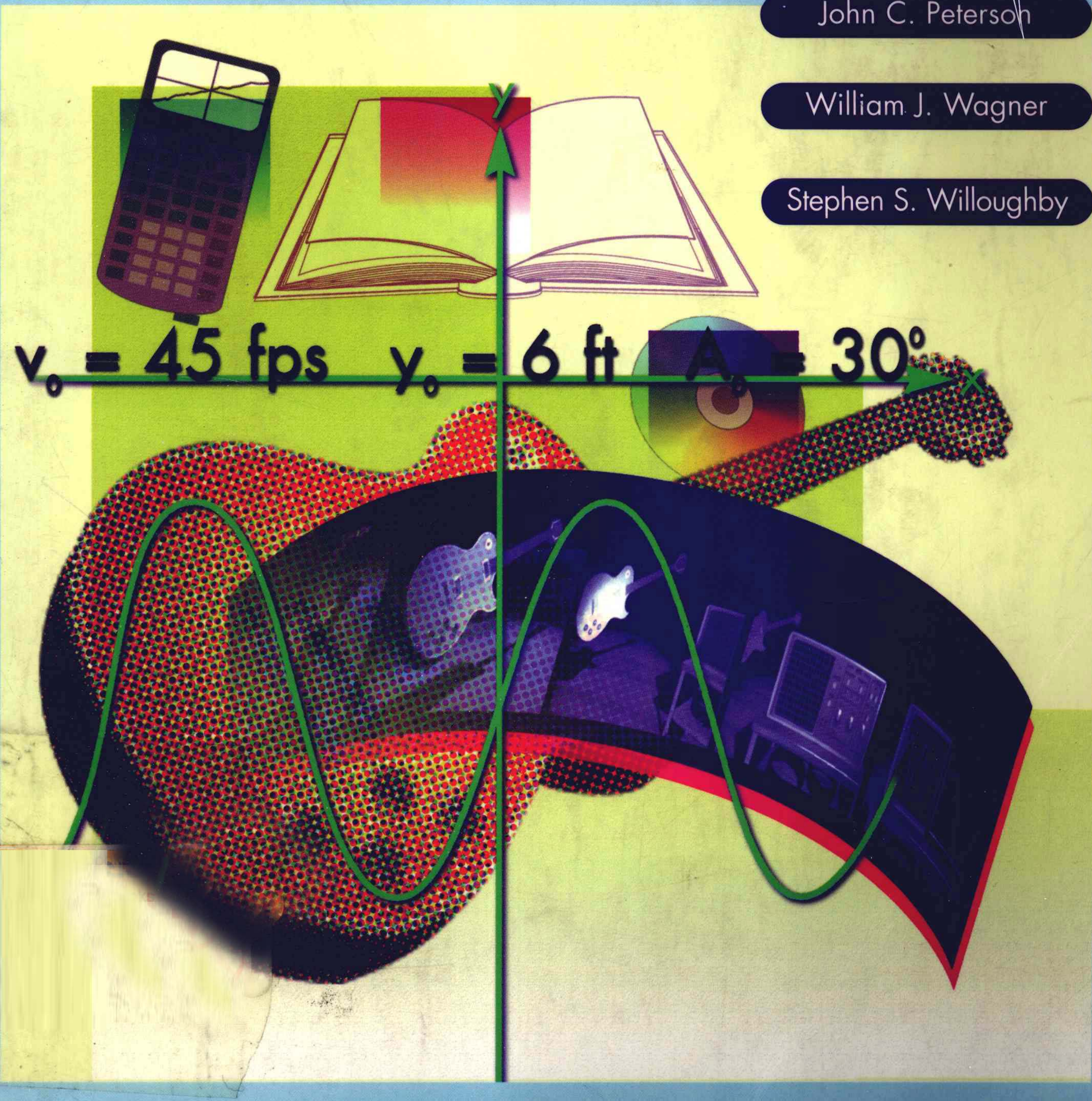

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PREFACE

Introduction

College Mathematics Through Applications was written to address the needs of today's technical mathematics and precalculus students. Inspired equally by the world of work and the current reform movement in mathematics education, we have looked hard at the traditional content of these courses and have chosen topics that are used in a variety of technical and scientific fields and that are intellectually rich. We believe that these students require a mathematics curriculum that focuses on the real environments in which they will apply their knowledge and the tools they will employ there, without degenerating into a set of rules and algorithms. Their mathematics education should be intellectually challenging and should lay a solid foundation for further learning and development.

The text covers advanced algebra, trigonometry, geometry, and intuitive calculus and explores these topics through applications. The presentation uses workplace-based applications as the cornerstone of the instruction and involves students in developing solutions and methods. The presentation and classroom activities have been designed to be accessible and interesting to all students.

The text is built on the following philosophical and pedagogical foundation:

- Learning in the context of real applications promotes retention and understanding.
- Mathematical content should reflect actual workplace needs.
- Students learn better by doing, writing, and discussing.
- Mathematical instruction should use the power of the technology to do traditional computations.
- Calculators should take over much of the machinery of calculations, allowing students to concentrate on a problem and focus on a concept.
- Content should be presented using the "rule of four": ideas are presented and students work in symbolic, graphic, and numeric methods and are then asked to express their ideas and answers in writing.
- Students who communicate their mathematical understanding through written and oral responses to well-designed, thought-provoking questions and problems will gain valuable workplace skills.

The Nature of the Classroom

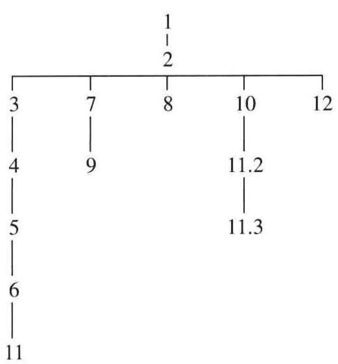
First and foremost, we expect that each student will have a graphing calculator, but just giving calculators to students or letting calculators creep into the classroom doesn't bring the benefits. Therefore, we have fully integrated the calculator into *College Mathematics Through Applications*. We believe that technology should not replace thinking, but it should help reduce mathematical error and provide additional mathematical resources.

The day-to-day work in the classroom will be different with this text, because it presents many projects, activities, and labs that can be completed individually or in groups. We believe that a mathematics text should present the material in a lively way in order to engage students in the study of the topic. By engaging students, our goal is to make them active learners and challenge them to

- Develop the ability to construct and evaluate mathematical models for real phenomena.
- Understand the limitations of tools, simulations, and mathematical methods.
- Develop intuition about the results that do and do not make sense.
- Not be held back by traditional prerequisites.
- Review prerequisite concepts in context.
- Use available technology to develop a deep understanding of concepts.

To aid teachers in implementing this approach in their classrooms, we have woven these common threads throughout the text:

- Applications and real data are used whenever possible.
- Equations and functions are used as models of phenomena.
- Technology provides alternative methods for approximating solutions.
- Technology is used in the classroom every day.
- Intuitive calculus is woven throughout.



Features of the Text

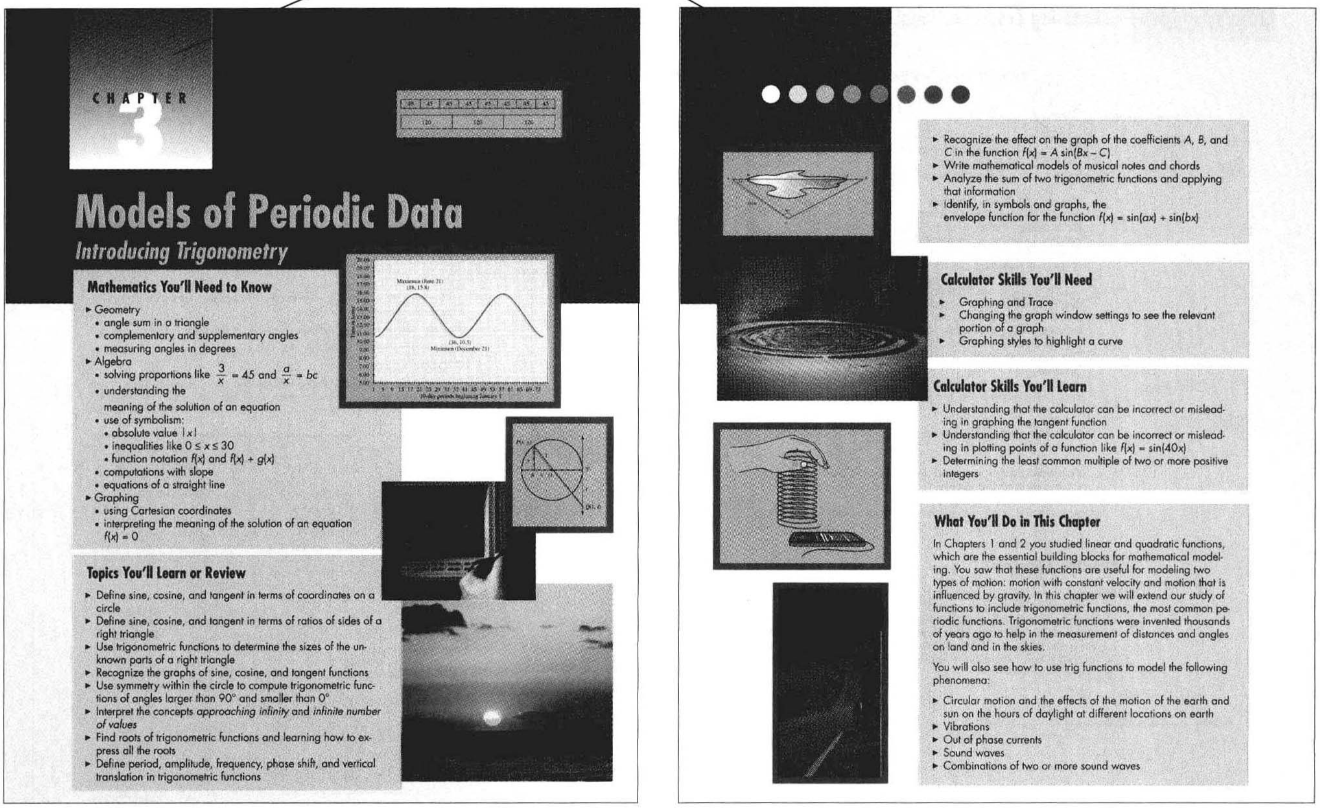
Order of Topics

A hierarchy diagram of the chapters in *College Mathematics Through Applications* is shown at left. The figure should be read from the top down. If you begin with Chapters 1 and 2, you then have the option of going to Chapter 3, 7, 8, 10, or 12. If you intend to cover Chapter 9, then you must first cover the material in Chapter 7.

Chapter Opener

The first two pages of each chapter provide a complete advance organizer for students. Here they'll find *Mathematics You'll Need to Know*, *Topics You'll Learn or Review*, *Calculator Skills You Need*, and *Calculator Skills You'll Learn*. Finally, the chapter opener ends with a short description in plain English of *What You'll Do in this Chapter*.

The Chapter Opener acts as an advanced organizer, so students know what's expected of them



CHAPTER 3

Models of Periodic Data

Introducing Trigonometry

Mathematics You'll Need to Know

- Geometry
  - angle sum in a triangle
  - complementary and supplementary angles
  - measuring angles in degrees
- Algebra
  - solving proportions like $\frac{3}{x} = 45$ and $\frac{a}{x} = bc$
  - understanding the meaning of the solution of an equation
  - use of symbolism:
    - absolute value $|x|$
    - inequalities like $0 \le x \le 30$
    - function notation $f(x)$ and $f(x) + g(x)$
  - computations with slope
  - equations of a straight line
- Graphing
  - using Cartesian coordinates
  - interpreting the meaning of the solution of an equation $f(x) = 0$

Topics You'll Learn or Review

- Define sine, cosine, and tangent in terms of coordinates on a circle
- Define sine, cosine, and tangent in terms of ratios of sides of a right triangle
- Use trigonometric functions to determine the sizes of the unknown parts of a right triangle
- Recognize the graphs of sine, cosine, and tangent functions
- Use symmetry within the circle to compute trigonometric functions of angles larger than 90° and smaller than 0°
- Interpret the concepts *approaching infinity* and *infinite number of values*
- Find roots of trigonometric functions and learning how to express all the roots
- Define period, amplitude, frequency, phase shift, and vertical translation in trigonometric functions
- Recognize the effect on the graph of the coefficients *A*, *B*, and *C* in the function $f(x) = A\sin(Bx - C)$
- Write mathematical models of musical notes and chords
- Analyze the sum of two trigonometric functions and applying that information
- Identify, in symbols and graphs, the envelope function for the function $f(x) = \sin(ax) + \sin(bx)$

Calculator Skills You'll Need

- Graphing and Trace
- Changing the graph window settings to see the relevant portion of a graph
- Graphing styles to highlight a curve

Calculator Skills You'll Learn

- Understanding that the calculator can be incorrect or misleading in graphing the tangent function
- Understanding that the calculator can be incorrect or misleading in plotting points of a function like $f(x) = \sin(40x)$
- Determining the least common multiple of two or more positive integers

What You'll Do in This Chapter

In Chapters 1 and 2 you studied linear and quadratic functions, which are the essential building blocks for mathematical modeling. You saw that these functions are useful for modeling two types of motion: motion with constant velocity and motion that is influenced by gravity. In this chapter we will extend our study of functions to include trigonometric functions, the most common periodic functions. Trigonometric functions were invented thousands of years ago to help in the measurement of distances and angles on land and in the skies.

You will also see how to use trig functions to model the following phenomena:

- Circular motion and the effects of the motion of the earth and sun on the hours of daylight at different locations on earth
- Vibrations
- Out of phase currents
- Sound waves
- Combinations of two or more sound waves

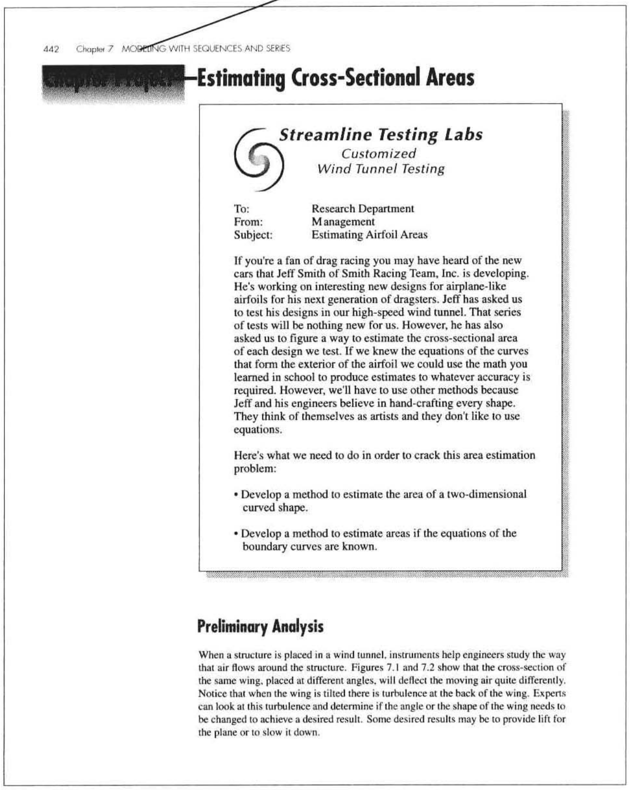
Chapter Project

Each chapter begins with a Project, and the goal of the chapter is to learn the mathematics necessary to solve this project. The world of work doesn't present problems in a neat, organized way, so to better prepare students for the workplace these projects are designed to force students to organize their thoughts and decide what the problem is asking them to do. Good problem solvers get information and skills as they need them, so at several points in the chapter students are asked to relate the mathematics they have learned to the solution of the project. By the end of the chapter they will have learned how to complete the entire project.

Activities and Calculator Labs

Frequent Activities and Calculator Labs are designed to get students involved in performing experiments, collecting and analyzing data, and forming conclusions—all skills they will need in their future careers.

Chapter Projects give students experience with interesting real-world problems. These projects are featured in the Presentation Software



442 Chapter 7 MODELING WITH SEQUENCES AND SERIES

Chapter Project—Estimating Cross-Sectional Areas

Streamline Testing Labs
Customized Wind Tunnel Testing

To: Research Department
From: Management
Subject: Estimating Airfoil Areas

If you're a fan of drag racing you may have heard of the new cars that Jeff Smith of Smith Racing Team, Inc. is developing. He's working on interesting new designs for airplane-like airfoils for his next generation of dragsters. Jeff has asked us to test his designs in our high-speed wind tunnel. That series of tests will be nothing new for us. However, he has also asked us to figure a way to estimate the cross-sectional area of each design we test. If we knew the equations of the curves that form the exterior of the airfoil we could use the math you learned in school to produce estimates to whatever accuracy is required. However, we'll have to use other methods because Jeff and his engineers believe in hand-crafting every shape. They think of themselves as artists and they don't like to use equations.

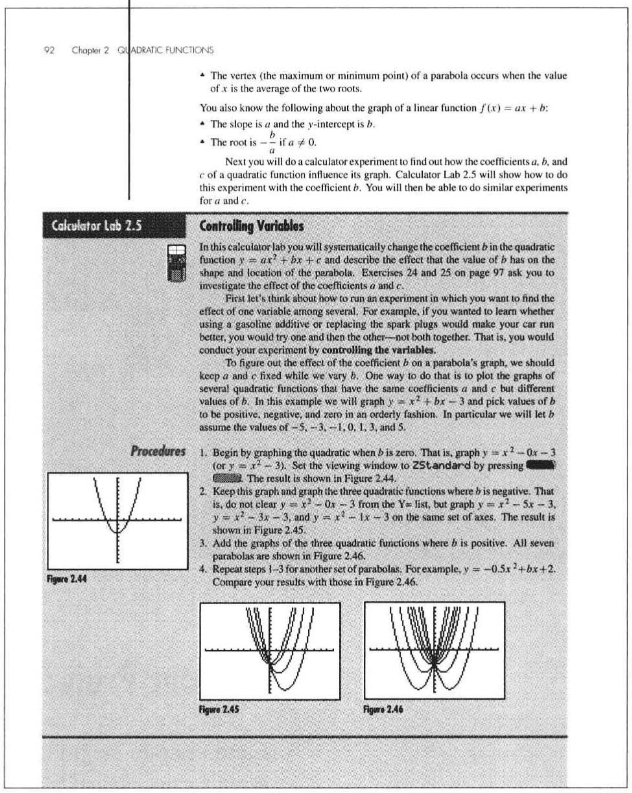
Here's what we need to do in order to crack this area estimation problem:

- Develop a method to estimate the area of a two-dimensional curved shape.
- Develop a method to estimate areas if the equations of the boundary curves are known.

Preliminary Analysis

When a structure is placed in a wind tunnel, instruments help engineers study the way that air flows around the structure. Figures 7.1 and 7.2 show that the cross-section of the same wing, placed at different angles, will deflect the moving air quite differently. Notice that when the wing is tilted there is turbulence at the back of the wing. Experts can look at this turbulence and determine if the angle or the shape of the wing needs to be changed to achieve a desired result. Some desired results may be to provide lift for the plane or to slow it down.

Calculator Labs show students how to use technology to solve problems. These labs can be completed individually or in groups.



92 Chapter 2 QUADRATIC FUNCTIONS

- The vertex (the maximum or minimum point) of a parabola occurs when the value of x is the average of the two roots.

You also know the following about the graph of a linear function $f(x) = ax + b$:

- The slope is a and the y-intercept is b.
- The root is $-\frac{b}{a}$ if $a \neq 0$.

Next you will do a calculator experiment to find out how the coefficients a, b, and c of a quadratic function influence its graph. Calculator Lab 2.5 will show how to do this experiment with the coefficient b. You will then be able to do similar experiments for a and c.

Calculator Lab 2.5 Controlling Variables

In this calculator lab you will systematically change the coefficient b in the quadratic function $y = ax^2 + bx + c$ and describe the effect that the value of b has on the shape and location of the parabola. Exercises 24 and 25 on page 97 ask you to investigate the effect of the coefficients a and c.

First let's think about how to run an experiment in which you want to find the effect of one variable among several. For example, if you wanted to learn whether using a gasoline additive or replacing the spark plugs would make your car run better, you would try one and then the other—not both together. That is, you would conduct your experiment by **controlling the variables.**

To figure out the effect of the coefficient b on a parabola's graph, we should keep a and c fixed while we vary b. One way to do that is to plot the graphs of several quadratic functions that have the same coefficients a and c but different values of b. In this example we will graph $y = x^2 + bx - 3$ and pick values of b to be positive, negative, and zero in an orderly fashion. In particular we will let b assume the values of $-5, -3, -1, 0, 1, 3$, and 5.

Procedures

1. Begin by graphing the quadratic when b is zero. That is, graph $y = x^2 - 0x - 3$ (or $y = x^2 - 3$). Set the viewing window to ZStandard by pressing [ ] [ ]. The result is shown in Figure 2.44.
2. Keep this graph and graph the three quadratic functions where b is negative. That is, do not clear $y = x^2 - 0x - 3$ from the Y= list, but graph $y = x^2 - 5x - 3$, $y = x^2 - 3x - 3$, and $y = x^2 - 1x - 3$ on the same set of axes. The result is shown in Figure 2.45.
3. Add the graphs of the three quadratic functions where b is positive. All seven parabolas are shown in Figure 2.46.
4. Repeat steps 1–3 for another set of parabolas. For example, $y = -0.5x^2 + bx + 2$. Compare your results with those in Figure 2.46.

Figure 2.44

Figure 2.45

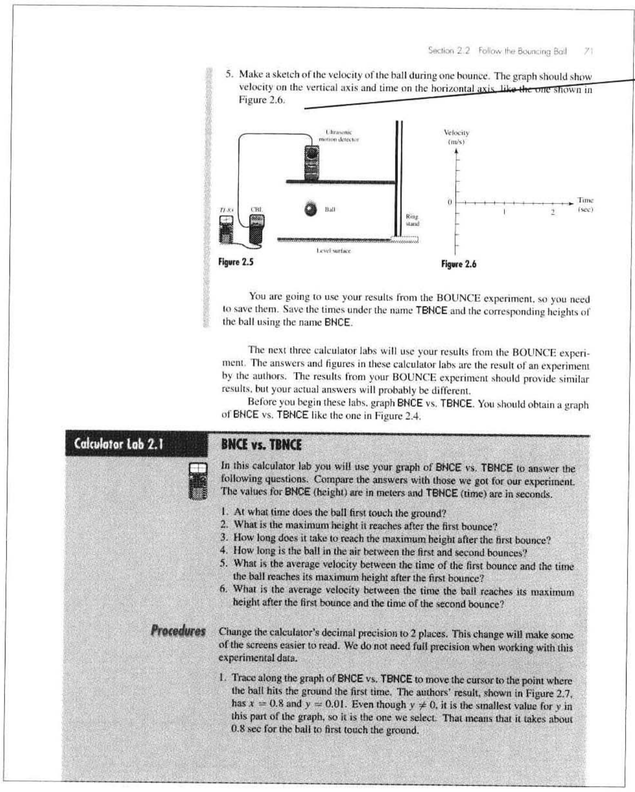
Figure 2.46

Technology

Technology—graphing calculators, the Calculator-Based Laboratory (CBL) System™, and presentation software—is integrated throughout the text to allow students to explore more advanced and interesting concepts.

Additional Features

- Important ideas, concepts, and definitions are prominently displayed, so students can find and read them easily.



Section 2.2 Follow the Bouncing Ball 71

5. Make a sketch of the velocity of the ball during one bounce. The graph should show velocity on the vertical axis and time on the horizontal axis, like the one shown in Figure 2.6.

Figure 2.5 Figure 2.6

You are going to use your results from the BOUNCE experiment, so you need to save them. Save the times under the name TBNCE and the corresponding heights of the ball using the name BNCE.

The next three calculator labs will use your results from the BOUNCE experiment. The answers and figures in these calculator labs are the result of an experiment by the authors. The results from your BOUNCE experiment should provide similar results, but your actual answers will probably be different.

Before you begin these labs, graph BNCE vs. TBNCE. You should obtain a graph of BNCE vs. TBNCE like the one in Figure 2.4.

Calculator Lab 2.1 **BNCE vs. TBNCE**

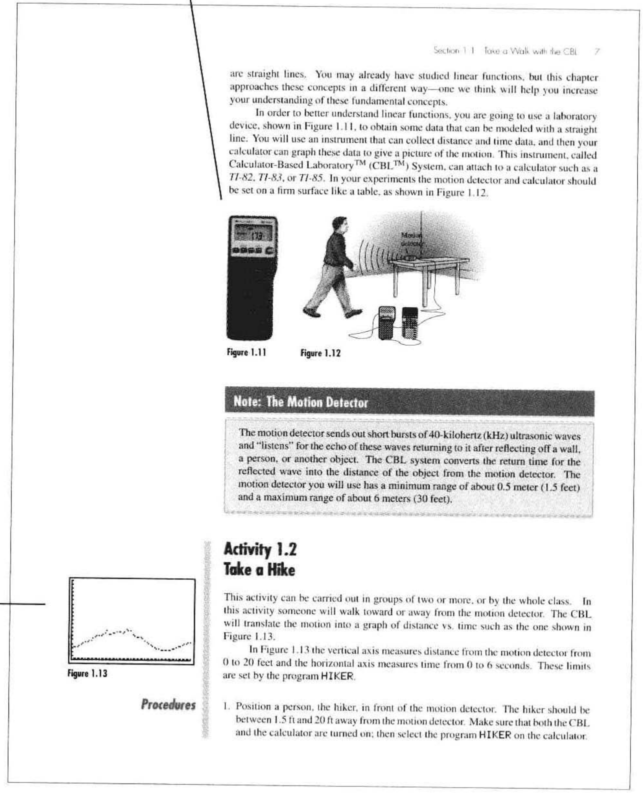
In this calculator lab you will use your graph of BNCE vs. TBNCE to answer the following questions. Compare the answers with those we got for our experiment. The values for BNCE (height) are in meters and TBNCE (time) are in seconds.

1. At what time does the ball first touch the ground?
2. What is the maximum height it reaches after the first bounce?
3. How long does it take to reach the maximum height after the first bounce?
4. How long is the ball in the air between the first and second bounces?
5. What is the average velocity between the time of the first bounce and the time the ball reaches its maximum height after the first bounce?
6. What is the average velocity between the time the ball reaches its maximum height after the first bounce and the time of the second bounce?

Procedures Change the calculator's decimal precision to 2 places. This change will make some of the screens easier to read. We do not need full precision when working with this experimental data.

1. Trace along the graph of BNCE vs. TBNCE to move the cursor to the point where the ball hits the ground the first time. The authors' result, shown in Figure 2.7, has $x = 0.8$ and $y = 0.01$. Even though $y \neq 0$, it is the smallest value for y in this part of the graph, so it is the one we select. That means that it takes about 0.8 sec for the ball to first touch the ground.

The Calculator-Based Laboratory System is used extensively to gather real data for students to analyze.



Section 1.1 Take a Walk with the CBL 7

are straight lines. You may already have studied linear functions, but this chapter approaches these concepts in a different way—one we think will help you increase your understanding of these fundamental concepts.

In order to better understand linear functions, you are going to use a laboratory device, shown in Figure 1.11, to obtain some data that can be modeled with a straight line. You will use an instrument that can collect distance and time data, and then your calculator can graph these data to give a picture of the motion. This instrument, called Calculator-Based Laboratory™ (CBL™) System, can attach to a calculator such as a *TI-82*, *TI-83*, or *TI-85*. In your experiments the motion detector and calculator should be set on a firm surface like a table, as shown in Figure 1.12.

Figure 1.11 Figure 1.12

Note: The Motion Detector

The motion detector sends out short bursts of 40-kilohertz (kHz) ultrasonic waves and "listens" for the echo of these waves returning to it after reflecting off a wall, a person, or another object. The CBL system converts the return time for the reflected wave into the distance of the object from the motion detector. The motion detector you will use has a minimum range of about 0.5 meter (1.5 feet) and a maximum range of about 6 meters (30 feet).

Activity 1.2
Take a Hike

Figure 1.13

This activity can be carried out in groups of two or more, or by the whole class. In this activity someone will walk toward or away from the motion detector. The CBL will translate the motion into a graph of distance vs. time such as the one shown in Figure 1.13.

In Figure 1.13 the vertical axis measures distance from the motion detector from 0 to 20 feet and the horizontal axis measures time from 0 to 6 seconds. These limits are set by the program HIKER.

Procedures

1. Position a person, the hiker, in front of the motion detector. The hiker should be between 1.5 ft and 20 ft away from the motion detector. Make sure that both the CBL and the calculator are turned on; then select the program HIKER on the calculator.

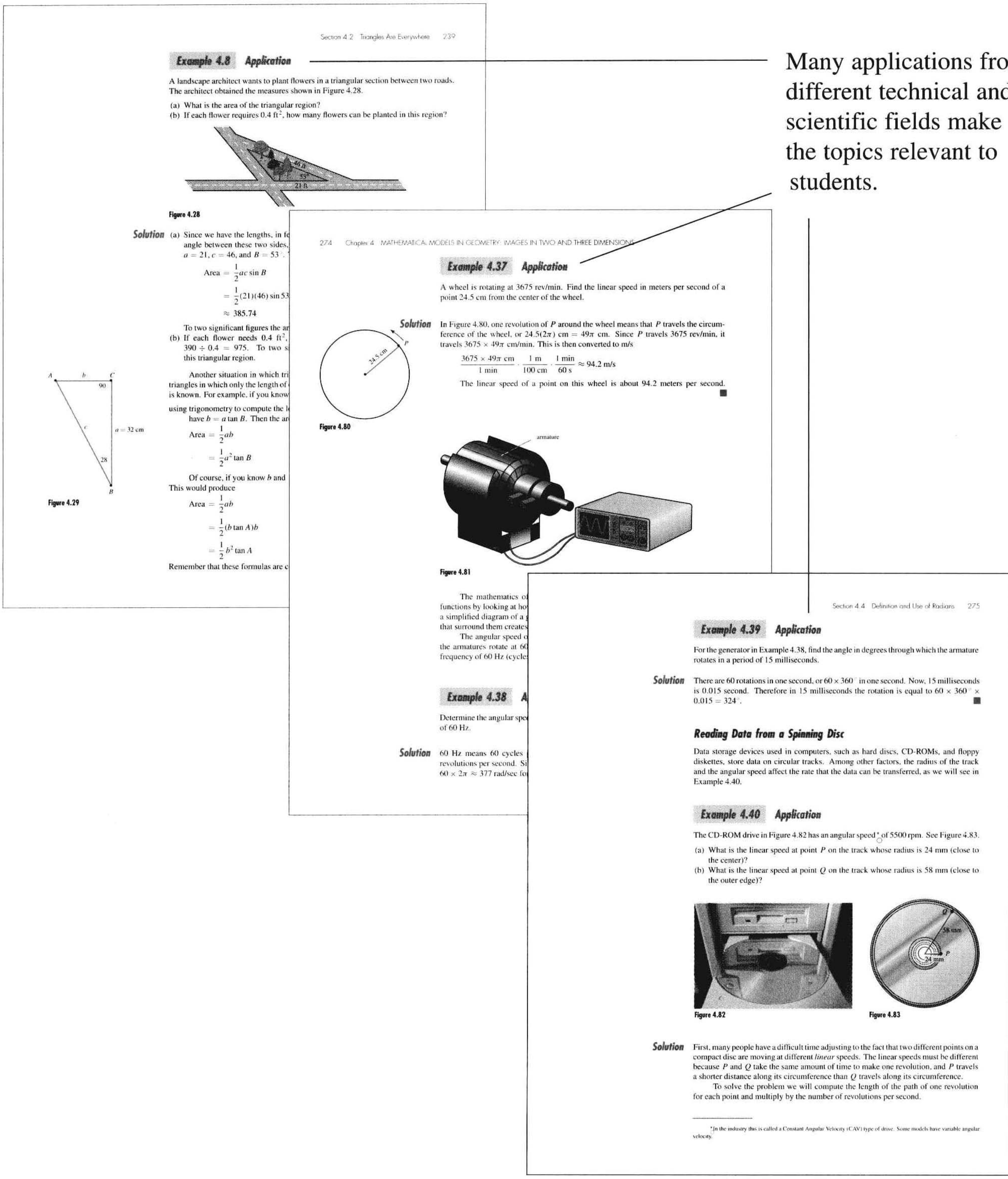
Frequent activites help students become active learners.

- Numerous examples and exercises show how the mathematics relates to different technical fields.
- A thorough chapter summary, extensive review exercises, and a model chapter test conclude every chapter.

Supplements

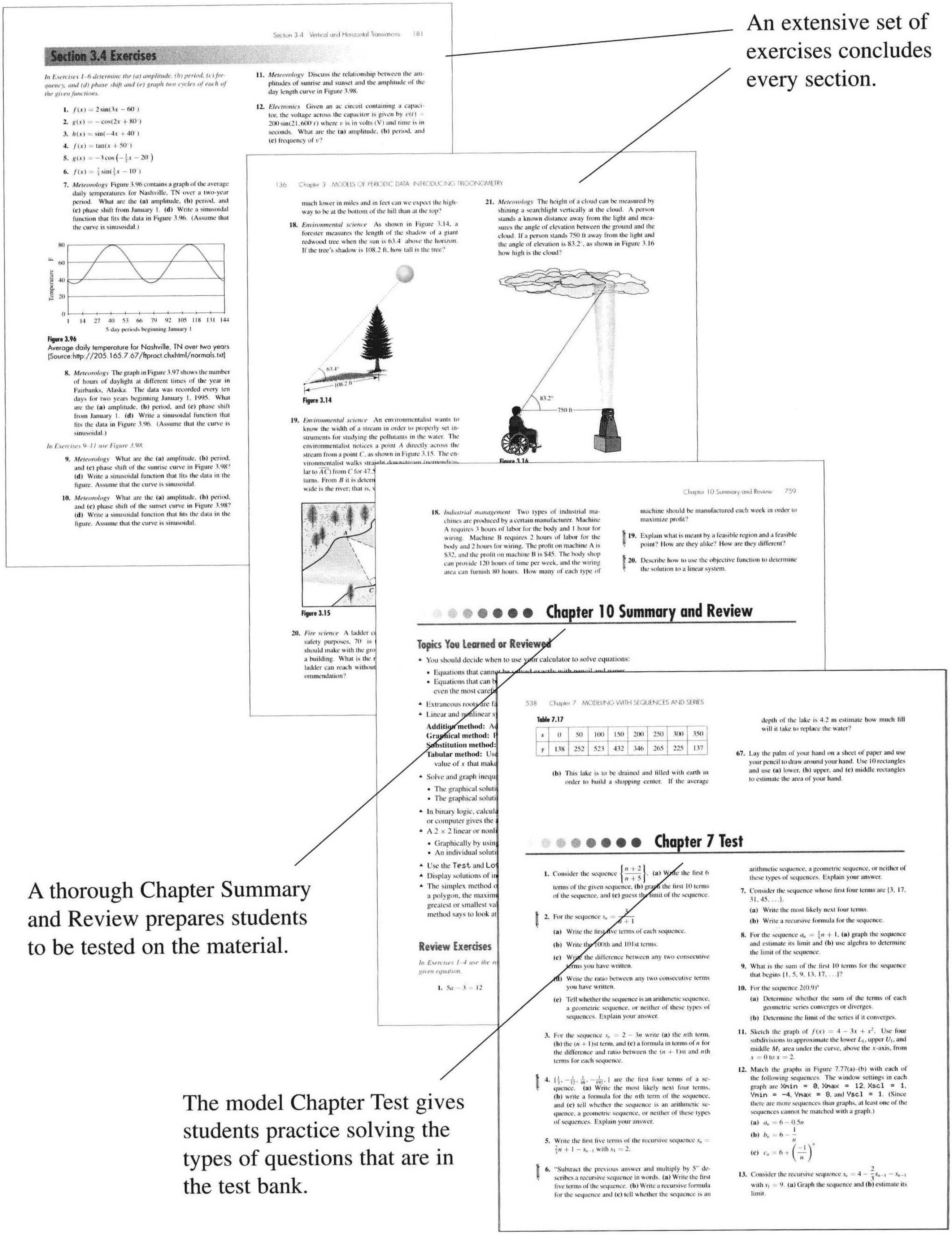
Presentation Software

The interactive Presentation Software is designed primarily as a classroom demonstration tool, although you will find that students who use the software individually will also benefit. In each of the 12 chapters a brief video shows the background and rationale of the mathematics in the chapter. Each chapter also includes a couple of simulations. You will find that any of the 25 simulations can form the basis for an



Many applications from different technical and scientific fields make the topics relevant to students.

An extensive set of exercises concludes every section.



A thorough Chapter Summary and Review prepares students to be tested on the material.

The model Chapter Test gives students practice solving the types of questions that are in the test bank.

interactive lecture. As you ask the students to provide input and to guess what will happen on the computer screen, they will become more actively involved in the subject matter. The "instructions" button in each screen tells how each simulation works and suggests some classroom activities and questions. The software runs in an easy to use web browser environment and comes with a free copy of Netscape® Navigator 4.0.

Computerized Test Bank

The Computerized Test Bank contains 500 questions based on the text and is in ITP's World-Class Test shell, which has the following features:

- Supports Essay, Fill in the Blank, Matching, Multiple Choice and True/False
- New question creation (World Class Test supplies you with a template for each question type)
- Custom question bank creation capability
- Ability to upload internet-based activities to testing server
- Page layout features (formatting)
- Multiple outputs (to Web, LAN or Printing)
- Graphic support
- Mathematical expression support
- Random questioning output
- Algorithm support
- Special character support

Solutions Manual

The Solutions Manual contains complete, worked out solutions to all the problems in the text, including Section Exercises, Chapter Review Exercises, Chapter Tests, and Calculator Labs.

Instructor's Manual

The Instructor's Manual is a comprehensive tool for giving you the information and tips you need to get the most for your students out of *College Mathematics Through Applications*. It gives you tips on utilizing each component of the text and supplements, including detailed chapter outlines and sample lesson plans for each chapter. Be sure to visit our web site for additional resources (www.cmta.delmar.com).

About the Authors

John C. Peterson is an Associate Professor of Mathematics at Chattanooga State Technical Community College (CSTCC). Dr. Peterson has been a recipient of CSTCC's Teaching Excellence Award. He received his BA and MA degrees from the University of Iowa and his Ph.D. degree from The Ohio State University.

Peterson has had articles published in journals such as the *Arithmetic Teacher*, *Mathematics Teacher*, *Journal for Research in Mathematics Education*, *School Science and Mathematics*, *Mathematics and Computer Education*, *The AMATYC Review*, and

Today's Education. Currently he is the Southeast Vice President of the American Mathematical Association of Two-Year Colleges (AMATYC). He has also authored *Technical Mathematics, Second Edition*, *Technical Mathematics with Calculus, Second Edition*, *Technical Calculus with Analytic Geometry*, and *Math for the Automotive Trade*.

William J. (Sandy) Wagner taught mathematics, science, computer science, and computer education for over 20 years from 7th grade through college. His other careers have included software manager at IBM and Computer Curriculum Corporation, and computer education coordinator for the 300 schools of Santa Clara County, California. He was early personal computer user in his classroom and in 1978 he founded Computer-Using Educators (CUE), a professional organization for California teachers. Recently he was happy to return to mathematics, his first true love.

Stephen S. Willoughby is a Professor of Mathematics at the University of Arizona. He previously taught at the University of Wisconsin and at New York University. He received bachelor's and master's degrees from Harvard and a doctorate from Columbia. Dr. Willoughby was President of the National Council of Teachers of Mathematics from 1982 through 1984 and was Chairman of the Council of Scientific Society Presidents in 1988. He chaired the United States National Commission on Mathematics Instruction from 1991 through 1994, was a member of the advisor board for SQUARE ONE TV, and has served on numerous other national commissions, boards, and committees.

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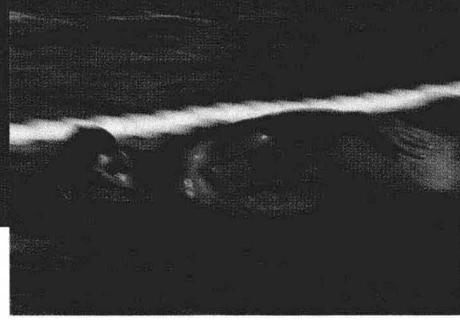
CHAPTER 1

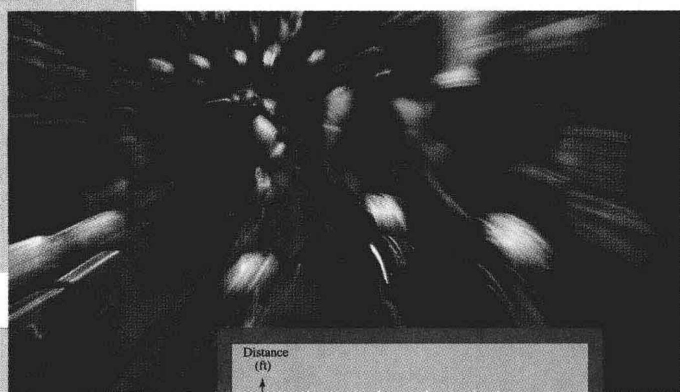
Linear Equations

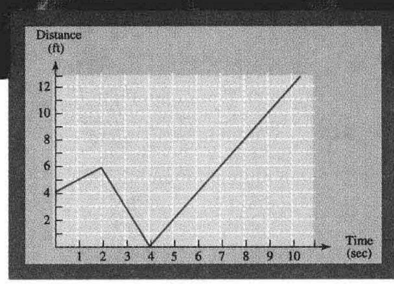
Making Mathematical Models of Data



Mathematics You'll Need to Know

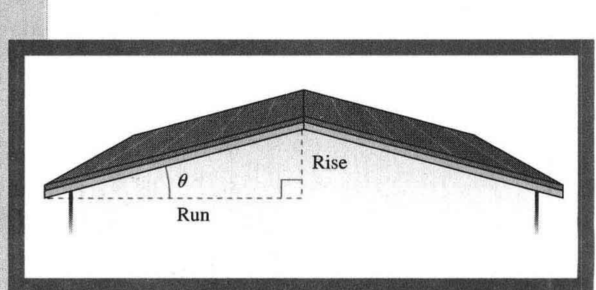
- Doing calculations involving formulas, including decimals and negative numbers
- Calculating average speed, given distance and time traveled
- Graphing on a Cartesian coordinate system
- Plotting points, reading graphs, and understanding the following terminology:
  - x-axis, y-axis
  - x-coordinate, y-coordinate
  - origin
  - y-intercept and x-intercept of a graph
- Making a graph from a table of values for x and y
- Calculating the slope of a line
- Calculating with positive and negative numbers





Topics You'll Learn or Review

- Describe how the slope of a line is related to the graph of the line and the equation of the line
- Describe the trigonometric connection between the slope of a line and the angle the line makes with the x-axis
- Use the connection between a constant velocity experiment and the slope and y-intercept of the distance vs. time graph
- Write the equation of a straight line given any of the following:
  - the slope and the y-intercept
  - one point and the slope
  - two points
  - one point and the inclination angle of the line
- Use a mathematical model to study real events and relationships; that is,
  - using a linear model of data to estimate missing values in the data
  - analyzing how well a linear model fits the data



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