LABORATORY MATHEMATICS

Medical and Biological Applications

June Mundy Campbell, M.Ed., M.T. (ASCP)

Joe Bill Campbell, Ph.D

LABORATORY MATHEMATICS
Medical and Biological Applications

June Mundy Campbell, *M.Ed., M.T. (ASCP)*
Formerly Chief Technologist,
Redman Park Hospital,
Rome, Georgia

Joe Bill Campbell, *Ph.D.*
Chemistry Professor,
Darlington School,
Rome, Georgia

Third Edition

With 52 illustrations

The C. V. Mosby Company
ST. LOUIS TORONTO 1984

MOSBY

A TRADITION OF PUBLISHING EXCELLENCE

Editor: **Don E. Ladig**
Manuscript editor: **Kathy Brown, Suzanne Harrawood**
Design: **Suzanne Oberholtzer**
Production: **Judith Bamert, Judith England, Mary Stueck**

Third edition

Printed in the United States of America

The C.V. Mosby Company
11830 Westline Industrial Drive, St. Louis, Missouri 63146

Library of Congress Cataloging in Publication Data

Campbell, June Blankenship
Laboratory mathematics.

Bibliography: p.
Includes index.
1. Medical technology—Mathematics. I. Campbell, Joe Bill. II. Title. [DNLM: 1. Diagnosis, Laboratory. 2. Mathematics. 3. Technology, Medical. QT 35 C188L]
RB38.2.C34. 1984 616.07'5'0151 83-8203
ISBN 0-8016-0800-7

GW/VH/VH 9 8 7 6 5 4 3 2 1 02/D/242

PREFACE

Mathematics is a science, a set of concepts, a way of thinking, a tool, and a mystery. It is all of these things to everyone. However, the importance of each property varies with the individual. To the people who work in laboratories, the two most significant are its function as a tool and its tendency to remain a mystery.

Laboratory Mathematics provides simplified explanations of the calculations commonly used in the clinical and general biological laboratories. Our rationale for this book has been to help the student who has an inadequate background in mathematics develop the essential skills necessary to meet the standards for clinical performance. It also will be useful to help refresh the laboratory worker's knowledge in specific areas.

We have attempted to present the explanations in such a manner that the reader can understand each step of the calculation. Also a brief explanation of the chemical and physical principles involved with the calculation is given. The explanations presented here are meant to show how to do the problems with enough theory to continue study. This text is not considered to be a complete book on the theory of mathematics; it is a beginning and not an end to the realm of laboratory mathematics.

In this third edition of **Laboratory Mathematics** the basic content of the book has not been changed. However, it has been expanded, reorganized, and updated, which will make the book more useful to the reader. The chapter on systems of measure contains the most current information on the SI measurement standards; new tables in this chapter provide conversions to nonmetric systems of measurement. Comprehensive new information on graphing has been added. An updated and expanded section on gastric acidity determination has been provided. Some other additions to this edition are: the inclusion of over a thousand new practice problems, a section on roman numerals, and new tables in the appendix. The sequence of the chapters has been rearranged to allow for more logical progression of information.

Many of the changes in this edition were made in response to suggestions from our readers. We continue to encourage suggestions and criticisms and always appreciate knowing ways to improve the usefulness of this book.

June Mundy Campbell
Joe Bill Campbell

CONTENTS

APPENDIXES

1 BASIC MATHEMATICS

GENERAL CONCEPTS

Students of laboratory technology have a varying degree of expertise in mathematics. This chapter considers some basic concepts of mathematics that anyone completing 12 years of public education should know. However, we realize that in many schools the goal is not that of disseminating useful knowledge but rather of sheltering incompetent personnel and frustrating competent teachers. This book cannot cover every aspect of mathematics and still remain within its designed scope. This chapter provides a review of the major concepts of mathematics needed for the technical aspects of most medical and biological laboratories. If the student does not understand these concepts after studying the review in this chapter, further sources of study should be sought to learn enough about the science and application of mathematics to use this book.

In so far as is possible, the student should not only understand how to do mathematical calculations but should also understand why the manipulations of the figures work as they do. When a formula is presented as a method of solution to a problem, the student should attempt to understand the principle on which the formula is based. Understanding the basis of a formula often allows one to modify the formula to better suit a particular situation.

In doing mathematical calculations there are several general considerations to make in order to determine the most efficient method of solving the problem and to reduce error. Some of these are as follows:

1. Read the problem carefully. Be sure the entire problem has been read and understood.
2. Determine what principles and relationships are involved.
3. Determine exactly what results are to be produced by the calculations.
4. Think about the possible methods to use in solving the problem.
5. Write the intermediate stages of the calculations clearly. Avoid writing one number on top of another as a method of correction. Make each digit legible.
6. Recognize different forms of the same value, such as ½, 1/2, 1 ÷ 2, 0.5, 50%, and 0.50.
7. Be extremely careful in positioning the decimal point.
8. Mentally estimate an answer before working the problem; compare the calculated result with the estimated answer. If the two figures disagree drastically, determine which is wrong.

NUMBER SYSTEMS

Two major number systems are used today. These are referred to as arabic numerals and roman numerals. The most useful are the arabic numerals.

Arabic numerals

The arabic system is the most widely used system of expressing values and in calculations. This is a base-10 system having 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. These digits express values when used alone or in combination. The system is positional in nature; the value of any numeral is affected by the position it occupies in the number. Each digit in a number has a place value beginning from a point called a decimal. Numbers to the left of the decimal point indicate values greater than 1. Digits to the right of the decimal point indicate values less than 1 but greater than zero. Every place is 10 times the next closest place to the right. In a number such as 245.679, 5 is in the unit place, expressing 5 × 1; 4 is in the tens place, expressing 4 × 10; and the 2 is in the hundreds place, expressing 2 × 100. Successive digits occupy a place value 10 times the preceding place value. Digits to the right of the decimal point follow the same pattern except that these indicate values less than 1. In the above number 6 is in the tenths place. This expresses a value of 6 × 1/10; 7 expresses 7 × 1/100, and 9 expresses 9 × 1/1000 (p. 11).

This system allows for relatively simple manipulation of numbers for the four basic computational processes: addition, subtraction, multiplication, and division. These processes will be discussed in detail in this chapter.

Roman numerals

The roman numeral system uses letters to represent numbers. It does not have a constant base system and hence makes calculations difficult or impossible. This system is used only to identify values. Roman numerals are used sporadically in writing. For example, in medicine roman numerals are used to indicate dosages of drugs that are written in apothecary units. Hence one needs to understand the current system of roman numerals even though the arabic numerals are much more common and useful.

Roman numerals are expressed as seven capital or lower case letters.

Roman numerals	Arabic equivalent
I, i	1
V, v	5
X, x	10
L, l	50
C, c	100
D, d	500
M, m	1000

A capital letter with a bar over it indicates 1000 times the simple letter.

$\overline{V}$	5,000
$\overline{C}$	100,000
$\overline{D}$	500,000
$\overline{M}$	1,000,000

These letters are combined in specific ways to indicate the magnitude of a value. There are some general rules for the combining of roman numerals.

1. When a numeral of lesser value follows one of greater value, add the two values.

EXAMPLE:

VI = 5 + 1 = 6
LX = 50 + 10 = 60
MDC = 1000 + 500 + 100 = 1600

2. When numerals of the same value are repeated in sequence, add the values.

EXAMPLE:

XX = 10 + 10 = 20
iii = 1 + 1 + 1 = 3

3. When a numeral of lesser value precedes one of greater value, the value of the first is subtracted from the value of the second. The numerals V, L, and D are never used as a subtracted number.

EXAMPLE:

IV = 5 − 1 = 4
XL = 50 − 10 = 40
XLV = 50 − 10 + 5 = 45 (not VL)

4. When a number of lesser value is placed between two numerals of greater value, the numeral of lesser value is subtracted from the numeral following it.

EXAMPLE:

XIV = 10 + 5 − 1 = 14
XXIX = 10 + 10 + 10 − 1 = 29

5. Numerals are never repeated more than three times in sequence.

EXAMPLE:

XXX = 30
XL = 40 (not XXXX)

6. Arrange the letters in order of decreasing value from left to right except for letters having values to be subtracted from the next letter.

EXAMPLE:

MCM = 1900
MCMXLIV = 1944

ARITHMETIC

The fundamental area of mathematics is arithmetic. This is the manipulation of real numbers by use of the four basic operations of mathematics: addition, subtraction, multiplication, and division. Nearly everyone can carry out these operations with simple problems. However, most people have some difficulty with specific parts of arithmetic. This section provides a general review of the major concepts of this area of mathematics.

Terminology of the basic operations

The four basic operations of arithmetic can be described using the following terminology.

Addition. Addition is the combination of numbers to obtain an equivalent single quantity. A number added to others is called an *addend*. When two or more addends are combined the result is called a *sum*.

Subtraction. Subtraction is the operation of deducting one number from another. The number from which another is deducted is called the *minuend*. The number that is deducted from the minuend is called the *subtrahend*. The result of a subtraction is called the *difference*.

Multiplication. Multiplication is the adding of a number to itself a specified number of times. The *multiplicand* is the number that is multiplied. The *multiplier* is the number of times the multiplicand is added to itself. The result of a multiplication operation is called the *product*.

Division. Division is the operation of finding how many times one number or quantity is contained in another. The *dividend* is the number that is to be divided. The *divisor* is the number that is divided into the dividend. The result of a division operation is called the *quotient*.

Use of + and − signs

The + and − signs have two uses in mathematics: (1) to indicate the addition and subtraction operations and (2) to determine the direction of progression of a number from zero. The first use is the generally known use. The second use is frequently not well understood. The following is an explanation of this use.

Numbers are used to denote a progression of values from a beginning point. The beginning point is zero. The progression of values moves away from zero in two directions.

Infinity −7 −6 −5 −4 −3 −2 −1 0 +1 +2 +3 +4 +5 +6 +7 Infinity

Customarily numbers to the right of the zero point are spoken of as *positive numbers*, whereas numbers to the left of zero are spoken of as *negative numbers*.

1. Addition of positive and negative numbers.
   a. Addition of positive numbers: add all numbers and give the sum a positive sign.

$$\begin{array}{r} +4 \\ \underline{+\ +2} \\ +6 \end{array}$$

   b. Addition of negative numbers: add all numbers and give the sum a negative sign.

$$\begin{array}{r} -4 \\ \underline{+\ -2} \\ -6 \end{array}$$

   c. Addition of positive and negative numbers: add the following numbers.

$$\begin{array}{r} +6 \\ -3 \\ -2 \\ \underline{+\ +1} \end{array}$$

   First, add all the positive numbers.

$$\begin{array}{r} +6 \\ \underline{+\ +1} \\ +7 \end{array}$$

   Second, add all the negative numbers.

$$\begin{array}{r} -3 \\ \underline{+\ -2} \\ -5 \end{array}$$

   Subtract the smaller sum from the larger sum and give the answer the sign of the larger sum.

$$\begin{array}{r} +7 \\ \underline{-\ -5} \\ +2 \end{array}$$

2. Subtraction with positive and negative numbers. To subtract, replace the sign of the number being subtracted with its opposite and then add.
   a. Subtraction of positive numbers from positive numbers: replace the sign of the number being subtracted with its opposite, subtract the smaller number from the larger number, and give the difference the sign of the larger number.

Sample 1

$$\begin{array}{r} +8 \\ \underline{-\ +6} \end{array} \qquad \begin{array}{r} +8 \\ \underline{+\ -6} \\ +2 \end{array}$$

Sample 2

$$\begin{array}{r} +22 \\ \underline{-\ +50} \end{array} \qquad \begin{array}{r} +22 \\ \underline{+\ -50} \end{array} \qquad \begin{array}{r} -50 \\ \underline{+\ +22} \\ -28 \end{array}$$

b. Subtraction of a positive number from a negative number: replace the sign of the number being subtracted with its opposite, add all numbers, and give the sum a negative sign.

Sample 1

$$\begin{array}{r} -4 \\ -\ +2 \\ \hline \end{array} \qquad \begin{array}{r} -4 \\ +\ -2 \\ \hline -6 \end{array}$$

Sample 2

$$\begin{array}{r} -9 \\ -\ +15 \\ \hline \end{array} \qquad \begin{array}{r} -9 \\ +\ -15 \\ \hline -24 \end{array}$$

c. Subtraction of a negative number from a positive number: replace the sign of the number being subtracted with its opposite, add all numbers, and give the sum a positive sign.

Sample 1

$$\begin{array}{r} +9 \\ -\ -3 \\ \hline \end{array} \qquad \begin{array}{r} +9 \\ +\ +3 \\ \hline +12 \end{array}$$

Sample 2

$$\begin{array}{r} +6 \\ -\ -10 \\ \hline \end{array} \qquad \begin{array}{r} +6 \\ +\ +10 \\ \hline +16 \end{array}$$

d. Subtraction of a negative number from a negative number: replace the sign of the number being subtracted with its opposite, subtract the lesser number from the larger number, and give the difference the sign of the larger number.

Sample 1

$$\begin{array}{r} -9 \\ -\ -3 \\ \hline \end{array} \qquad \begin{array}{r} -9 \\ +\ +3 \\ \hline -6 \end{array}$$

Sample 2

$$\begin{array}{r} -7 \\ -\ -13 \\ \hline \end{array} \qquad \begin{array}{r} -7 \\ +\ +13 \\ \hline \end{array} \qquad \begin{array}{r} +13 \\ +\ -7 \\ \hline +6 \end{array}$$

3. Multiplication of positive and negative numbers.

a. A positive number × a positive number = a positive number

$$+4 \times +4 = +16$$

b. A positive number × a negative number = a negative number

$$+4 \times -4 = -16$$

c. A negative number × a positive number = a negative number

$$-4 \times +4 = -16$$

d. A negative number × a negative number = a positive number

$$-4 \times -4 = +16$$

4. Division of positive and negative numbers.
   a. A positive number ÷ a positive number = a positive number

$$+20 \div +5 = +4$$

   b. A positive number ÷ a negative number = a negative number

$$+20 \div -5 = -4$$

   c. A negative number ÷ a positive number = a negative number

$$-20 \div +5 = -4$$

   d. A negative number ÷ a negative number = a positive number

$$-20 \div -5 = +4$$

FRACTIONS

The word "fraction" refers to a part of a whole. In mathematics fractions refer to the division of some value into any number of equal parts.

There are two common forms of expression of fractions, common and decimal.

Common fractions

A *common fraction* is written in two parts, one over the other. The lower number is the *denominator;* the upper number is the *numerator.*

The denominator is the number of parts into which 1 is divided. The numerator is the number of these parts in the fraction.

A *proper fraction* is one in which the numerator is smaller than the denominator. The value of any proper fraction will always be less than 1.

$$\frac{1}{2} \quad \frac{4}{5} \quad \frac{6}{13} \quad \frac{227}{2000}$$

An *improper fraction* is one in which the numerator is equal to or greater than the denominator. The value of any improper fraction is always 1 or greater than 1.

$$\frac{4}{4} \quad \frac{7}{5} \quad \frac{18}{16}$$

A *whole number* is any number in which the denominator is 1. This denominator is rarely expressed but is understood.

$$4 = \frac{4}{1} \quad 16 = \frac{16}{1} \quad 2000 = \frac{2000}{1}$$

An improper fraction can be changed to a whole or mixed number by dividing the numerator by the denominator.

$$\frac{20}{5} = 20 \div 5 = 4 \quad \frac{9}{5} = 1\frac{4}{5}$$

A *mixed number* is a whole number plus a fraction. This number will always have a value of more than 1.

$$2\frac{1}{8} \qquad 10\frac{6}{9} \qquad 15\frac{1}{3}$$

A mixed number can be changed to an improper fraction by multiplying the whole number by the denominator of the fraction, adding the product to the numerator, and using the result as a new numerator over the original denominator.

$$2\frac{2}{5} = \frac{2 \times 5 + 2}{5} = \frac{10 + 2}{5} = \frac{12}{5}$$

A *complex fraction* is one in which the numerator, the denominator, or both are fractions.

$$\frac{2\,{}^{1}/_{2}}{3} \qquad \frac{6}{{}^{2}/_{5}} \qquad \frac{{}^{3}/_{8}}{{}^{4}/_{9}}$$

Basic operations with common fractions

1. Addition and subtraction. To add or subtract fractions, it is necessary that the denominators of all fractions be the same. The term "like fractions" has been used to describe fractions having a common denominator. To add or subtract like fractions, add or subtract the numerators and place the result over the denominator. Do not change (add or subtract) the denominator.

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$$

$$\frac{9}{16} + \frac{5}{16} = \frac{14}{16}$$

$$\frac{19}{33} - \frac{12}{33} = \frac{7}{33}$$

$$\frac{8}{13} - \frac{5}{13} = \frac{3}{13}$$

Unlike fractions are fractions with denominators that are not alike. To add or subtract unlike fractions, it is necessary to change the unlike fractions to like fractions. This means that a *common denominator* must be found for all fractions in the calculations. It is better if this common denominator is the *least common denominator* (LCD). The LCD is the smallest denominator into which all the denominators can be divided evenly. Frequently the least common denominator can be found easily.

EXAMPLE: Find the sum of $^1/_2$, $^3/_4$, and $^3/_8$.

Since these are unlike fractions, find the least common denominator. It is obvious that both 2 and 4 can be evenly divided into 8. Thus the LCD for this problem is 8. The next step is to find the fraction having a denominator of 8 equivalent to each of the fractions in the problem. To do this, divide each denominator into the LCD and multiply the result by the numerator. This will give the new numerator and hence equivalent fraction.

$$\frac{1}{2} \quad 8 \div 2 = 4 \quad 4 \times 1 = 4 \quad \frac{4}{8}$$

$$\frac{3}{4} \quad 8 \div 4 = 2 \quad 2 \times 3 = 6 \quad \frac{6}{8}$$

Hence $$\frac{4}{8} + \frac{6}{8} + \frac{3}{8} = \frac{13}{8} = 1\frac{5}{8}$$

In those problems in which the least common denominator is not obvious, the following procedures may be used.

Place all the denominators in a line. If possible, divide the numbers in this line by a number that will divide at least two of the denominators evenly. Bring down all numbers that cannot be divided. Continue this procedure until all denominators are reduced to a quotient of 1. Multiply all numbers used as divisors together. This number is the least commom denominator.

EXAMPLE: Find the LCD for $^1/_2$, $^2/_3$, $^5/_6$, and $^4/_9$.

```
2 )2 3 6 9
3 )1 3 3 9
3 )1 1 1 3
         1
2 × 3 × 3 = 18
```

The LCD is 18 for these fractions.

Another method to find a common denominator is to multiply all denominators together. This will not always produce the least common denominator but it can be used to produce equivalent like fractions.

2. Multiplying common fractions. To multiply one fraction by another, simply multiply the numerators together and multiply the denominators together. This can be done either with like or unlike fractions.

$$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

$$\frac{4}{11} \times \frac{7}{8} = \frac{28}{88}$$

If mixed numbers are involved in a multiplication, convert them to an improper fraction before multiplying the numbers.

3. Dividing common fractions. As in multiplication, unlike fractions can be divided without being converted to like fractions. To divide one fraction by another, invert the fraction doing the dividing (divisor) by making the numerator the denominator and vice versa. Multiply the resulting fractions.

$$\frac{3}{6} \div \frac{4}{5} = \frac{3}{6} \times \frac{5}{4} = \frac{15}{24}$$

To divide mixed numbers, convert them to improper fractions and proceed as directed.