Fourth Edition

SIGNALS & SYSTEMS

Continuous and Discrete

Rodger E. Ziemer
William H. Tranter
D. Ronald Fannin

Signals and Systems: Continuous and Discrete

FOURTH EDITION

Rodger E. Ziemer
University of Colorado—Colorado Springs

William H. Tranter
Virginia Polytechnic Institute and State University

D. Ronald Fannin
University of Missouri—Rolla

PRENTICE HALL
Upper Saddle River, NJ 07458

Library of Congress Cataloging-in-Publication Data
Ziemer, Rodger E.
Signals and systems : continuous and discrete / Rodger E. Ziemer. William H. Tranter, D. Ronald Fannin. —4th ed.
p. cm.
Includes bibliographical references (p. –) and index.
ISBN 0-13-496456-x
1. System analysis 2. Discrete-time systems. 3. Digital filters (Mathematics) I. Tranter, William H. II. Fannin, D. Ronald. III. Title.
QA402.Z53 1998
621.382′2—dc21 97–44182
CIP

Publisher: **TOM ROBBINS**
Acquisitions editor: **ALICE DWORKEN**
Production editor: **ROSE KERNAN**
Editor-in-chief: **MARCIA HORTON**
Cover designer: **BRUCE KENSELAAR**
Director of production and manufacturing: **DAVID W. RICCARDI**
Managing editor: **BAYANI MENDOZA DE LEON**
Manufacturing buyer: **JULIA MEEHAN**

THE MATHWORKS, INC.
24 Prime Park Way
Natick, Massachusetts 01760-1500
Phone: (508) 647-7000
Fax: (508) 647-7001
Email: info@mathworks.com
http://www.mathworks.com

Printed in the United States of America

10 9 8 7 6 5 4 3

ISBN 0-13-496456-X

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Prentice-Hall, *Upper Saddle River, New Jersey*

SIGNALS AND SYSTEMS:
Continuous and Discrete

Preface

The philosophy of the previous edition of *Signals and Systems* is retained in the fourth edition by continuing to stress the systems approach so that students are provided the tools and techniques for understanding and analyzing both continuous-time and discrete-time linear systems. While the systems approach is applicable to a very broad class of problems, liberal examples based on traditional circuit theory are included in the book to illustrate the various systems analysis techniques that are introduced.

The most obvious difference between this edition of *Signals and Systems* and previous editions is the inclusion of MATLAB for solving many of the in-chapter worked examples and the addition of problems and exercises at the end of chapters that require the use of computational aids. The selection of MATLAB as a computer tool was an easy choice for us. Over the past several years MATLAB has become widely used within the engineering curricula and has been integrated into a number of courses. MATLAB provides very powerful computational and graphical capabilities. MATLAB code is very concise and as a result it is possible to express complex ideas using very few lines of code. The many toolboxes available provide the support necessary for investigating problems in a variety of areas. As a result, once a student has learned to use MATLAB effectively as a computational and visualization tool, the student can continue to use it throughout his or her career. The availability of a Student Edition that is low-cost, windows-based, and well-documented was considered a very important attribute of MATLAB for this application. All of the exercises included in this book can be worked using the Student Edition, Version 5.

In developing the MATLAB examples, problems, and computer exercises that appear in this edition we kept two considerations in mind. First we strongly believe that computer support can aid learning by allowing students to attack problems in different ways and to consider a wider variety of problems. Students still need to understand basic analysis and synthesis techniques and therefore need to work a large number of problems using traditional (pencil and paper) methods. We therefore chose to use MATLAB as a *supplement* with the in-chapter example problems rather than as a replacement for the traditional analysis techniques. Taking this approach also allows instructors to use the book without committing to the use of MATLAB. In this case, students having interest in the MATLAB applications can pursue the MATLAB applications as individual study. Students are encouraged to use MATLAB for problem solving where appropriate. For the most part, the end-of-chapter problems do not designate the use of MATLAB or any other tool for problem solving. We have, however, placed a section entitled "Computer Exercises" at the end of each chapter. Most of these exercises extend the text material and give the student an opportunity to bring some originality into the problem solving effort.

The second consideration was a very strong desire to keep it simple. MATLAB, as stated previously, is a very compact language. As a result it is possible to combine many operations into a single line of code. Such operations often lead to code that is very difficult to comprehend without considerable effort. We have therefore avoided complicated statements and data structures. In addition we have avoided the use of graphical user interfaces. The code contained in this book, therefore, is not very elegant but we hope that it is easy for the student to understand. The main effort of the student at this point should be to understand the concepts of signal and system theory. Spending considerable time writing code or trying to understand code is not the best use of student time.

The authors have developed a software supplement to *Signals and Systems: Continuous and Discrete,* 4e. This supplement contains copies of all m-files used in this book. This software is available free of charge via file transfer protocol (FTP) from The Mathwork's world wide web site. The files may be found at

```
ftp://ftp.mathworks.com/pub/books/tranter/
```

Another feature of the fourth edition of *Signals and Systems* is that all of the end-of-chapter summaries are written as a point-by-point review of the important topics covered in the chapter. Such a listing can be a valuable study guide for the student. In addition, a number of new problems are contained in the fourth edition.

The organization of the fourth edition is identical to that of the third edition. The first seven chapters deal with continuous-time linear systems in both the time domain and the frequency domain. References to discrete-time signals and systems are made where appropriate so that the students can appreciate the relationship between discrete-time and continuous-time signals and systems. The principal tool developed for time-domain analysis is the convolution integral. Frequency-domain techniques include the Fourier and the Laplace transforms. An introduction to state-variable techniques is also included, along with treatments of both continuous-time and discrete-time state equations. The remainder of the book deals with discrete-time systems, including z-transform analysis techniques, digital filter analysis and synthesis, and the discrete Fourier transform and fast Fourier transform (FFT) algorithms. A new section on computer-aided filter design has been added to Chapter 9.

This organization allows the book to be covered in two three-semester-hour courses, with the first course being devoted to continuous-time signals and systems and the second course to discrete-time signals and systems. Alternatively, the material can be used as a basis for three quarter-length courses. With this format, the first course could cover time- and frequency-domain analysis of continuous-time systems. The second course could cover state variables, sampling, and an introduction to the z-transform and discrete-time systems. The third course could deal with the analysis and synthesis of digital filters and provide an introduction to the discrete analysis and synthesis of digital filters and provide an introduction to the discrete Fourier transform and its applications. Other groupings of topics are possible.

The assumed background of the student is mathematics through differential equations and the usual introductory circuit theory course or courses. Knowledge of the basic concepts of matrix algebra would be helpful but is not essential. An appendix is included to bring together the pertinent matrix relations that are used in Chapters 6 and 7. We feel that in most electrical engineering curricula the material presented in this book is best taught at the junior level.

Plan of the Text

We begin the book by introducing the basic concepts of signal and system models and system classifications. The idea of spectral representations of periodic signals is first introduced in Chapter 1 because we feel that it is important for the student to think in terms of both the time and the frequency domains from the outset.

The convolution integral and its use in fixed, linear system analysis by means of the principle of superposition are treated in Chapter 2. This chapter deals with system modeling and analysis in the time domain. The evaluation of the convolution integral is treated in detailed examples to provide reinforcement of the basic concepts. Calculation of the impulse response and its relation to the step and ramp responses of a system are discussed. The concepts of system modeling and system simulation are also treated in Chapter 2. This chapter concludes with a section devoted to the numerical solution of system equations using SIMULINK.

The Fourier series is introduced in Chapter 3. We have emphasized the elementary approach of approximating a periodic function by means of a trigonometric series and obtaining the expansion coefficients by using the orthogonality of sines and cosines. We do this because this is the first time many students are introduced to the Fourier series. The alternative generalized orthogonal function approach is included as a section at the end of this chapter for those who prefer it. The concept of the transfer function in terms of sinusoidal steady-state response of a system is discussed in relation to signal distortion.

The Fourier transform is the subject of Chapter 4, with its applications to spectral analysis and system analysis in the frequency domain. The concept of an ideal filter, as motivated by the idea of distortionless transmission, is also introduced at this point. The Gibbs phenomenon, window functions, and convergence properties of the Fourier coefficients are also included. In addition, the DFT has been introduced in Chapter 4 to be covered at the option of the instructor.

The Laplace transform and its properties are introduced in Chapter 5. Again, we have tried to keep the treatment as simple as possible because this is assumed to be a first exposure to the material for a majority of students, although a summary of complex variable theory is provided in Appendix C so that additional rigor may be used at the instructor's option. The derivation of Laplace transforms from elementary pairs is illustrated by example, as is the technique of inverse Laplace transformation using partial-fraction expansion. Optional sections on the evaluation of inverse Laplace transforms by means of the complex inversion integral and an introduction to the two-sided Laplace transform are also provided.

The application of the Laplace transform to network analysis is treated in detail in Chapter 6. The technique of writing Laplace-transformed network equations by inspection is covered and used to review the ideas of impedance and admittance matrices, which the student will have learned in earlier circuits courses for resistive networks. The concepts of zero-state response and zero-input response are discussed along with their relationship to transient and steady-state responses. The transfer function is treated in detail, and the Routh test for determining stability is presented. The chapter closes with a treatment of Bode plots and block diagram algebra for fixed, linear systems.

In Chapter 7 the concepts of a state variable and the formation of the state-variable approach to system analysis are developed. The state equations are solved using both time-domain and Laplace transform techniques, and the important properties of the solution are examined. As an example, we show how the state-variable method can be applied to the analysis of circuits. Discrete-time state equations and the concepts of controllability and observability are briefly introduced.

The final three chapters provide coverage of the topics of discrete-time signal and system analysis. Chapter 8 begins with a study of sampling and the representation of discrete-time systems. The sampling operation is covered in considerable detail. This is accomplished in the context of formulating a model for an analog-to-digital (A/D) converter so that the operation of quantizing can be given some physical basis. A brief analysis of the effect of quantizing sample values in the A/D conversion process is included as an introduction to quantizing errors. As a bonus, the student is given a basis upon which to select an appropriate wordlength of an A/D converter. Both ideal and approximate methods for reconstructing a signal from a sequence of samples are treated in detail. The z-transform, difference equations, and discrete-time transfer functions are developed with sufficient rigor to allow for competent problem solving but without the complications of contour integration. The material on the classification of discrete-time systems has been written in a way that parallels the similar material for continuous-time systems.

Chapter 9 allows students to use their knowledge of discrete-time analysis techniques to solve an important class of interesting problems. The idea of system synthesis, as opposed to system analysis, is introduced. Discrete-time integration is covered in considerable detail for several reasons. First, the idea of integration will be a familiar one. Thus students can appreciate the different information gained by a frequency-domain analysis as opposed to a time-domain analysis. In addition, the integrator is a basic building block for many analog systems. Finally, the relationship between trapezoidal integration and the bilinear z-transform is of sufficient importance to warrant a discussion of trapezoidal integration. The synthesis techniques for digital filters covered in this chapter are the standard ones. These are synthesis by time-domain invariance, bilinear z-transform syntheses, and synthesis through Fourier series expansion. Through the application of these techniques, the student is able to gain confidence in the previously developed theory. Since several synthesis techniques depend on knowledge of analog filter prototypes, Appendix E, which discusses several different filter prototypes, is included. Computer-aided digital filter design is also included in this chapter for the first time in this edition.

The discrete Fourier transform (DFT) and its realization through the use of fast Fourier transform (FFT) algorithms is the subject of Chapter 10. Both decimation-in-time and decimation-in-frequency algorithms are discussed. Several examples are provided to give the student practice in performing the FFT operations. We believe that this approach best leads to a good understanding of the FFT algorithms and their function. Basic properties of the DFT are summarized and a comparison of the number of operations required for the FFT as compared to the DFT is made. Several applications of the DFT are summarized and the use of windows in supressing leakage is discussed. This chapter closes with a discussion and illustration of FFT algorithms with arbitrary radixes and the chirp-z transform.

The book also contains a number of optional sections, denoted by an asterisk, that go deeper into specific topics than is often customary for a junior-level course. These topics can be eliminated without loss of continuity.

A complete solutions manual, which contains solutions to all problems and computer exercises, is available from the publisher as an aid to the instructor. The solutions manual for the fourth edition is in a typeset format and makes extensive use of MATLAB for a large number of problems and for solution of computer exercises. Answers to selected problems are provided in Appendix H as an aid to the student.

Acknowledgments

We wish to thank the many people who have contributed, both knowingly and unknowingly, to the development of this textbook. Special thanks go to our many students who have provided a living laboratory within which we could test-teach many sections of this textbook. Their comments and criticisms have been very valuable and are gratefully appreciated. Special thanks are also due to our many faculty colleagues, now too many to mention, who taught from the first and second editions of this book and have offered many suggestions for improvement. Thanks are also due to the reviewers who provided many insightful comments and suggestions for improvement for the fourth edition.

The reviewers for the fourth edition were:

Ronald H. Brown, Marquette University
Malur K. Sundareshan, University of Arizona at Tucson

We also greatly appreciate the assistance of those who reviewed all or parts of previous editions, and we acknowledge their help and contributions to our efforts. These include:

Peter L. Balise, University of Washington; W. A. Blackwell, Virginia Tech; Peter Bofah, Howard University; Gordon E. Carlson, University of Missouri-Rolla; Ralph S. Carson, University of Missouri—Rolla; Hua Ting Chieh, Howard University; Mickey D. Cox, Louisiana Technological University;

F. Dianat, Rochester Institute of Technology; Jimmy L. Dodd, Mississippi State University; John L. Fleming, Texas A&M University; Victor Frost, University of Kansas; Neal Gallagher, Purdue University; Russell A. Hannen, Wright State University; Brian Harms, Kansas State University; Dennis W. Herr, Ohio Northern University; R. Dennis Irwin, Mississippi State University; K. Ross Johnson, Michigan Technological University; Bruce Johansen, Ohio Northern University; Mark Jong, Wichita State University; Fabien Josse, Marquette University; Saleem Kassam, University of Pennsylvania; John M. Liebetreu, University of Colorado—Colorado Springs; John H. Lilly, University of Kentucky; Wu-sheng Lu, University of Victoria; Rushdi Muammar, University of Illinois—Chicago; D. M. Petrela, SUNY Farmingdale; William A. Porter, Louisiana State University; Arifur Rahman, University of South Alabama; Michael Rudko, Union College; John V. Wait, University of Arizona; Mark Wickert, University of Colorado—Colorado Springs; and Chu Huiw Wu, University of Missouri—Rolla.

Last, but certainly not least, we wish to thank our wives, Sandy, Judy, and Sara, and our families for the patience shown as we worked on the fourth edition of this book. They have by now become well accustomed to our endless writing projects.

R. E. Z.
W. H. T.
D. R. F.

Contents

CHAPTER 1

Signal and System Modeling Concepts

1-1 Introduction

This book deals with *systems* and the interaction of *signals* in systems. A system, in its most general form, is defined as a combination and interconnection of several components to perform a desired task.† Such a task might be the measurement of the acceleration of a rocket or the transmission of a message from New York to Los Angeles. The measurement of the acceleration might make use of visual observation of its position versus time. An equally unsophisticated solution to the message delivery problem might use a horse and rider. Obviously, more complex solutions are possible (and probably better). Note, however, that our definition is sufficiently general to include them all.

We will be concerned primarily with *linear* systems. Such a restriction is reasonable because many systems of engineering interest are closely approximated by linear systems and very powerful techniques exist for analyzing them. We consider several methods for analyzing linear systems in this book. Although each of the methods to be considered is general, not all of them are equally convenient for any particular case. Therefore, we will attempt to point out the usefulness of each.

A *signal* may be considered to be a function of time that represents a physical variable of interest associated with a system. In electrical systems, signals usually represent currents and voltages, whereas in mechanical systems, they might represent forces and velocities (or positions).‡ In the example mentioned above, one of the signals of interest represents the acceleration, but this could be integrated to yield a signal proportional to velocity. Since electrical voltages and currents are relatively easy to process, the original signal representing acceleration, which is a mechanical signal, would probably be converted to an electrical one before further *signal processing* takes place. Examples illustrating these remarks will be given in the next section.

Just as there are several methods of systems analysis, there are several different ways of representing and analyzing signals. They are not all equally convenient in any particular situation. As we study methods of signal representation and system analysis we will attempt to point out useful applications of the techniques.

So far, the discussion has been rather general. To be more specific and to fix more clearly the ideas we have introduced, we will expand on the acceleration measurement and the message delivery problems already mentioned.

†The Institute of Electrical and Electronics Engineers dictionary defines a system as "an integrated whole even though composed of diverse, interacting structures or subjunctions."

‡More generally, a signal can be a function of more than one independent variable, such as the pressure on the surface of an airfoil, which is a function of three spatial variables *and* time.