DESIGN OF PRESTRESSED CONCRETE

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PREFACE

Although the first proposal to apply prestressing to concrete was made as early as 1886, in the United States, it was only as a result of the studies of the renowned French engineer Eugene Freyssinet, in the 1930s, that prestressed concrete became a practical reality. In Europe, in the period of acute material shortages following World War II, Freyssinet and other pioneers such as Finsterwalder and Magnel demonstrated the remarkable possibilities of this new concept of design and set the stage for the development that was to take place in the years that followed.

Largely for economic reasons, the evolution of prestressed concrete in the United States has taken place along very different lines than in Europe. Until recently, the main interest had been in pretensioned precast units of short to medium span, which could be mass-produced at great savings in labor costs. Used for floors, roofs, and walls, these units have accounted for a significant fraction of new construction, and undoubtedly will continue to do so.

However, changing economic conditions are producing important changes in U.S. practice. Construction labor is not in such short supply as before. The cost of materials is constantly increasing, and there is serious concern for conservation of resources. Under such circumstances, it is natural that engineers should consider the suitability of more sophisticated designs, which more fully exploit the capability of prestressing. It has been found that prestressed concrete now competes successfully with other forms of construction for medium- and long-span bridges, tall buildings, long-span roofs, and other types of one-of-a-kind construction.

Such changes in conditions of practice have created the need for engineers who have a firm understanding of the fundamental principles of prestressed concrete behavior and design, who can not only act effectively to optimize existing forms of construction, but who can also apply fundamental concepts with confidence in unusual and challenging situations.

I hope that this textbook may be effective in developing that basic understanding. The book has grown from a set of lecture notes that I developed while teaching prestressed concrete to civil engineering students at Cornell University over a 15-year period. Every effort has been made to insure a thorough understanding of basic mechanics and behavior. Although the book is intended mainly as a college textbook at the fourth- or fifth-year level, a special effort has been made to develop a clear, self-contained presentation, so that the book may be useful to engineers who wish to improve their knowledge of this relatively new field by self-study. The material has been carefully coordinated with codes and specifications governing U.S. practice, notably the ACI Building Code, but also the AASHTO Specification for highway structures and the AREA Design Manual for railway construction.

It is assumed that the student has had prior exposure to the basic aspects of reinforced concrete behavior and design. Certain fundamentals, encountered first in the design of reinforced concrete, are not developed fully here; in such cases references are given to other sources.

The arrangement of the material follows that of my lectures. After an introduction to basic concepts and properties of materials, in Chapters 1 and 2, the analysis and design of beams is presented in Chapters 3 to 5. Losses of prestress force are considered in Chapter 6. It may be argued that analysis of losses should precede beam analysis and design, but I have concluded that, from a pedagogical point of view, there are advantages to getting on with the business of design early. In many practical cases, losses must be considered in no more detail than in Chapters 3 and 4.

The study of deflections (Chapter 9) and the design of slabs (Chapter 10) are fundamental and should be included in a first course of study. However, the teacher may not find time to cover composite beams or continuous members (Chapter 7 and 8, respectively). These topics, as well as the treatment of axially loaded members (Chapter 11), may be deferred until a later course or taken up through self-study.

Chapters 12 and 13, which deal, respectively, with precast construction and applications, have been written to permit their assignment as outside reading.

Appendix A contains a variety of design aids. These are useful in connection with examples and problems to be assigned, and may also make the book a useful desk aid for the practicing engineer. Appendix B contains engineering data for certain common post-tensioning systems. No attempt has been made to be encyclopedic here, but only to present sufficient detail to permit realistic proportioning of members in practice problems.

A word is in order relative to the units of measurement used. Nationwide, there is a movement toward adoption of the International System (SI) of metric units. In many cases, basic science and engineering science courses are now taught in SI units. Certain industries have already converted. However, in current U.S. structural practice, the familiar "English" or "customary" units are still almost universally used. Conversion to metric units will follow by at least several years the metrication of design codes and specifications. Yet the new edition of the ACI Code, governing the greatest part of U.S. concrete design and construction, is written entirely in customary units.

Recognizing that the users of this textbook may have become familiar with SI units in preparatory courses, but will soon enter design offices in which customary units prevail, I have proceeded as follows: (1) all graphs and tabulated information of a fundamental nature are given in dual units; (2) all dimensionally inconsistent equations are given in customary units, but SI equivalents are given in the separate Appendix C; (3) examples are given in customary units, but SI equivalents are provided, in parentheses, for input data and key answers; and (4) design aids in Appendix A are given in customary units only. This is a reasonable compromise between encouragement to adopt the obviously superior International System of units, and recognition of the probable facts of professional practice over the next 5 to 10 years.

Many persons and organizations have contributed to this volume. Significant contributions have been made by former students, particularly by Charles Dolan, of ABAM-Engineers, Inc., who offered valuable comments and arranged for much illustrative material. Other illustrations were obtained through the cooperation of George Nasser, of the Prestressed Concrete Institute, Gene Corley, of the Portland Cement Association, Cliff Freyermuth, of the Post-Tensioning Institute, and many others. A substantial contribution was made by Edward Nawy, of Rutgers University, who reviewed the final manuscript.

Secretarial and other essential support was provided by Cornell University.

Finally, I would like to acknowledge the influence of George Winter, with whom an earlier book on reinforced concrete was coauthored. A long professional and personal association with him has had a profound effect in developing a point of view that I hope is reflected in the following pages.

Ithaca, New York
March 1978

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CONTENTS

CHAPTER 1

BASIC CONCEPTS

1.1 INTRODUCTION

Prestressing can be defined in general terms as the preloading of a structure, before application of the required design loads, in such a way as to improve its overall performance. Although the principles and techniques of prestressing have been applied to structures of many types and materials, the most common application is in the design of structural concrete.

Concrete is essentially a compression material. Its strength in tension is much lower than that in compression, and in many cases in design the tensile resistance is discounted altogether. The prestressing of concrete, therefore, naturally involves application of a compressive loading, prior to applying the anticipated design loads, so that tensile stresses that otherwise would occur are reduced or eliminated.

In fact, the original concept of prestressing concrete was to introduce sufficient axial precompression in beams so that all tension in the concrete was eliminated in the loaded member. However, as knowledge of this relatively new form of construction has developed, it has become clear that this view is unnecessarily restrictive, and in present design practice tensile stress in the concrete, even some limited cracking, is permitted. By varying the amount of compressive prestress, the number and width of cracks can be limited to the desired degree. Of equal importance, the deflection of the member may be controlled. Beams may even be designed to have zero deflection at a specified combination of prestress and external loading. In the sense of improved serviceability, such partial prestressing represents a substantial improvement, not only over conventional reinforced concrete construction, but also over the original form of full prestressing which, while eliminating service-load cracking, often produced troublesome upward camber.

But it is not only through improved serviceability that prestressing has achieved its position of importance. By crack and deflection control at service loads, prestressing makes it possible to employ economical and efficient high tensile strength steel reinforcement and high strength concrete.

Crack widths in conventional reinforced concrete beams are roughly proportional to the stress in the tensile reinforcement, and for this reason steel stresses must be limited to values far less than could otherwise be used. In prestressed beams, high steel stress is not accompanied by wide concrete cracks, because much of the strain is applied to the steel before it is anchored to the concrete, and before the member is loaded.

Deflection of ordinary reinforced concrete beams is also linked directly to stresses. If very high stresses were permitted, the accompanying high strains in the concrete and steel would inevitably produce large rotations of the cross sections along the member, which translate directly into large deflections. By prestraining the high tensile reinforcement of prestressed beams, the large rotations and deflections that would otherwise occur are avoided. In addition, the essentially uncracked concrete member is stiffer, for given section dimensions, than it would be if cracking were permitted to the extent typical of reinforced concrete construction.

Thus it is not only because of improvement of service load behavior, by controlling cracking and deflection, that prestressed concrete is attractive, but also because it permits utilization of efficient high strength materials. Smaller and lighter members may be used. The ratio of dead to live load is reduced, spans increased, and the range of possible application of structural concrete is greatly extended.

The dramatic improvements in the performance of concrete structures that could be obtained by prestressing were first recognized by the renowned French engineer Eugene Freyssinet. His studies of the time-dependent effects of shrinkage and creep of concrete, which began as early as 1911, led him to realize the importance of using steel at a high initial stress to prestress concrete members. In 1940 he introduced a system for prestressing using wedge-anchored high strength cables, an arrangement of great practicality that is still in wide use.

The remarkable bridge over the river Marne at Luzancy, France, shown in Figs. 1.1 and 1.2, illustrates the innovation and daring that was to be typical of Freyssinet's later designs. Built in 1941, this very flat, two-hinged portal frame structure has a span of 180 ft and a depth at midspan of only 4.17 ft, a ratio of span to depth of 43. The hinged supports of the bridge were provided with adjustments in order to compensate for the effects of shrinkage and creep.

The I-shaped bridge segments were precast. The flanges were cast first, and were connected by wires that were tensioned prior to casting the web, by jacking the flanges apart. After the webs were cast, the jacking force was released, precompressing the webs to counteract diagonal tensile stresses resulting from loads. Individual segments were then assembled into larger components, which were placed in final position by cableways, and the entire structure then post-tensioned. This structure, and five other nearly identical spans in the same region, provide the model for segmentally precast bridges now widely used.

Prestressing has been applied to great advantage in a wide variety of situations, a few of which are illustrated by the following photographs. Figure 1.3 shows the use of precast "double-tee" beams carrying a floor with clear span of about 20 ft. End support is provided by the precast L-section beam over the

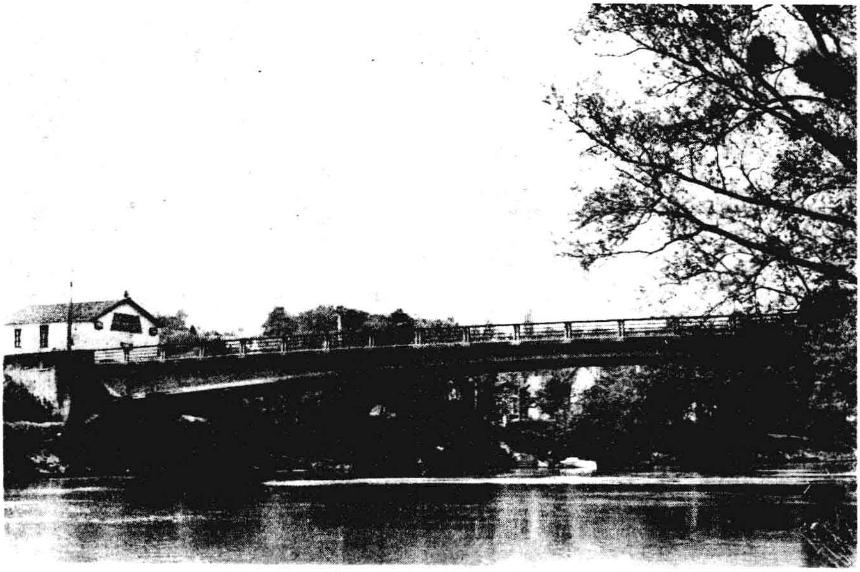

Figure 1.1 Bridge of 180 ft span over the river Marne at Luzancy designed by Freyssinet and built in 1941.

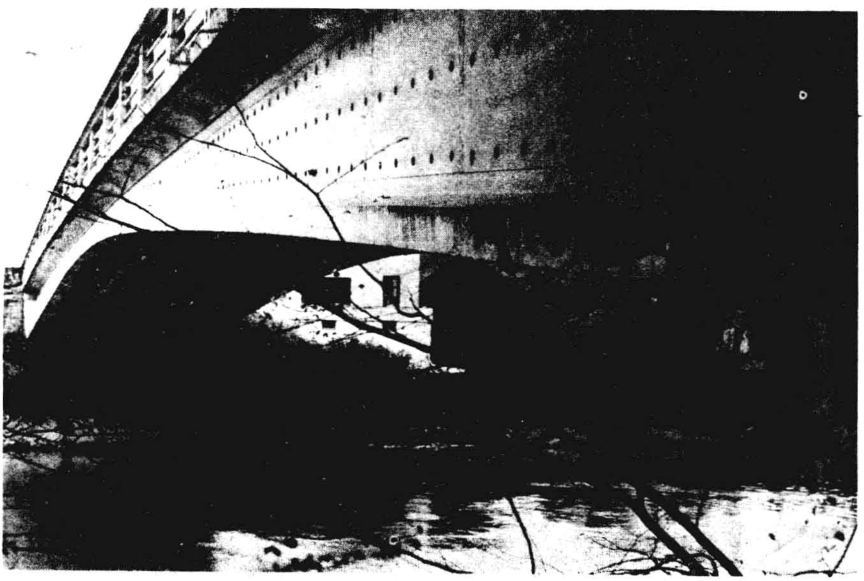

Figure 1.2 View of the Luzancy bridge.

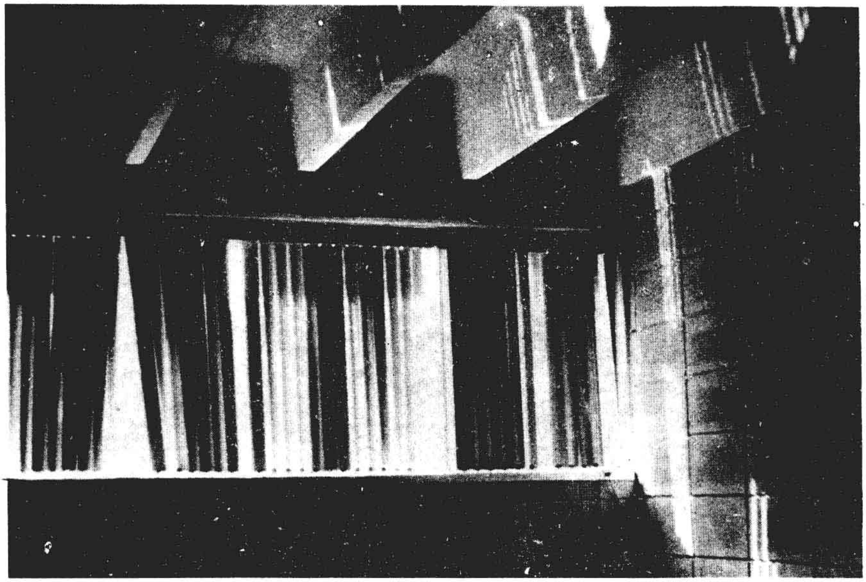

Figure 1.3 Precast prestressed double-tee floor beams.

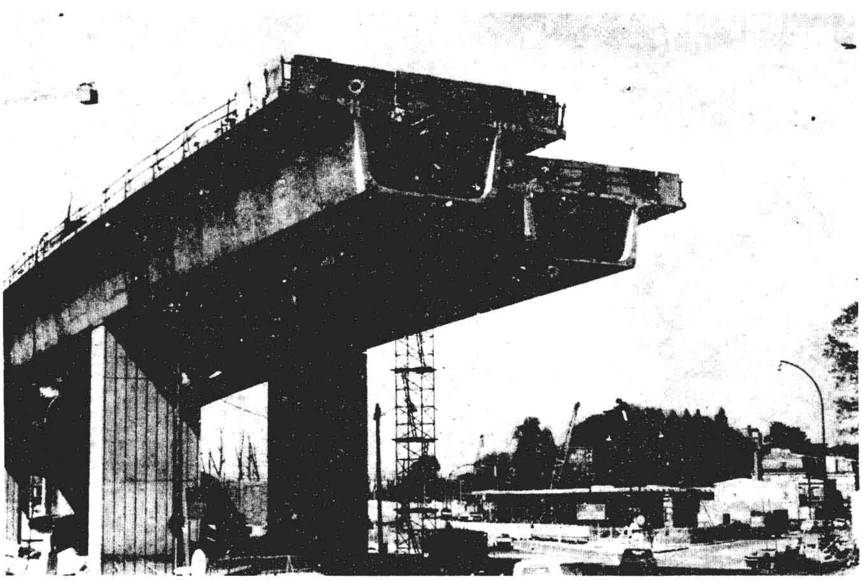

Figure 1.4 Twin box girder bridge under construction using the segmentally cast cantilever method.

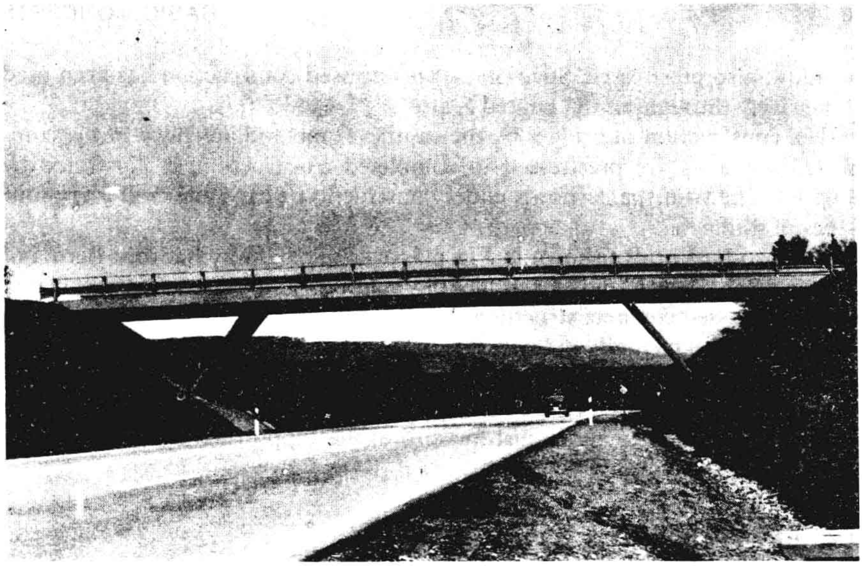

Figure 1.5 Highway crossing in Switzerland, continuous over three spans.

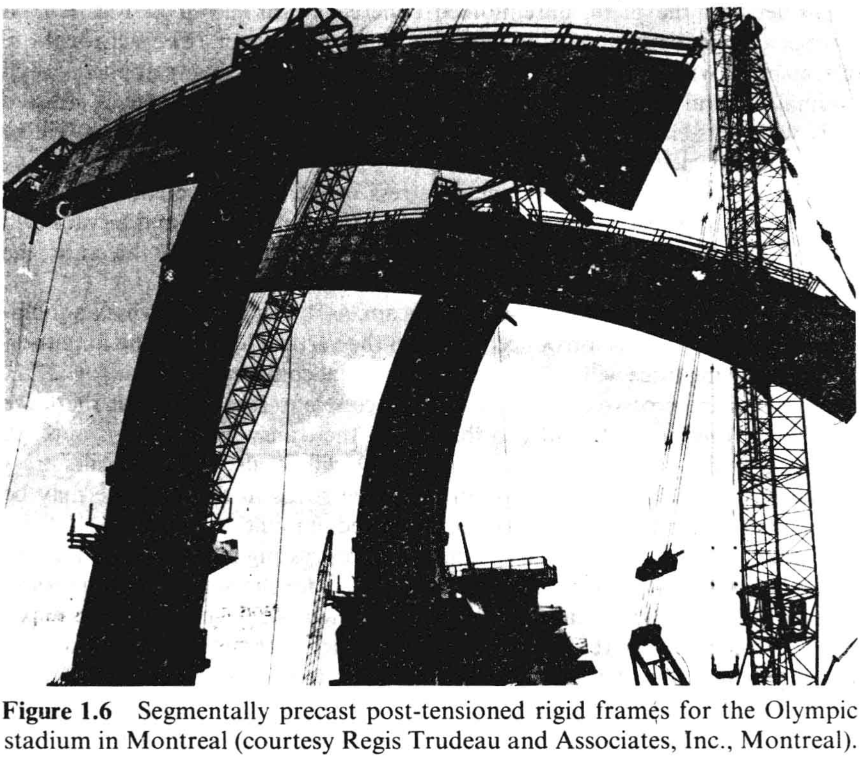

Figure 1.6 Segmentally precast post-tensioned rigid frames for the Olympic stadium in Montreal (courtesy Regis Trudeau and Associates, Inc., Montreal).

window, also prestressed. Such precast prestressed construction has been used extensively throughout the United States.

The construction of bridges by the cantilever method, in which newly completed segments are prestressed to completed construction, is illustrated by Fig. 1.4. The twin spans shown under construction, near Paris, will carry four lanes of traffic.

The two-lane bridge shown in Fig. 1.5, over the highway between Bern and Lausanne in Switzerland, illustrates the lightness and grace often associated with prestressed concrete structures.

The huge, segmentally precast frames shown in Fig. 1.6, recently completed for the 1976 Olympic Games in Montreal illustrate the versatility of prestressed concrete. To provide a sense of scale, note the construction worker atop the catwalk of the farther frame, just forward of the supporting leg.

1.2 EXAMPLE

Many important features of prestressing can be illustrated by a simple example. Consider first the plain, unreinforced concrete beam shown in Fig. 1.7*a*. It carries a single concentrated load at the center of its span. (The self-weight of the member will be neglected here.) As the load W is gradually applied, longitudinal flexural stresses are induced. Assuming that the concrete is stressed only within its elastic range, the flexural stress distribution at midspan will be linear, as shown.

At a relatively low load, the tensile stress in the concrete at the bottom of the member will reach the tensile strength of the material, f'_r, and a crack will form. Since no restraint is provided against upward extension of the crack, the member will collapse without further increase of load.

Now consider an otherwise identical beam, as in Fig. 1.7*b*, in which a longitudinal axial force P is introduced prior to the vertical loading. The longitudinal prestressing force will produce a uniform axial compressive stress $f_c = P/A_c$, where A_c is the cross-sectional area of the concrete. It is clear that the force can be adjusted in magnitude, so that, when the transverse load Q is applied, the superposition of stresses due to P and Q will result in zero tensile stress at the bottom of the beam, as shown. Tensile stress in the concrete may be eliminated in this way, or reduced to a specified amount.

But it would be more logical to apply the prestressing force near the bottom of the beam, so as to compensate more effectively for the load-induced tension. A possible design specification, for example, might be to introduce the maximum compression at the bottom of the member without causing tension at the top, when only the prestressing force acts. It is easily shown that, for a rectangular cross section beam, the corresponding point of application of the force is at the lower third point of the section depth. The load P, with the same

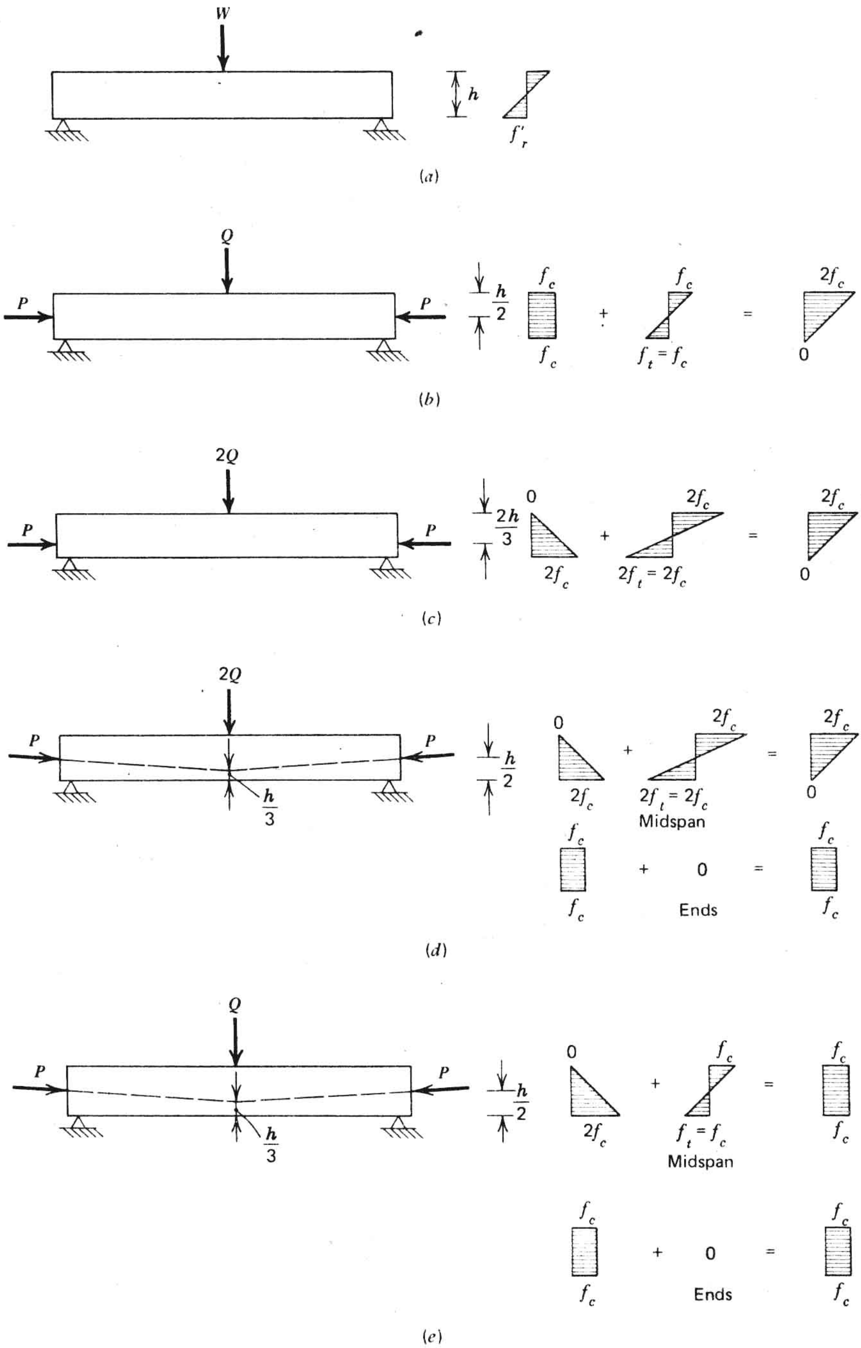

Figure 1.7 Alternative schemes for prestressing a rectangular concrete beam. (*a*) Plain concrete beam. (*b*) Axially prestressed beam. (*c*) Eccentrically prestressed beam. (*d*) Beam prestressed with variable eccentricity. (*e*) Balanced load stage for beam with variable eccentricity.

value as before, but applied with eccentricity $e = h/6$ relative to the concrete centroid, will produce a longitudinal compressive stress distribution varying from zero at the top surface to a maximum value of $2f_c = (P/A_c) + (Pec_2/I_c)$, at the bottom, where f_c is the concrete stress at the section centroid, c_2 is the distance from concrete centroid to the bottom face of the concrete, and I_c is the moment of inertia of the cross section. This is shown in Fig. 1.7*c*. The stress at the bottom will be exactly twice the value produced before by axial prestressing.

Consequently the transverse load may now be twice as great as before, or $2Q$, and still cause no tensile stress. In fact, the final stress distribution resulting from the superposition of load and prestressing force in Fig. 1.7*c* is identical to that of Fig. 1.7*b*, although the load is twice as great. The advantage of eccentric prestressing is obvious.

The methods by which concrete members are prestressed will be discussed in some detail in Art. 1.6 with further details given in Appendix B. For present purposes, it is sufficient to know that one common method of prestressing uses high strength steel wires passing through a conduit embedded in the concrete beam. The tendon is anchored to the concrete at one end, and is stretched at the far end by a hydraulic jack that reacts against the concrete. When the desired tension in the tendon is obtained, it is anchored against the concrete at the jacking end as well, and the jack is removed. The result is a self-contained system by which the force P of Fig. 1.7 may be applied.

If such a system is used, a significant improvement over the arrangement of Figs. 1.7*b* or 1.7*c* can be made, by using a variable eccentricity of prestress force, with respect to the centroid of the concrete section, along the length of the member. The load $2Q$ produces a bending moment that varies linearly along the span, from zero at the supports to maximum at the center. Intuitively, one suspects that the best arrangement of prestressing would produce a countermoment, acting in the opposite sense, which would vary in the same way. This is easily done, because the prestress moment is directly proportional to the eccentricity of the tendon, measured from the steel centroid to the concrete centroid. Accordingly, the tendon is now given an eccentricity that varies linearly from zero at the supports to maximum at the center of the span. Such an arrangement is shown in Fig. 1.7*d*. The stresses at midspan are the same as before, both when the load $2Q$ acts and when it does not. At the supports, where only the prestress force acts, with zero eccentricity, a uniform compressive stress f_c is obtained as shown.

It should be clear that, for each characteristic load arrangement, there is a "best" tendon profile in the sense that it produces a prestress moment diagram which corresponds to that of the applied load. It is of further interest to note that, if the prestress countermoment should be made exactly equal and opposite to the moment from the loads, all along the span, the result is a beam that is subject only to uniform axial compressive stress throughout for that particu-