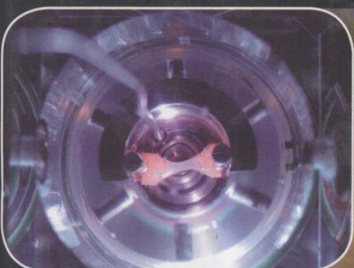
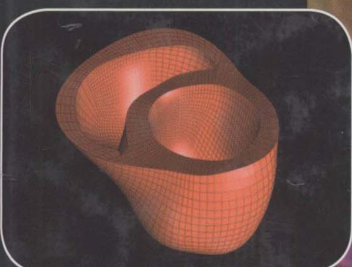
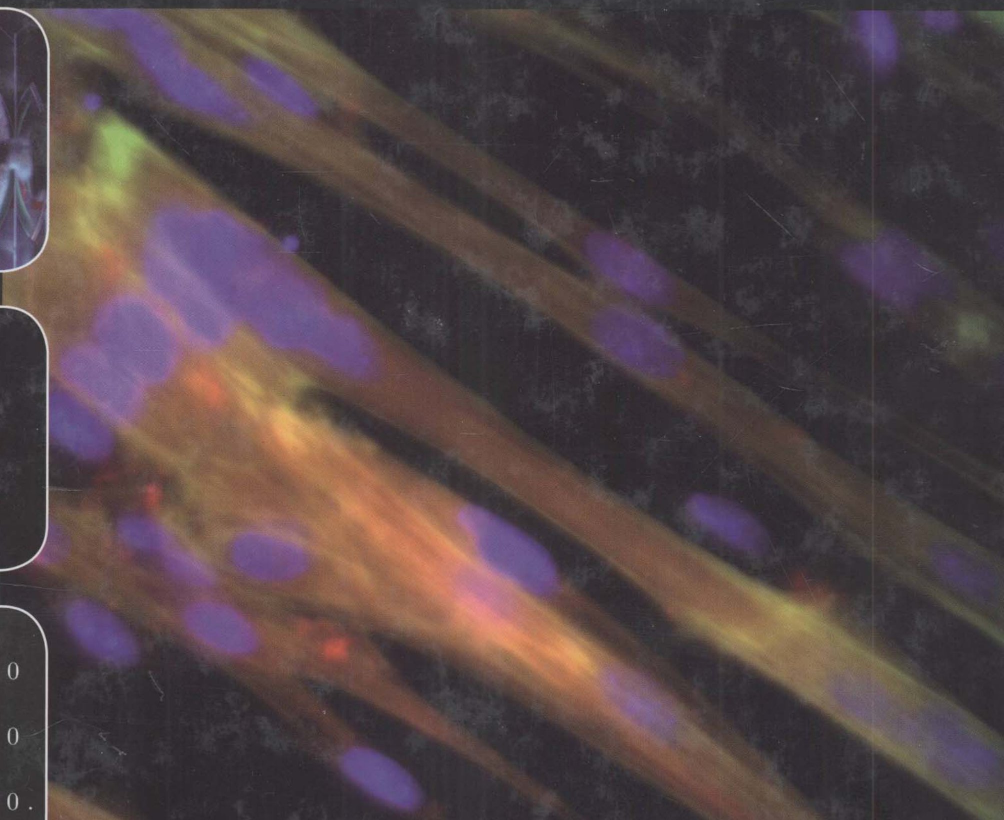
CAMBRIDGE TEXTS IN
BIOMEDICAL
ENGINEERING

Biomechanics

Concepts and Computation







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Cees Oomens, Marcel Brekelmans
and Frank Baaijens

Biomechanics

Concepts and Computation

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Biomechanics: Concepts and Computation

This quantitative approach integrates the classical concepts of mechanics and computational modelling techniques, in a logical progression through a wide range of fundamental biomechanics principles. Online MATLAB-based software, along with examples and problems using biomedical applications, will motivate undergraduate biomedical engineering students to practise and test their skills. The book covers topics such as kinematics, equilibrium, stresses and strains, and also focuses on large deformations and rotations and non-linear constitutive equations, including visco-elastic behaviour and the behaviour of long slender fibre-like structures. This is the first textbook that integrates both general and specific topics, theoretical background and biomedical engineering applications, as well as analytical and numerical approaches. This is the definitive textbook for students.

Cees Oomens is Associate Professor in Biomechanics and Continuum Mechanics at the Eindhoven University of Technology, the Netherlands. He has lectured many different courses ranging from basic courses in continuum mechanics at bachelor level, to courses on mechanical properties of materials and advanced courses in computational modelling at masters and postgraduate level. His current research focuses on damage and adaptation of soft biological tissues, with emphasis on skeletal muscle tissue and skin.

Marcel Brekelmans is Associate Professor in Continuum Mechanics at the Eindhoven University of Technology. Since 1998 he has also lectured in the Biomedical Engineering Faculty at the University; here his teaching addresses continuum mechanics, basic level and numerical analysis. He has published a considerable number of papers in well-known journals, and his research interests in continuum mechanics include the modelling of history-dependent material behaviour (plasticity, damage and fracture) in forming processes.

Frank Baaijens is Full Professor in Soft Tissue Biomechanics and Tissue Engineering at the Eindhoven University of Technology, where he has also been a part-time Professor in the Polymer Group of the Division of Computational and Experimental Mechanics since 1990. He is currently Scientific Director of the national research program on BioMedical Materials (BMM), and his research focuses on soft tissue biomechanics and tissue engineering.

CAMBRIDGE TEXTS IN BIOMEDICAL ENGINEERING

Cambridge Texts in Biomedical Engineering provides a forum for high-quality accessible textbooks targeted at undergraduate and graduate courses in biomedical engineering. It will cover a broad range of biomedical engineering topics from introductory texts to advanced topics including, but not limited to, biomechanics, physiology, biomedical instrumentation, imaging, signals and systems, cell engineering, and bioinformatics. The series blends theory and practice, aimed primarily at biomedical engineering students, it also suits broader courses in engineering, the life sciences and medicine.

About the cover

The cover contains images reflecting biomechanics research topics at the Eindhoven University of Technology. An important aspect of mechanics is experimental work to determine material properties and to validate models. The application field ranges from microscopic structures at the level of cells to larger organs like the heart. The core of biomechanics is constituted by models formulated in terms of partial differential equations and computer models to derive approximate solutions.

- *Main image*: Myogenic precursor cells have the ability to differentiate and fuse to form multinucleated myotubes. This differentiation process can be influenced by means of mechanical as well as biochemical stimuli. To monitor this process of early differentiation, immunohistochemical analyses are performed to provide information concerning morphology and localization of characteristic structural proteins of muscle cells. In the illustration, the sarcomeric proteins actin (red), and myosin (green) are shown. Nuclei are stained blue. Image courtesy of Mrs Marloes Langelaan.
- *Left top*: To study the effect of a mechanical load on the damage evolution of skeletal tissue an in-vitro model system using tissue engineered muscle was developed. The image shows this muscle construct in a set-up on a confocal microscope. In the device the construct can be mechanically deformed by means of an indentor. Fluorescent identification of both necrotic and apoptotic cells can be established using different staining techniques Image courtesy of Mrs Debby Gawlitta.
- *Left middle*: A three-dimensional finite element mesh of the human heart ventricles is shown. This mesh is used to solve the equations of motion for the beating heart. The model was used to study the effect of depolarization waves and mechanics in the paced heart. Image courtesy of Mr Roy Kerckhoffs.
- *Left bottom*: The equilibrium equations are derived from Newton's laws and describe (quasi-)static force equilibrium in a three-dimensional continuum. Chapter 9 of the present book.

Preface

In September 1997 an educational programme in Biomedical Engineering, unique in the Netherlands, started at the Eindhoven University of Technology, together with the University of Maastricht, as a logical step after almost two decades of research collaboration between both universities. This development culminated in the foundation of the Department of Biomedical Engineering in April 1999 and the creation of a graduate programme (MSc) in Biomedical Engineering in 2000 and Medical Engineering in 2002.

Already at the start of this educational programme, it was decided that a comprehensive course in biomechanics had to be part of the curriculum and that this course had to start right at the beginning of the Bachelor phase. A search for suitable material for this purpose showed that excellent biomechanics textbooks exist. But many of these books are very specialized to certain aspects of biomechanics. The more general textbooks are addressing mechanical or civil engineers or physicists who wish to specialize in biomechanics, so these books include chapters or sections on biology and physiology. Almost all books that were found are at Masters or post-graduate level, requiring basic to sophisticated knowledge of mechanics and mathematics. At a more fundamental level only books could be found that were written for mechanical and civil engineers.

We decided to write our own course material for the basic training in mechanics appropriate for our candidate biomedical engineers at Bachelor level, starting with the basic concepts of mechanics and ending with numerical solution procedures, based on the Finite Element Method. The course material assembled in the current book, comprises three courses for our biomedical engineers curriculum, distributed over the three years of their Bachelor studies. Chapters 1 to 6 mostly treat the basic concepts of forces, moments and equilibrium in a discrete context in the first year. Chapters 7 to 13 in the second year discuss the basis of continuum mechanics and Chapters 14 to 18 in the third year are focussed on solving the field equations of mechanics using the Finite Element Method.

What makes this book different from other basic mechanics or biomechanics treatises? Of course there is the usual attention, as in standard books, focussed on kinematics, equilibrium, stresses and strains. But several topics are discussed that are normally not found in one single textbook or only described briefly.

- Much attention is given to large deformations and rotations and non-linear constitutive equations (see Chapters 4, 9 and 10).
- A separate chapter is devoted to one-dimensional visco-elastic behaviour (Chapter 5).
- There is special attention to long slender fibre-like structures (Chapter 4).
- The similarities and differences in describing the behaviour of solids and fluids and aspects of diffusion and filtration are discussed (Chapters 12 to 16).
- Basic concepts of mechanics and numerical solution strategies for partial differential equations are integrated in one single textbook (Chapters 14 to 18).

Because of the usually rather complex geometries (and non-linear aspects) found in biomechanical problems hardly any relevant analytical solutions can be derived for the field equations and approximate solutions have to be constructed. It is the opinion of the authors that at Bachelor level at least the basis for these numerical techniques has to be addressed.

In Chapters 14 to 18 extensive use is made of a finite element code written in Matlab by one of the authors, which is especially developed as a tool for students. Applying this code requires that the user has a licence for the use of Matlab, which can be obtained via MathWorks (www.mathworks.com). The finite element code, which is a set of Matlab scripts, including manuals, is freely available and can be downloaded from the website: www.mate.tue.nl/biomechanicsbook.

Contents

1 Vector calculus

1.1 Introduction

Before we can start with biomechanics it is necessary to introduce some basic mathematical concepts and to introduce the mathematical notation that will be used throughout the book. The present chapter is aimed at understanding some of the basics of vector calculus, which is necessary to elucidate the concepts of force and momentum that will be treated in the next chapter.

1.2 Definition of a vector

A **vector** is a physical entity having both a magnitude (length or size) and a direction. For a vector $\vec{a}$ it holds, see Fig. 1.1:

$$\vec{a} = a\vec{e}. \tag{1.1}$$

The **length** of the vector $\vec{a}$ is denoted by $|\vec{a}|$ and is equal to the length of the arrow. The length is equal to a, when a is positive, and equal to $-a$ when a is negative. The **direction** of $\vec{a}$ is given by the unit vector $\vec{e}$ combined with the sign of a. The unit vector $\vec{e}$ has length 1. The vector $\vec{0}$ has length zero.

1.3 Vector operations

Multiplication of a vector $\vec{a} = a\vec{e}$ by a positive scalar α yields a vector $\vec{b}$ having the same direction as $\vec{a}$ but a different magnitude $\alpha|\vec{a}|$:

$$\vec{b} = \alpha\vec{a} = \alpha a\vec{e}. \tag{1.2}$$

This makes sense: pulling twice as hard on a wire creates a force in the wire having the same orientation (the direction of the wire does not change), but with a magnitude that is twice as large.

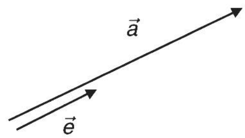

Figure 1.1

The vector $\vec{a} = a\vec{e}$ with $a > 0$.

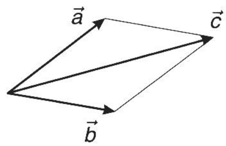

Figure 1.2

Graphical representation of the sum of two vectors: $\vec{c} = \vec{a} + \vec{b}$.

The **sum** of two vectors $\vec{a}$ and $\vec{b}$ is a new vector $\vec{c}$, equal to the diagonal of the parallelogram spanned by $\vec{a}$ and $\vec{b}$, see Fig. 1.2:

$$\vec{c} = \vec{a} + \vec{b}. \tag{1.3}$$

This may be interpreted as follows. Imagine two thin wires which are attached to a point P. The wires are being pulled at in two different directions according to the vectors $\vec{a}$ and $\vec{b}$. The length of each vector represents the magnitude of the pulling force. The net force vector exerted on the attachment point P is the vector sum of the two vectors $\vec{a}$ and $\vec{b}$. If the wires are aligned with each other and the pulling direction is the same, the resulting force direction is clearly coinciding with the direction of the two wires and the length of the resulting force vector is the sum of the two pulling forces. Alternatively, if the two wires are aligned but the pulling forces are in opposite directions and of equal magnitude, the resulting force exerted on point P is the zero vector $\vec{0}$.

The **inner product** or **dot product** of two vectors is a scalar quantity, defined as

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\phi), \tag{1.4}$$

where ϕ is the smallest angle between $\vec{a}$ and $\vec{b}$, see Fig. 1.3. The inner product is **commutative**, i.e.

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}. \tag{1.5}$$

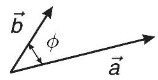

Figure 1.3

Definition of the angle ϕ.

The inner product can be used to define the length of a vector, since the inner product of a vector with itself yields ($\phi = 0$):

$$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}| \cos(0) = |\vec{a}|^2. \tag{1.6}$$

If two vectors are perpendicular to each other the inner product of these two vectors is equal to zero, since in that case $\phi = \frac{\pi}{2}$:

$$\vec{a} \cdot \vec{b} = 0, \text{ if } \phi = \frac{\pi}{2}. \tag{1.7}$$

The **cross product** or **vector product** of two vectors $\vec{a}$ and $\vec{b}$ yields a new vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a right-handed system. The vector $\vec{c}$ is denoted as

$$\vec{c} = \vec{a} \times \vec{b}\,. \tag{1.8}$$

The length of the vector $\vec{c}$ is given by

$$|\vec{c}| = |\vec{a}||\vec{b}| \sin(\phi)\,, \tag{1.9}$$

where ϕ is the smallest angle between $\vec{a}$ and $\vec{b}$. The length of $\vec{c}$ equals the area of the parallelogram spanned by the vectors $\vec{a}$ and $\vec{b}$. The vector system $\vec{a}$, $\vec{b}$ and $\vec{c}$ forms a right-handed system, meaning that if a corkscrew is used rotating from $\vec{a}$ to $\vec{b}$ the corkscrew would move into the direction of $\vec{c}$.

The vector product of a vector $\vec{a}$ with itself yields the zero vector since in that case $\phi = 0$:

$$\vec{a} \times \vec{a} = \vec{0}. \tag{1.10}$$

The vector product is **not** commutative, since the vector product of $\vec{b}$ and $\vec{a}$ yields a vector that has the opposite direction of the vector product of $\vec{a}$ and $\vec{b}$:

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}. \tag{1.11}$$

The **triple product** of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is a scalar, defined by

$$\vec{a} \times \vec{b} \cdot \vec{c} = (\vec{a} \times \vec{b}) \cdot \vec{c}. \tag{1.12}$$

So, first the vector product of $\vec{a}$ and $\vec{b}$ is determined and subsequently the inner product of the resulting vector with the third vector $\vec{c}$ is taken. If all three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-zero vectors, while the triple product is equal to zero then the