CSIR REPORT CENG 467

A DISCRETE MODEL FOR THE FLOW DISTRIBUTION IN MANIFOLDS

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CHEMICAL ENGINEERING RESEARCH GROUP – CSIR

COUNCIL for SCIENTIFIC and INDUSTRIAL RESEARCH

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SYNOPSIS

A review of the state of the art indicated the need for a widely applicable discrete model for the flow and pressure distribution in manifolds, allowing for friction in the headers and laterals as well as for flow profile asymmetries, taking interaction between laterals (downstream and upstream) into account and also covering coupled manifolds.

The developed model meets these requirements. It is based on the integral form of the momentum balance and requires flow profile information. The model results in a large system of non-linear equations solvable by an iterative matrix procedure using the multidimensional Newton rootfinder method.

'N DISKRETE MODEL VIR DIE VLOEIVERDELING IN SPRUITSTUKKE

SINOPSIS

'n Oorsig van die stand van sake het die behoefte aangedui vir 'n wyd toepasbare diskrete model vir die vloei- en drukverdeling in spruitstukke, wat wrywing in hoofpype en sytakke, asook vloeiprofiel-asimmetrieë toelaat, interaksie tussen sytakke (stroomaf en stroomop) in aanmerking neem, en ook gekoppelde spruitstukke insluit.

Die ontwikkelde model voldoen aan hierdie vereistes. Dit is op die integrale vorm van die momentumbalans gebaseer en vereis inligting oor die vloeiprofiele. Die model lewer 'n groot stelsel van nie-liniêre vergelykings op wat met 'n iteratiewe matriksprosedure, wat die meerdimensionale Newton-wortelbepalingsmetode gebruik, oplosbaar is.

KEYWORDS: Distribution, flow, pressure, manifold, discrete, model.

File No: 660–43–2

C O N T E N T S

1. INTRODUCTION

Manifolds are widely used in the process industry, in irrigation systems and in various other fields. They can be classified into two main types:

i. the dividing flow, or "blowing" manifolds for the distribution of a fluid to multiple laterals, and

ii. the combining flow, or "sucking" manifolds in which fluid is collected from laterals.

Often manifolds of both types are coupled through a system of mutual laterals with, for instance, a unit of process equipment in each lateral. The coupled manifolds form either a "Z" or a "U" formation (see Figure 1).

Design problems – A generally valid design procedure for manifolds is not available. Efforts have been made to develop correlations for the head loss coefficients of single branching junctions. However, a wide spread exists in the available data because different geometrical factors prevailed in the various sets of experiments.

Further complications may be encountered when multiple laterals are present. This is the case in manifolds where the laterals are spaced so closely that the flow pattern caused by the presence of one lateral influences the flow division at the next lateral and vice versa. This problem arises to a great extent in, for instance, plate heat exchangers and electrodialysis stacks where many laterals are separated by only a few millimetres. The existence of coupled manifolds, as present in such equipment, causes additional complications.

Flow patterns – Attempts to account for the mutual interaction of adjacent laterals, using data obtained from pressure tappings close to the junctions, were not very successful. The reason for this is that each lateral distorts the velocity profile in the header so that it becomes asymmetrical. The flow at each junction comprises separation, recirculation and reattachment zones, each depending on the geometry and on the approaching flow profile. The flow situation is extremely complex.

A simple model adding the effects of each individual junction will lead to unacceptable deviations in many cases. Therefore, a general manifold flow model, which is capable of accounting for the flow phenomena mentioned previously, is needed. This requires insight into the local flow structure in junctions.

Turbulent flow prevails in most of the manifolds in industry. General flow modelling has the potential of providing information on the turbulent flow structure in simple geometries. This might be applicable to individual T-junctions, but, at present, it is not feasible to model the turbulent flow structure in a complex system such as a manifold. For an overall flow distribution model for the whole manifold that covers the mutual interaction of junctions, a different approach must be used.

Discrete versus continuum models – Another important aspect is the discontinuous character of manifold flow. A discrete flow distribution model describes the physical situation better than a continuum flow distribution model. The latter may be advantageous because of simplicity, but in most cases it will be only an approximation. Furthermore, geometrical non-uniformities cannot be dealt with.

In this report a novel discrete manifold flow distribution model is developed.

FIGURE 1a Blowing or dividing flow manifold

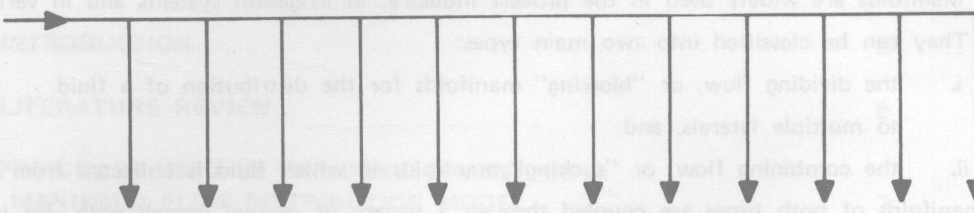


FIGURE 1b Sucking or combining flow manifold

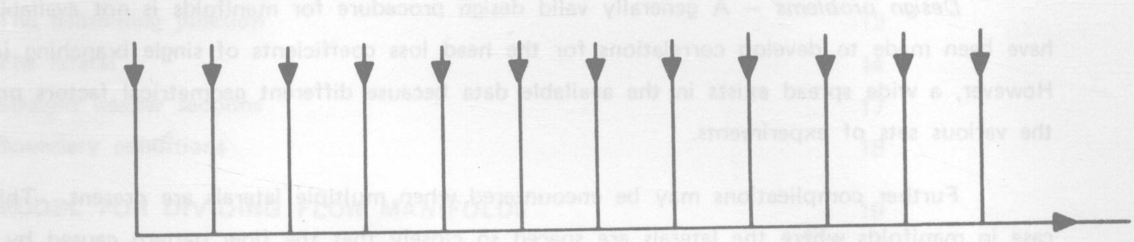


FIGURE 1c Combined manifolds, Z configuration

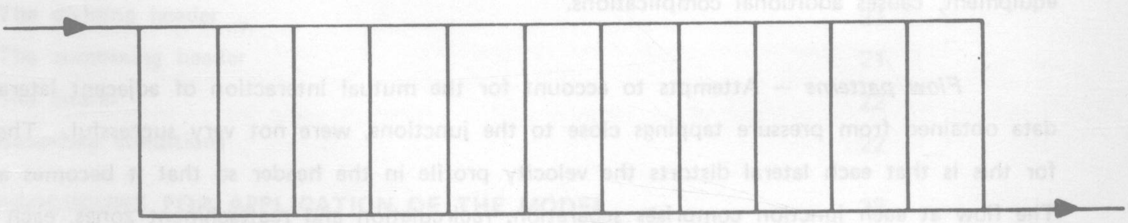
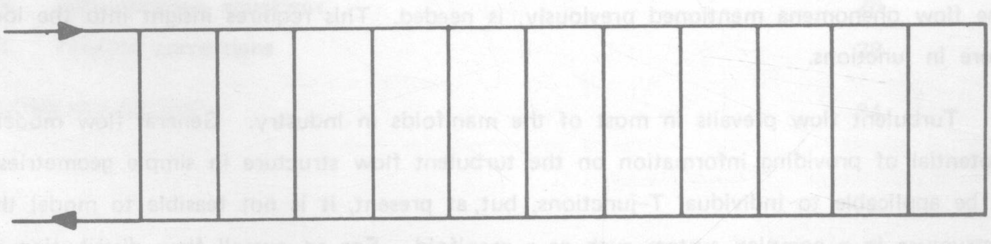


FIGURE 1d Combined manifold, U configuration



2. LITERATURE REVIEW

This review covers mainly mathematical models for flow distribution in manifolds and not the literature on experimental work on junctions and manifolds. A summary of these models is given in Table 1.

An extensive review of data on combining and dividing junctions with a variety of geometries is given by Miller[1].

Aspects covered by models – The mathematical models which have been proposed are either continuum models, in which the laterals are assumed to be spread over the length of a header as in a porous tube, or discrete models, in which the flow splits at individual laterals are considered. Interactions between junctions may or may not be accounted for. Some models take flow profile distortion, which is caused by the asymmetrical branching of the flow, into account. Most of the models make allowance for friction in the header and/or laterals. Some models deal with dividing flow, others with combining flow or with both.

Basis of models – Early models were based on an energy balance using head loss coefficients at each junction. The Bernoulli equation is used for the energy balance, but this is not entirely correct as it is a condition for the application of this equation that a relevant streamline is defined. This condition cannot be fulfilled in branching flow.

Therefore, there was a shift to the use of a momentum equation to express a local momentum balance. For the momentum equation a simplified version of the equations of motion (Navier-Stokes equations) was used.

Further progress was made by the use of the integral form of the momentum balance. This integral form can be derived from the Navier-Stokes equations in combination with the continuity equation and applying Gauss' divergence theorem. The result is a surface integral over a control volume. Such "global" description of branching flow is much more useful than that derived from a local momentum balance.

The point about the incorrect application of the Bernoulli equation in branching flow made earlier is not as important when considering that all models have to be complemented by experimental correlations which account for the deviation from the basic model.

Discrete versus continuum models – A discrete model results in non-linear difference equations. These are more difficult to solve than the differential equations obtained when using a continuum model. However, the latter considers pressures and velocities as variables changing gradually along the header whereas a real manifold is physically a discontinuous system. A continuum model applies physically only in the case of a porous header.

Discussion – The comparison with experiments was reasonable to good in all cases. This does not imply that these models are generally applicable, because all of them were fitted to the experimental data.

TABLE 1 Summary of manifold flow distribution models

Authors	Reference	Type of model	Basis	Configuration	Friction	Junction interaction	Profile parameters	Comparison with experiments
Miller	(1)	Discrete	Energy balance	Dividing/ combining	No	No	No	No
Enger and Levy	(2)	Continuum	Local momentum balance	Dividing	No	–	No	No
V/d Hegge-Zijnen Markland Keller	(3) (4) (5)	Continuum	Local momentum balance	Dividing/ combining	Yes	–	No	No
Acrivos et al.	(6)	Discrete/ continuum	Local momentum balance	Dividing/ combining	Only in header	No	One overall factor	Yes
Kubo and Ueda	(7)	Discrete	Local momentum balance	Dividing/ combining	Only in laterals	Yes	One overall factor	Yes
Bajura	(8)	Continuum	Integral momentum balance	Dividing/ combining	Yes	Yes	One overall factor	Yes
Bajura and Jones	(9)	Continuum	Integral momentum balance	Coupled	Yes	Yes	Yes	Yes
Majumdar	(10)	Discrete	Local momentum balance	Dividing/ combining	Yes	Yes	One obtained from Ref. (7)	With those of References (6) and (7)
Datta and Majumdar	(11)	Discrete	Local momentum balance	Coupled	Yes	Yes	One obtained from Ref. (8)	With those of Ref. (9)
Hudson et al.	(12)	Discrete	Energy balance	Dividing	No	Yes	No	No

Two major trends can be recognized: references (8) and (9) give much attention to the actual flow profile behaviour in their continuum models, whereas in references (10) and (11) the main point is the discrete formulation, more or less neglecting flow profile asymmetries.

Conclusion – The real problem in manifold flow modelling is the complexity of the flow which is present, independent of which model is chosen. Using the Bernoulli equation the correlations of the experimental observations give expressions for the losses owing to viscous dissipation. These losses are difficult to quantify generally, except by numerous experiments to cover the geometrical factors as well. Using the equations of motion, a local momentum balance is obtained as the basis for the global information sought; this is not very suitable.

A generally applicable discrete model, allowing for friction in headers and laterals, allowing for profile asymmetries, taking interactions between laterals into account and covering coupled manifolds as well is the real requirement.

3. PRINCIPLES USED IN THE DEVELOPMENT OF A MANIFOLD FLOW DISTRIBUTION MODEL

The basic principle used in this model is to structure the whole system of manifolds and laterals in adjoining momentum control volumes.

The position and shape of these control volumes is such that:

- the only external forces working on the fluid inside a control volume are wall shear stresses,
- the information is concentrated at positions which need good resolution, ie, the junction ports.

The integral form of the momentum balance is applied to each of these control volumes. From each momentum balance an equation with velocity and pressure variables can be found. The adjoining structure of control volumes results in continuity of velocity and pressure at the boundaries. This results in a system of equations that must be solved.

From each momentum balance a number of coefficients arise; these are the momentum correction factors. They represent the actual flow profile in the branching junctions. These factors are not a part of the solution, they must be known beforehand.

The model covers the four configurations pointed out in Section 1:

- dividing flow manifold
- combining flow manifold
- coupled manifolds, Z configuration
- coupled manifolds, U configuration

and the following points are allowed:

- non-uniform lateral spacing
- different length, cross-section and friction factor for each lateral
- different cross-section for dividing and combining header
- different friction factor for each header section
- arbitrary angles between headers and laterals (see Figure 7)
- dividing and combining headers need not be geometrically parallel.

For simplicity, however, only parallel headers and right-angled junctions are drawn in the figures.

Because of the formulation in terms of integral momentum balances the flow profiles can be arbitrary and, therefore, the junction shapes can be arbitrary too. Momentum correction factors account for this. They are evaluated at the positions where good resolution is needed, ie, at the junction ports. At each junction port about four variables and four correction factors should be considered. This is approximately twice as many as in other available methods.

The correction factors are the link between the manifold flow distribution model and the actual flow behaviour. They can easily be quantified by local velocity information. The latter can be obtained by Laser-Doppler velocimetry methods and by turbulent flow modelling.

At this stage the model is restricted to systems which do not include any local resistances such as valves, bends or reducers in the headers and the laterals. Thus, the manifold(s) and laterals are all in one plane. However, the model may be extended to include local resistances at a later stage.

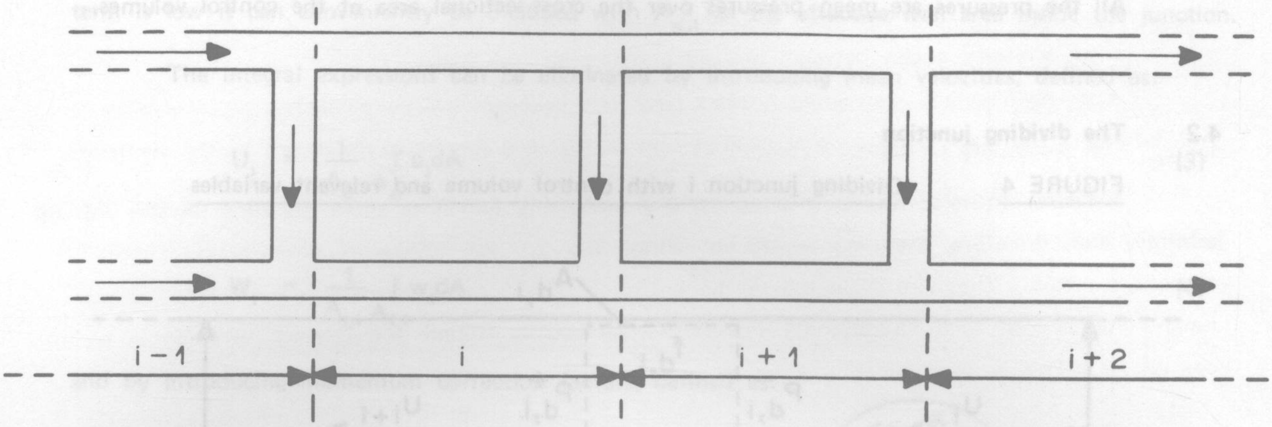
4. MODEL FOR COUPLED MANIFOLDS IN Z CONFIGURATION

This configuration consists of a dividing header and a combining header, coupled by laterals. The flow in the two headers is parallel (see Figure 2).

4.1 Conventions

The two manifolds are coupled by N laterals, numbered 1,2, ... i, ... N. Therefore, the system is divided into N sections, also numbered 1,2, ... i, ... N. Each section i includes lateral i, junctions connected to lateral i, and the two header sections directly upstream of lateral i (see Figure 2). The subscript i on a variable refers to section i.

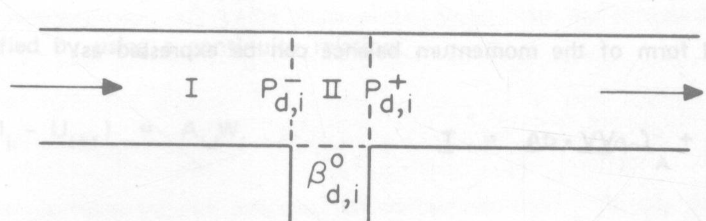
FIGURE 2 Subdivision of the Z configuration in sections



Some quantities, such as U_i, V_i, W_i (velocities), $D_{l,i}$, $A_{l,i}$, L_i (areas, lengths) or $f_{d,i}$, $f_{c,i}$, $f_{l,i}$ (friction factors) are related to straight sections, eg, header sections and laterals.

Other quantities, such as $P^-_{d,i}$, $P^+_{d,i}$, $P^-_{c,i}$, $P^+_{c,i}$ (pressures), $\beta^-_{d,i}$, $\beta^+_{c,i}$, $\gamma_{d,i}$ (correction factors) and u^-_i, u^0_i, u^+_i (junction branch velocities) are related to the junctions themselves. Because these variables are different at each branch of the junction superscripts −, 0, and + are introduced to indicate that the variable is valid at the upstream header branch (−), the lateral branch (0) and the downstream header branch (+) at the boundary of the control volumes. Therefore, $P^-_{d,i}$ is the pressure just before junction i at the boundary of the two control volumes I and II (see Figure 3).

FIGURE 3 Dividing junction i



The subscripts d, l and c are introduced to indicate that the variables refer to the dividing header (d), lateral (l) and combining header (c) respectively.

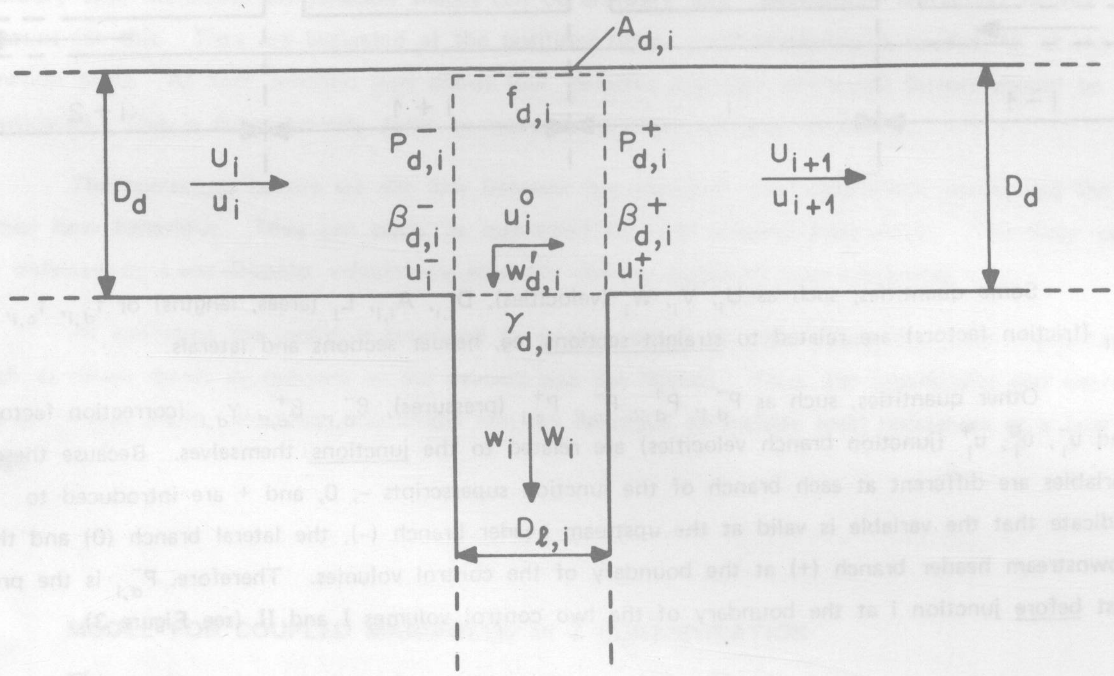
Therefore, $f_{c,i}$ is the friction factor in section i of the combining header. However, the symbols U (dividing header), V (combining header) and W (lateral) are used for velocities.

The fluids are considered as incompressible; this is valid for liquids and also for gases if the velocities are lower than approximately one third of the sound velocity.

All the pressures are mean pressures over the cross-sectional area of the control volumes.

4.2 The dividing junction

FIGURE 4 Dividing junction i with control volume and relevant variables



The integral form of the momentum balance can be expressed as:

$$\int_A P d\underline{A} + \int_A \rho \underline{V}\underline{V} \bullet d\underline{A} = \underline{T} \tag{1}$$

where the vector $\underline{T}$ is the total force working on the fluid inside the control volume. The integrations of pressure and momentum flow are over the surface of the control volume, where $d\underline{A}$ is the outward pointing vector.

When the component of Equation (1) is taken in the direction of the flow in the dividing header, the following equation is obtained:

$$-P^-_{d,i}A_d + P^+_{d,i}A_d - \rho \int_A \overline{u^-_i}^2 dA + \rho \int_A \overline{u^+_i}^2 dA + \rho \int_{A_{,i}} u^0_i w'_{d,i} dA + \frac{f_{d,i}}{8} \rho U_i^2 A_{d,i} = 0 \tag{2}$$

where the last term represents the shear stress inside the junction, which is the only force working on the fluid in the control volume. The shear stress is evaluated with the upstream mean velocity and with the friction factor of the upstream pipe section. Although the relative importance of this term is low it can conveniently be included with $A_{d,i}$ as the effective wall area inside the junction.

The integral expressions can be eliminated by introducing mean velocities, defined as:

$$U_i = \frac{1}{A_d} \int_{A_d} u_i dA \tag{3}$$

$$W_i = \frac{1}{A_{l,i}} \int_{A_{l,i}} w_i dA \tag{4}$$

and by introducing momentum correction factors, defined as:

$$\beta^-_{d,i} = \frac{1}{A_d U^2} \int_{A_d} u_i^{-2} dA \tag{5}$$

$$\beta^+_{d,i} = \frac{1}{A_d U^2_{i+1}} \int_{A_d} u_i^{+2} dA \tag{6}$$

$$\gamma_{d,i} = \frac{1}{A_{l,i} U_i W_i} \int_{A_{l,i}} u_i^0 w'_{d,i} dA \tag{7}$$

Now Equation (2) can be written as:

$$A_d(P^+_{d,i} - P^-_{d,i}) + \rho A_d(\beta^+_{d,i}U^2_{i+1} - \beta^-_{d,i}U^2_i) + \rho A_{l,i}\gamma_{d,i}U_iW_i + \frac{f_{d,i}}{8}\rho U^2 A_{d,i} = 0 \tag{8}$$

This can be simplified by using a continuity relation:

$$A_d(U_i - U_{i+1}) = A_{l,i}W_i \tag{9}$$

which gives:

$$\beta^+_{d,i}U^2_{i+1} - \gamma_{d,i}U_iU_{i+1} + U_i^2\left(\gamma_{d,i} - \beta^-_{d,i} + \frac{f_{d,i}}{8}\frac{A_{d,i}}{A_d}\right) + \frac{1}{\rho}(P^+_{d,i} - P^-_{d,i}) = 0 \tag{10}$$

Equation (10) expresses the exchange between momentum and pressure inside the junction influenced by losses owing to shear stresses and to turning effects (terms with $\gamma_{d,i}$).

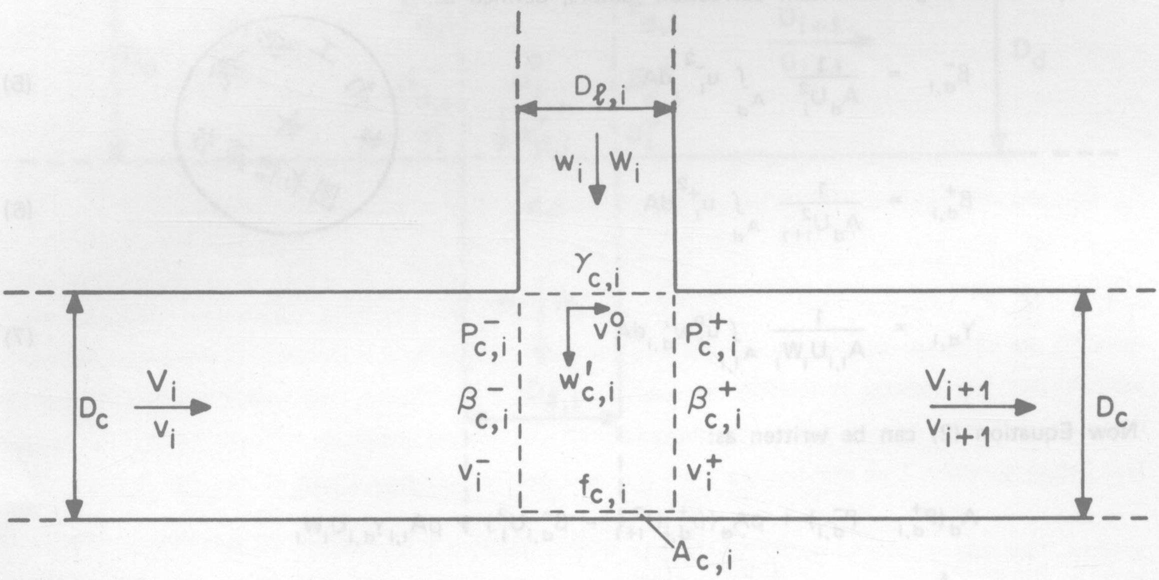
Because mean velocities are used momentum correction factors arise. These express the influence of the actual velocity profiles at the ports of the junction.

NOTE: The surface $A_{l,i}$ in Equation (7) can be a curved surface, dependent on the shape of the junction; in fact, the total axial momentum outflux is considered.

4.3 The combining junction

A similar equation can be derived for the combining junction, where, opposite to the dividing junction, there is inflow from the lateral (see Figure 5).

FIGURE 5 Combining junction i with control volume and relevant variables



Applying the momentum balance on the control volume in the direction of the header flow gives:

$$-P^-_{c,i}A_c + P^+_{c,i}A_c - \rho\int_{A_c} v_i^{-2}\,dA + \rho\int_{A_c} v_i^{+2}\,dA - \rho\int_{A_{l,i}} v_i^o w'_{c,i}\,dA$$

$$+ \frac{f_{c,i}}{8}\rho V^2 A_{c,i} = 0 \qquad (11)$$

where the shear stresses are also evaluated with the upstream velocity. The area $A_{c,i}$ is the effective wall area inside the combining junction.

Again, introducing a mean velocity V_i and momentum correction factors, defined as:

$$V_i = \frac{1}{A_c}\int_{A_c} v_i\,dA \qquad (12)$$

$$\beta^-_{c,i} = \frac{1}{A_c V_i^2} \int_{A_c} v_i^{-2} \, dA \qquad (13)$$

$$\beta^+_{c,i} = \frac{1}{A_c V_{i+1}^2} \int_{A_c} v_i^{+2} \, dA \qquad (14)$$

$$\gamma_{c,i} = \frac{1}{A_{l,i} V_i W_i} \int_{A_{l,i}} v_i^o w'_{c,i} \, dA \qquad (15)$$

Equation (11) can now be written as:

$$A_c(P^+_{c,i} - P^-_{c,i}) + \rho A_c(\beta^+_{c,i} V^2_{i+1} - \beta^-_{c,i} V_i^2) - \rho A_{l,i} \gamma_{c,i} V_i W_i$$
$$+ \frac{f_{c,i}}{8} \rho V_i^2 A_{c,i} = 0 \qquad (16)$$

Using a second continuity relation:

$$A_c(V_{i+1} - V_i) = A_{l,i} W_i \qquad (17)$$

gives:

$$\beta^+_{c,i} V^2_{i+1} - \gamma_{c,i} V_i V_{i+1} + V_i^2(\gamma_{c,i} - \beta^-_{c,i} + \frac{f_{c,i}}{8} \frac{A_{c,i}}{A_c})$$
$$+ \frac{1}{\rho}(P^+_{c,i} - P^-_{c,i}) = 0 \qquad (18)$$

In the case of the Z configuration, Equation (18) for the combining junction is exactly the same as Equation (10) for the dividing junction. However, this does not imply that the effects of pressure and velocity distribution always are equal for both junctions, and thus for both headers. It can be shown that Equations (10) and (18) depend strongly on the factors $U_{i+1} - U_i$ (dividing junction) or $V_{i+1} - V_i$ (combining junction) which have an opposite sign. Therefore, in cases where friction plays a minor role, the pressure distribution is opposite in both headers. This causes the non-uniform flow distribution which is well known in the case of a Z configuration.

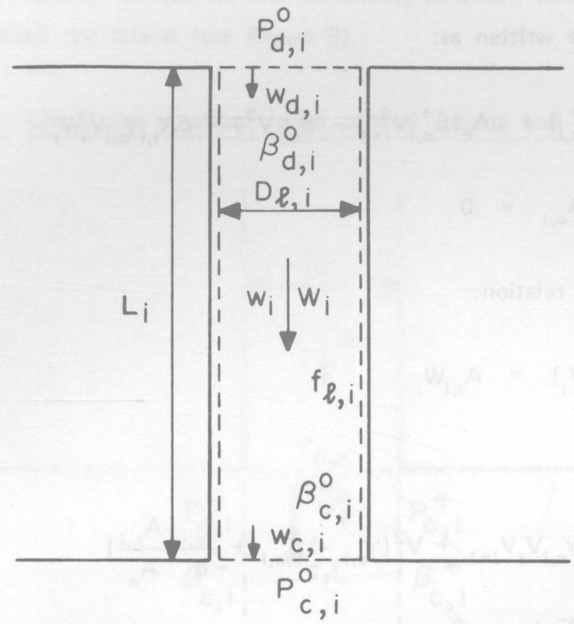
When friction is neglected the influence of the terms with γ can be shown in the following way. In the dividing junction these terms represent the fact that the decrease in momentum is not fully compensated for by the increase in pressure. The rise in pressure is not high enough; therefore, these terms can be regarded as turning losses. This can be explained by the fact that some fluid is leaving the control volume taking momentum in the header direction with it. In the combining header the result is that the increase in momentum is not fully compensated for by the decrease in pressure. Thus the pressure drop is too little, so these terms can be regarded here as turning gains. In the latter case some fluid enters the control volume already having momentum in the header direction. These different effects also result from the opposite sign of $U_{i+1} - U_i$ and $V_{i+1} - V_i$.

NOTE: Surface $A_{l,i}$ in Equation (15) can be a curved surface as well, dependent on the shape of the junction. Here the total axial momentum influx is considered.

4.4 The lateral

The lateral i is the connection between dividing junction i and combining junction i. The momentum balance in the direction of the lateral flow is applied on the control volume, shown in Figure 6.

FIGURE 6 Lateral i



This gives:

$$-P^o_{d,i} A_{l,i} + P^o_{c,i} A_{l,i} - \rho \int_{A_{l,i}} w^2_{d,i}\, dA + \rho \int_{A_{l,i}} w^2_{c,i}\, dA$$

$$+ \frac{f_{l,i}}{8} \pi D_{l,i} L_i \rho W_i^2 = 0 \quad (19)$$

and by introducing another two momentum correction factors:

$$\beta^o_{d,i} = \frac{1}{A_{l,i} W_i^2} \int_{A_{l,i}} w^2_{d,i}\, dA \quad (20)$$

$$\beta^o_{c,i} = \frac{1}{A_{l,i} W_i^2} \int_{A_{l,i}} w^2_{c,i}\, dA \quad (21)$$