INTRODUCTION TO STATISTICAL ANALYSIS

Wilfrid J. Dixon
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Fourth Edition

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McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

This book was set in Times Roman by Santype-Byrd.
The editors were John J. Corrigan, Stephen Wagley, and James S. Amar;
the production supervisor was Dennis J. Conroy.
The cover was designed by Jo Jones.
R. R. Donnelley & Sons Company was printer and binder.

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234567890 DOCDOC 8987654

ISBN 0-07-017073-8

Library of Congress Cataloging in Publication Data

Dixon, Wilfrid Joseph, date
Introduction to statistical analysis.

Bibliography: p.
Includes index.
1. Statistics. I. Massey, Frank Jones.
II. Title.
QA276.12.D58 1983 519.5 81-19313
ISBN 0-07-017073-8 AACR2

To
Eva, Janet, Kitty
Mildred, Frank, Laura

PREFACE

Major changes to the fourth edition include a new half chapter on multiple regression, additional material on multiple comparisons, a large section on chi-square tests and their power, and expanded treatment of orthogonal contrasts. Other changes have been made throughout.

This textbook is written for a basic course in statistics to be taken by students from all fields in which statistics finds application. We have attempted to present the fundamental concepts of the subject in a manner which will show the student how general is the application of the statistical method. It is intended that interested students continue this type of training in courses giving special applications in their own fields after one, two, or three quarters of this course.

We have found that the contents of this revised text can easily be covered in a one-year course having either three lectures per week or two lectures with one laboratory per week. For shorter courses the following topics are suggested: Chapters 1 to 8 plus selections from any of Chapters 10, 11, 13, 17, and 20. Except for parts of Chapter 20 the only mathematical ability assumed of the student is a knowledge of algebraic addition, subtraction, and multiplication. We feel that the topic of probability is more meaningful for students with a minimum of mathematical background if it is presented late in the year course. With students who have the equivalent of two years of high school algebra some teachers may wish to present Chapter 20 quite early, and it has been prepared with this in mind. It may be desirable to introduce part or all of Chapter 20 immediately following Chapter 4. We have avoided conventional gambling games, dice problems, etc., in Chapter 20 and have stressed the statistical applications of the theory.

The concepts of distribution, sample, and population are introduced early. The elementary descriptive procedures of statistics are introduced as they are needed in the development of the ideas of sampling, tests of hypotheses, and design of experiments. The analysis of variance is introduced sufficiently early for its inclusion in a one-semester or two-term course. Nonparametric statistics have been included because of their wide applicability and because of their validity under general conditions. The sampling distributions of the various statistics are introduced by means of experimental sampling. Experimental verifications of tabled distributions have been carried out by comparing percentiles of observed sampling distributions with the mathematical results. The sampling experiments indicated at the end of the chapters are organized so that computations on the samples drawn are used in several following class exercises.

Many changes have been made to improve or to change emphasis, and several sections presenting new or different methods have been added. We now have two sets of data at the end of Chapter 2 to provide realistic problems of interest in themselves and for use in computer assisted analysis. Exercises using these data are included at the end of many of the chapters. When statistical computer packages such as BMDP, SAS, or SPSS are available, assignments can be made from these exercises. Teachers using BMDP will find suggestions for further analysis since the second data set is used extensively in the BMDP manual. The data in Table 2-2*a* and *b* are observations on 200 men chosen from the Los Angeles Heart Study, by courtesy of Dr. John M. Chapman, U.C.L.A. The data in Table 2-3*a* and *b* are observations on 188 women chosen from the San Francisco Health Fair Study, by courtesy of Dr. M. Werner.

Additional discussion questions have been added relating to points raised by students. Some formulas have been modified in recognition of the widespread use of hand calculators.

Following Chapter 20 we have included a section of "General Comments" which provides notes for extended reading either into various areas of application or into a study in greater depth of special topics.

For the preparation of the fourth edition we acknowledge the contributions of Linda Moody Chilingar for modifications to the questions and exercises and some portions of the text. We appreciate the many useful ideas for improvement provided by the reviewers: Stephen Meeks of Boston University, Michael L. Deaton of Virginia Polytechnic Institute and State University, Ralph Catanese of Villanova University, Allan P. Sampson of the University of Pittsburgh, Thomas O'Connor of the University of Louisville, and David Finkel of Bucknell University.

Excellent assistance with manuscript preparation was provided by Anne Eiseman.

We wish to express our appreciation to Professor E. S. Pearson for permission to reprint from *Biometrika* parts of Tables A-7, A-8, A-9, A-13, A-18, and A-30; to S. K. Banerjee for permission to reprint from *Sankhyā* Table A-25; to the RAND Corporation for permission to print the random-number tables; to A. Hald and John Wiley & Sons for permission to copy certain

percentiles forming part of Table A-7*c*. We are indebted to Sir Ronald A. Fisher, Frank Yates, and to Messrs. Oliver & Boyd, Ltd., Edinburgh, for permission to reprint, in part, Table III from their book *Statistical Tables for Biological, Agricultural and Medical Research*. For other tables in the Appendix we are indebted to C. Colcord, L. S. Deming, C. Eisenhart, M. W. Hastay, L. A. Knowler, R. F. Link, F. Mosteller, E. G. Olds, F. Swed, W. A. Wallis, J. E. Walsh, and E. K. Yost.

We wish to take this opportunity to express our appreciation to the many friends and colleagues who have made helpful criticisms and suggestions on the earlier editions.

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CONTENTS

CHAPTER

ONE

INTRODUCTION

The term *statistics* pertains to a listing of facts, to systematic methods of arranging and describing the data, and finally to a science of inferring generalities from specific observations. The emphasis in this book will be on the problem of inference, but in order to build up the necessary background the first few chapters will be of a descriptive nature. The laws of physical, biological, and social science have their proof in statistical facts. The study of statistics here will not stop with the mere description of an existing situation but will continue to the study of procedures of scientific inference and proof.

1-1 TYPES OF PROOF

You, no doubt, have heard the statement "You can prove anything with statistics." We shall determine just what sort of things can be proved by statistics, and furthermore, just what we mean by "proof." Natural and physicals *laws* are *hypotheses* which have been subjected to various tests and have become accepted, or, as some say, proved. The proof of a hypothesis is the *testing* of the hypothesis. If the tests show the hypothesis to be satisfactory, it is accepted; if the tests show the hypothesis unsatisfactory, it is rejected. When have we tested a hypothesis sufficiently to reject it? The standard procedure will be to collect information in the form of numerical observations and to base our decision on these observations. For example, a person who tosses a coin 100 times and obtains heads every time may feel that the hypothesis that this is an unbiased coin is no longer acceptable, and will reject that hypothesis. It is well known that it is possible for this result to occur with a true coin, but if we were to demand that we be completely positive before making a decision, we could never decide that the coin was biased, even if it had two heads, unless we were allowed to examine it. We would not be able to accept the hypothesis of gravity as a law until all the apples of all time had fallen. The procedures of *statistical inference* will make it possible (under certain assumptions) for us to state just what the probability is that we will accept false hypotheses or reject true hypotheses. We will never know for sure in any particular case, of course, whether the hypothesis is true or not. This *statistical proof* is the basic form of proof used in the investigations of all sciences. We must make a distinction here between the methods of statistical proof and the methods of mathematical proof. *Mathematical proof* is available only within the framework of mathematics itself and cannot be applied outside that field. A hypothesis in mathematics may be declared false by the presentation of a single example which violates it. However, a single example which does not agree with the hypothesis cannot usually by itself cause us to reject a hypothesis outside of mathematics.

The methods of proof used to develop the statistical procedures presented in this book will in a few cases be *mathematical proofs* in the adaptation of various formulas. For the most part, however, the development will be that of statistical proof, or *experimentation*. These statistical procedures can all be developed by mathematical means, although in most cases it will not be possible to present these mathematical developments because of the limited mathematical background assumed of the reader of this text. The development by experimentation will serve a double purpose, for here we are interested not only in the results of the experiments, but also in a study of the process of proof by experiment.

1-2 GENERALITY OF APPLICATIONS OF STATISTICS

There does not exist a theory of statistics applicable only to economics, only to medicine, or only to education. There is a *general* theory of statistics which is applicable to any field of study in which observations are made. Statistical procedures now form an important part of all fields of science, and procedures which have been developed for use in one field have almost invariably found important application in a number of other fields. There are, however, statistical procedures which are more frequently used in one field than in another. We shall concentrate on those procedures which are most widely used.

1-3 EXAMPLES OF STATISTICAL PROBLEMS

We need not look far to find problems using statistical ideas. The concept of *average* is used in referring to a person of average height, to a ballplayer's batting average, or to the average number of cigarettes smoked per day. We use the single figure of an average to represent the whole group of people, or the ballplayer's performance in all his games, or the many different speeds on a trip between towns. We use the single quantity, the average, to describe one characteristic of the group. For example, when we say that the average height of a group of men is 69 inches, we do not mean that all men in the group are 69 inches tall; the average describes the whole *group* of men, so that if we were to pick a man at random from this group and were asked what height we would *expect* him to be, we would give as our estimate 69 inches. Only if all individuals in the group were exactly the same height could we *guarantee* that the chosen one would have the same height as the average.

Another concept with which we are all familiar is that of *dispersion*, or variability. A teacher may say that one class is more uniform in ability than another class. An engineer may say that one batch of electric light bulbs is more variable in quality than another batch. A textile worker may say that one type of yarn is more variable in breaking strength than it should be. A manufacturer who wishes to use mass production must reduce the variability in the dimensions of parts if any shaft is to fit into any sleeve. The manufacturer of propellant for a missile must produce propellant of sufficiently uniform burning time that the total propellant burning time can be adjusted. If the propellant burns too rapidly, the missile will explode before it leaves the ground; if it burns too slowly, the missile cannot be launched.

Another commonly used statistical description of data is that of *correlation*, or association. The teacher may say of a class that the faster readers are also better at arithmetic and the slower readers poorer at arithmetic. As an example of a negative relationship, a teacher may say of a particular class that the older children read more slowly and the younger chidren more rapidly.

These concepts of proof, average, dispersion, and correlation will be developed more fully, and their combined use in various types of statistical investigations will be illustrated in the ensuing chapters.

A major area of statistical theory is concerned with the design of experiments and the efficient collection of information to aid in this design. Many experimental materials are so expensive that it is essential that the desired information be obtained with a minimum number of observations. Statistics is also concerned with problems which arise from the necessity of designing experiments to investigate several factors at the same time, either because of the great length of time required for the experiment or because of the difficulty in reproducing the experimental treatments or conditions. In such cases statistical methods must be used to separate the effects of the separate treatments.

Whenever anything is measured numerically, even though the attempt to make an assessment results in numbers no more refined than simple counting, there arises a desire to judge the significance of the data and to make maximum use of the information gathered. These are the principal problems with which statistical methods are concerned.

CHAPTER

TWO

DISTRIBUTIONS

In the investigation of any phenomenon, whether it is the study of forces of attraction by the physicist, the study of the effects of anxiety by the psychologist, the study of radiation effects on animals by the biologist, or any other research, it is necessary to observe and record some characteristic of the objects under consideration.

2-1 OBSERVATIONS

We must have observations of some form, even if they are of a very rudimentary sort. It is perhaps easiest to think of *measurements* as such simple properties as length, weight, volume. For such properties it is generally easy to establish a measuring stick to compare objects or individuals. For example, we may mark off a pole in units of equal length and use it to show that an object 60 inches long is twice as long as another 30 inches long.

Let us consider, for example, such a characteristic as intelligence. It may be possible to construct an examination which will indicate whether one individual has more than another of something that might be called intelligence, but unlike the measuring stick it may be more difficult to say that a particular individual is twice as intelligent as some other individual. In fact, we may not be able to attach an absolute interpretation to the differences between individuals IQs; for example, the difference between IQs of 100 and 110 may not be comparable to the difference between 140 and 150.

The physical sciences also deal with quantities for which only a *scale* is established, for example, the common measurement called temperature. We may say that today is hotter than yesterday, but we cannot decide that today is twice as hot as yesterday.