

OPERATIONAL AMPLIFIER CHARACTERISTICS AND APPLICATIONS

ROBERT G. IRVINE

California State Polytechnic University, Pomona

Prentice-Hall, Inc., Englewood Cliffs, NJ 07632

Library of Congress Cataloging in Publication Data

Irvine, Robert, 1931-

Operational amplifier characteristics and applications.

Bibliography: p.

Includes index.

I. Operational amplifiers. I. Title.

TK7871.58.06178 621.3815'35 80-25860

ISBN 0-13-637751-3

Editorial/production supervision and interior design by Mary Carnis
Manufacturing buyer: Anthony Caruso

© 1981 by Prentice-Hall, Inc., Englewood Cliffs, N. J. 07632

All rights reserved. No part of this book
may be reproduced in any form or
by any means without permission in writing
from the publisher.

Printed in the United States of America

10 9 8 7 6 5

Prentice-Hall International, Inc., *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall of Canada, Ltd., *Toronto*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall of Southeast Asia Pte. Ltd., *Singapore*
Whitehall Books Limited, *Wellington, New Zealand*

Preface

For years during the 1940s and 1950s, George Philbrick had developed his company around the vacuum-tube operational amplifier. It was a self-contained device with two dual triodes in a phenolic housing, which in turn plugged into an octal socket. It was a mystery to many electrical and electronics engineers just what this device did when used in an instrumentation circuit.

In 1948, the transistor was developed; by 1954 it was in some general use by specific industries but was still a novelty to most engineers. Companies such as Philbrick, Burr-Brown, and others began to develop operational amplifiers in small modules using transistors. The use of an operational amplifier in an electronic circuit was still a specialized application not frequently employed.

Then, in 1965, an engineer named Bob Widlar, working for Fairchild, developed an operational amplifier as a monolithic integrated circuit chip. It was called the μA 709. Almost immediately, engineers who had no intentions of ever using an operational amplifier found one in a neat little 8-pin TO-5 can. It was an overnight success, and the 709 became famous along with Fairchild and Mr. Widlar, who later joined National Semiconductor and led them into the forefront of integrated-circuit technology.

Today, the operational amplifier is in almost as common usage as the transistor was in 1965. And as the technology progresses, the operational amplifier will surely overshadow the transistor in new designs.

It therefore behooves anyone interested in electronics, to become as familiar with the integrated operational amplifier as he or she is with the transistor. His or her future value to the industry depends upon this capability.

This text provides a thorough investigation of the design rules for operational amplifiers. Even though the reader will seldom, if ever, be required to design a monolithic operational amplifier, it is imperative that the reasons for design criteria be understood so that quantitative as well as qualitative judgments can be made by the user. When feedback is placed around an operational amplifier, it tends to mask all but the most obvious of failures. If the user is not equipped to make quantitative judgments about the operation, a failure exhibiting a subtle change in operational characteristics may go unnoticed.

Material usually omitted from texts at the technology level is included in this text. The examples and problems are written around real-life applications and situations. An extensive group of derivations, which support the equations in the text, is provided in an appendix. The use of this appendix actually upgrades the level of the text to include material not usually found in an engineering technology or engineering-level text. It also allows the more inquisitive student to seek out the roots of the equations given without searching through innumerable references. It is the intention of the author that this text should stand alone, at its level, and include all the supporting reference material on the subject of operational amplifiers.

The book was written for two groups of people. First, for the student pursuing a formal education at a college or university. Second, for the person practicing in the field of engineering. The book is a teaching text. It was not specifically meant for self-study, but the presentation and worked examples make that a viable option for those inclined to do so.

This text is taught by the author in two quarters: Chapters 1 through 6 are taught in the first quarter, and Chapters 7 through 11 are taught in the second quarter, both at the junior level.

The text could be used for a semester course by either including Chapter 7 for the more comprehensive course or by beginning at Chapter 2 and proceeding through Chapter 11, excluding Chapter 9 (active filters). Chapter 9 should be used only in the more comprehensive course. It takes between three and four weeks to present this chapter as it is written. Section 2.6 should be excluded from the presentation in Chapter 2; it should then be presented just before the phase-locked-loop section in Chapter 11.

The problems are written to correlate closely with the presentation and examples in the chapters. Wherever possible, problems are presented which parallel the examples in method of solution. Where the problem is difficult, or an ambiguity seems to exist, the problem is worded exactly as in the example to assure clarity. SI metrics are used throughout the book. This only affects electrical students in nomenclature and in conductance, where mho (Ω) is replaced by the SI metric unit Siemen (S). No mathematics beyond algebra is required for solution of any of the problems or examples. Where differentiation or integration is required, the solution is given and the student is then asked to understand examples or work problems using that solution.

Chapter 9 is presented in such a manner as to enhance the student's understanding without diminishing the quality of presentation. The six major filter types are all presented, and examples and problems using these ensure a feeling of ease by the student after finishing the chapter. A special method of performing the required

mathematics has been developed to prevent the student from becoming bogged down in math and not seeing the function being analyzed. In every case, the pole-zero diagram is shown opposite the filter's frequency response to give the student a "feeling" for the numbers in the equations and the pole locations for a particular filter type.

The book has been written to give students the tools with which to analyze and understand unfamiliar or special op amp circuits which will be encountered during the course of a career. Almost all circuits encountered by the author in industry were combinations or variations of the classical circuits presented in this book. A solutions manual, with all problems worked in detail, is available to instructors and those practicing engineering.

ACKNOWLEDGMENTS

This book began as a one-man effort. Very soon, though, it became evident that outside help would speed its completion. Thus the following people have contributed to the manuscript during the 27 months of its development.

Typists: Vivian Peeters, Arlene Marino, Betty Lord, and Sally Wiger.

Drawings: Geoffrey Blaha and Henry Hanzen.

Professional consulting: on derivations and references

Don B. Smedley, Earl E. Schoenwetter, Mark W. Dunbar, and

Dr. Alan Felzer.

Professors who have taught from the manuscript and provided invaluable critiques:

Mark W. Dunbar, Kenneth Finster, Richard Cockrum, and

Earl E. Schoenwetter.

Over 300 students have provided written critiques of the manuscript, which have been incorporated to correct errors and enhance understanding.

Safetran Systems Corporation, specifically Richard Peel, Chief Engineer, for the circuit-board photographs in Chapters 6 and 9.

To my wife, Joan G. Irvine, for infinite patience.

To my sister, Jane I. Stibal, for guiding me on English grammar in Chapters 1 and 2.

To Dr. Beaumont Davison, Dr. Richard J. Hermsen, Professor James T. Todd and Dr. David L. Clark for providing an atmosphere conducive to the preparation of this text.

*Robert G. Irvine
Claremont, California*

Contents

PREFACE

xiii

1 CONSTRUCTION OF THE OPERATIONAL AMPLIFIER

1

- 1.1 Introduction, 1
- 1.2 Notation, 1
- 1.3 SI Metric Units, 2
- 1.4 Differential-Amplifier Operation, 2
 - 1.4.1 Common-Mode Voltages, 3
 - 1.4.2 Differential Voltages, 6
 - 1.4.3 Differential and Common-Mode Voltages, 8
 - 1.4.4 Common-Mode Rejection Quotient, 14
- 1.5 Bipolar Transistor Differential Amplifier, 15
 - 1.5.1 Common-Mode Gain Equation, 15
 - 1.5.2 Differential Gain Equation, 17
 - 1.5.3 Common-Mode Rejection Quotient, 19
- 1.6 Common-Mode Input Voltage Range, 20
- 1.7 Operational Amplifier Realization, 21
 - 1.7.1 Monolithic Circuit Elements, 21
 - 1.7.2 Differential Amplifier, 23
 - 1.7.3 Level Shifter, 23
 - 1.7.4 Output Stage, 23
 - 1.7.5 The Operational Amplifier, 24
 - 1.7.6 Complementary MOSFET Operational Amplifier, 25
 - 1.7.7 Operational Amplifier Symbol, 27
- 1.8 Common-Mode Rejection Ratio, 27

vii

2	USING THE OPERATIONAL AMPLIFIER AS A CIRCUIT ELEMENT	35
2.1	Introduction, 35	
2.2	Operational Amplifier, 35	
2.3	Inverting Amplifier Gain, 37	
2.4	Noninverting Amplifier Gain, 40	
	2.4.1 <i>Noninverting Amplifier with Voltage-Divider Input</i> , 45	
	2.4.2 <i>Rule 5 Combines Rules 2 and 3 or 4</i> , 46	
2.5	Excess Loop Gain, 50	
2.6	Representing the Op Amp Circuit as a Control System, 52	
	2.6.1 <i>Inverting Amplifier</i> , 52	
	2.6.2 <i>Noninverting Amplifier</i> , 55	
2.7	Special Cases of Op Amp Amplifiers, 56	
	2.7.1 <i>Noninverting Amplifier with Unity Gain (Voltage Follower)</i> , 56	
	2.7.2 <i>Inverting Summer</i> , 57	
	2.7.3 <i>Noninverting Averager</i> , 59	
2.8	Op Amp-Circuit Impedance Modification through Feedback, 61	
	2.8.1 <i>Input Impedance of the Noninverting Amplifier</i> , 61	
	2.8.2 <i>Output Impedance of the Op Amp Circuit</i> , 63	
3	OFFSETS AND OFFSET COMPENSATION	69
3.1	Introduction, 69	
3.2	Output Voltage Offsets Due to Input Voltage Offsets, 69	
	3.2.1 <i>Inverting Amplifier</i> , 70	
	3.2.2 <i>Noninverting Amplifier</i> , 73	
3.3	Output-Voltage Offsets Due to Input Currents, 74	
	3.3.1 <i>Output-Voltage Offsets Due to Bias Currents</i> , 74	
	3.3.2 <i>Output-Voltage Offsets Due to Offset Currents</i> , 76	
	3.3.3 <i>Current Compensation in Noninverting Amplifiers</i> , 78	
3.4	Output Changes Due to Input Voltage and Current Offsets, 79	
3.5	Offset Drifts, 80	
3.6	Offset Compensation, 82	
	3.6.1 <i>Inverting Amplifiers</i> , 82	
	3.6.2 <i>Noninverting Amplifiers</i> , 86	
4	POWER SUPPLIES FOR OP AMP CIRCUITS	91
4.1	Introduction, 91	
4.2	Dual-Polarity Power Supply, 91	
4.3	Filter Capacitor, 91	
4.4	Power-Supply Rejection Ratio, 96	
4.5	Voltage Regulator, 100	
	4.5.1 <i>Zener Diode Regulator</i> , 100	
	4.5.2 <i>Feedback Voltage Regulator</i> , 101	
	4.5.3 <i>Integrated-Circuit Regulators</i> , 104	
4.6	Slaving Plus and Minus Voltage Regulators, 104	

5 FREQUENCY-RELATED CHARACTERISTICS OF OPERATIONAL AMPLIFIERS

110

- 5.1 Introduction, 110
- 5.2 Open-Loop-Gain Characteristics, 110
 - 5.2.1 *The AC Equivalent Op Amp*, 110
 - 5.2.2 *Frequency Compensation for the Op Amp Circuit*, 116
 - 5.2.3 *AC Closed-Loop-Gain Equations*, 118
 - 5.2.4 *Feedforward Compensation*, 120
 - 5.2.5 *Frequency Response*, 121
 - 5.2.6 *Constant Gain-Bandwidth Product*, 122
- 5.3 Op Amp Slew Rate, 123
 - 5.3.1 *Output Voltage of Low Pass RC Section with Step Input*, 123
 - 5.3.2 *Rise Time and Slew Rate In Terms of Gain Bandwidth*, 125
 - 5.3.3 *Effects of Slew-Rate Limiting*, 127
 - 5.3.4 *Response Time*, 129
 - 5.3.5 *Feedback Loop: Open versus Closed*, 130
- 5.4 Noise, 131
 - 5.4.1 *Shot Noise*, 131
 - 5.4.2 *Thermal Noise*, 131
 - 5.4.3 *Flicker Noise*, 132
 - 5.4.4 *Combination of Noise on the Op Amp Input*, 132

6 LINEAR CIRCUIT APPLICATIONS

137

- 6.1 Introduction, 137
- 6.2 Differential-Mode Amplifiers, 138
 - 6.2.1 *Difference Amplifier*, 138
 - 6.2.2 *Differential-Mode Instrumentation Amplifier*, 138
 - 6.2.3 *Variable-Gain Differential-Mode Instrumentation Amplifiers*, 139
- 6.3 Op Amp Circuits Using Two-Port Networks, 141
- 6.4 High-Gain Amplifier, 143
 - 6.4.1 *Variable-Gain Amplifier*, 143
 - 6.4.2 *Variable-Gain Differential-Mode Instrumentation Amplifier*, 144
 - 6.4.3 *Dual-Phase Amplifier*, 145
- 6.5 Constant Current Amplifier, 148
 - 6.5.1 *Floating Load Resistor*, 148
 - 6.5.2 *Ground-Referenced Load Resistor*, 150
- 6.6 Integrator, 151
 - 6.6.1 *DC Input Voltages*, 152
 - 6.6.2 *AC Input Voltages*, 155
- 6.7 Differentiator, 156
 - 6.7.1 *High-Pass RC Filter Section*, 156
 - 6.7.2 *Op-Amp Differentiator*, 157
 - 6.7.3 *Practical Differentiator*, 158
 - 6.7.4 *Differentiator Operation*, 160
 - 6.7.5 *AC Input Voltages*, 161

6.8	Solving Differential Equations, 162	
6.8.1	<i>Differential Equation of Mechanical Systems, 162</i>	
6.8.2	<i>Differential Equation of Electrical Systems, 163</i>	
6.8.3	<i>Differential Equation of Damped, Oscillatory Systems, 163</i>	
6.8.4	<i>Analog-Computer Simulation of Differential Equation, 164</i>	
7	NONLINEAR APPLICATIONS	173
7.1	Introduction, 173	
7.2	Precision Rectifier, 173	
7.2.1	<i>Half-Wave Precision Rectifier, 173</i>	
7.2.2	<i>Full-Wave Precision Rectifier, 174</i>	
7.3	Curve Shapers, 177	
7.3.1	<i>Discrete Step Curve Shapers, 177</i>	
7.3.2	<i>Continuous Change Curve Shapers, 183</i>	
7.4	Clippers, 186	
7.5	Comparators, 188	
7.5.1	<i>Zero-Crossing Detector, 188</i>	
7.5.2	<i>Voltage Comparator, 190</i>	
7.6	Schmidt Trigger, 193	
7.7	Peak Holding Circuit, 196	
8	DIGITAL APPLICATIONS	203
8.1	Introduction, 203	
8.2	Interfacing Analog Devices to Logic Devices, 203	
8.2.1	<i>Interfacing of Analog and TTL Devices, 205</i>	
8.2.2	<i>Interfacing of Analog Devices and COS/MOS Gates, 206</i>	
8.3	Digital-To-Analog Converter, 207	
8.3.1	<i>Inverting Summer DAC, 207</i>	
8.3.2	<i>R/2R Ladder DAC, 214</i>	
8.4	Analog-To-Digital-Converter, 221	
8.4.1	<i>Counting A/D Converter, 221</i>	
8.4.2	<i>Digital Readouts, 223</i>	
8.4.3	<i>Tracking A/D Converter, 223</i>	
8.4.4	<i>Dual-Slope A/D Converter, 224</i>	
8.4.5	<i>Successive-Approximation ADC, 228</i>	
8.4.6	<i>Simultaneous ADC, 231</i>	
9	ACTIVE FILTERS	237
9.1	Introduction, 237	
9.2	Definition of Terms, 237	
9.2.1	<i>s-Plane, 237</i>	
9.2.2	<i>Poles and Zeros, 239</i>	
9.2.3	<i>The Elastic Membrane, 241</i>	
9.2.4	<i>The East Face, 242</i>	
9.2.5	<i>Frequency-Response Variations, 244</i>	
9.2.6	<i>Mathematical Pole and Zero, 244</i>	

9.3	Single Op-Amp Active Filter Sections, 254	
9.3.1	Low-Pass Filters, 254	
9.3.2	Normalization of Circuit Components, 260	
9.3.3	Active Low-Pass Frequency and Impedance Transformations, 261	
9.3.4	Characteristic Equations for Filters, 267	
9.3.5	High-Pass Filters, 279	
9.3.6	Band-Pass Filter, 287	
9.4	Biquadratic Filters, 296	
9.4.1	Notch Filter, 296	
9.4.2	Band-Reject-Filter, 298	
9.4.3	State-Variable Biquad Filters, 301	
9.4.4	All-Pass Equalizers, 314	
9.5	System Concepts, 320	
10	OSCILLATORS	338
10.1	Introduction, 338	
10.2	Creating an Oscillator, 338	
10.3	Bounding the Amplitude, 339	
10.4	Sine-Wave Oscillators, 339	
10.4.1	Twin-Tee Oscillator, 339	
10.4.2	Wien Bridge Oscillator, 341	
10.4.3	Phase-Shift Oscillator, 343	
10.5	Relaxation Oscillator, 347	
10.6	Triangle, Square Wave Oscillator, 350	
10.7	Voltage-Controlled Oscillators, 355	
11	LINEAR DEVICES	362
11.1	Introduction, 362	
11.2	Operational Transconductance Amplifier, 362	
11.3	Current-Differencing (Norton) Amplifier, 367	
11.4	Low-Voltage Op Amps, 370	
11.4.1	Op Amp and Voltage Reference, 370	
11.4.2	Low-Power MAXCMOS Op Amp, 374	
11.5	Stabilized Operational Amplifiers, 375	
11.5.1	Chopper-Stabilized Op Amp, 375	
11.5.2	Commutating Auto-Zero Op Amp, 379	
11.6	Phase-Locked Loops, 379	
Appendix A	DERIVATIONS	396
Appendix B	DEVICE SPECIFICATIONS	416
	ANSWERS TO SELECTED PROBLEMS	433
	Index	439

1

Construction of the Operational Amplifier

1.1 INTRODUCTION

The operational amplifier is a special-purpose amplifier intended to be used with external feedback components for proper operation. Only the operational amplifier itself will be discussed in this chapter. Its use with feedback elements will be reserved for later chapters, where an in-depth analysis will be presented.

1.2 NOTATION

The notation used in this text will be as follows:

1. Average (dc), maximum, and effective (root-mean-square or rms) values are represented using uppercase letters (V for voltage, I for current, and P for power).
2. Instantaneous values of quantities that vary with time are represented using lowercase letters (v , i , or p).
3. Instantaneous total values and average (dc) values are represented using an uppercase subscript letter for the appropriate terminal of the device being described (v_E).
4. Varying component values are indicated using a lowercase subscript letter for the appropriate device terminal (i_b).
5. A double subscript is used in two cases:
 - a. Where the power-supply voltage (V_{CC} or V_{SS}) is referenced.

- b. Where the voltage being represented is not ground-referenced; that is, it is the potential difference *between* two points in the circuit neither of which is at ground potential (zero volts). An example is V_{BE} for a transistor circuit with an emitter resistor.
6. E or e will be used in place of V or v for voltage sources. Power-supply sources are excluded from this notation.
7. The zero-volts reference plane for circuits will be represented using the terms “ground” and “common” interchangeably. These only differ in systems employing a chassis. There, the chassis is *ground* and the circuit *common* is a bus wire that is attached to the chassis at one point.

1.3 SI METRIC UNITS

The use of SI (International System of Units) metric units will be followed. As a result, the reader will encounter a few units that differ from the previous MKS or CGS systems. The symbols, together with the SI metric units, are listed in Table 1-1.

Table 1-1 SI Metric Units

<i>Quantity</i>	<i>Symbol</i>	<i>Unit Name</i>	<i>SI Metric Units</i>
Voltage	VE	(V) Volt	MV, kV, V, mV, μ V
Current	I	(A) Ampere	kA, A, mA, μ A, nA, pA
Power	P	(W) Watt	MW, kW, W, mW, μ W
Energy (work)	W	(J) Joule	MJ, kJ, J, mJ, μ J
Resistance	R	(Ω) Ohm	M Ω , k Ω , Ω , m Ω
Conductance	G	(S) Siemen	kS, S, mS, μ S
Capacitance	C	(F) Farad	F, mF, μ F, nF, pF
Inductance	L	(H) Henry	H, mH, μ H
Electric charge	Q	(C) Coulomb	kC, C, mC, μ C, nC, pC
Frequency	f	(Hz) Hertz	THz, GHz, MHz, kHz, Hz
Force	F	(N) Newton	MN, kN, N, mN, μ N
Length	l	(m) Meter	km, m, mm, μ m
Mass	M	(kg) Kilogram	Mg, kg, g, mg, μ g
Time	t	(s) Second	s, ms, μ s, ns, ps
Angular velocity	ω	(ω) rad/s	ω
Damping force ^a	d	(N) Newton	N
Spring constant ^a	K	(N/m) $\frac{\text{Newtons}}{\text{Meter}}$	N/m

^aNot actually SI units but represented as their SI equivalents.

1.4 DIFFERENTIAL-AMPLIFIER OPERATION

Much emphasis and time will be spent on the differential amplifier. Many of the good and most of the bad characteristics of the operational amplifier originate in the differential amplifier.

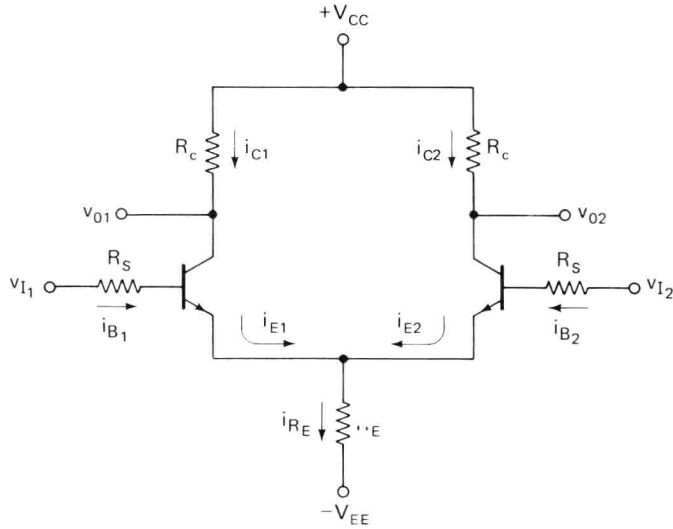


Fig. 1-1. Bipolar transistor differential amplifier.

A basic differential amplifier is shown in Fig. 1-1. It functions in such a manner that voltages common to both inputs (v_{I1} and v_{I2}) cause both collector voltages to move in unison, whereas voltages differing on the two inputs cause the two collector voltages to move in opposition. The basic rules of transistor action still apply; the collector voltage moves in opposition to the base voltage direction, regardless of whether the base change was caused by common or differing voltages.

1.4.1 Common-Mode Voltages

The voltage, common to both inputs, is labeled *common-mode input voltage* ($v_{I_{CM}}$). If v_{I1} is the voltage on base 1 and v_{I2} is the voltage on base 2, the common-mode input voltage is the average of these two voltages, as expressed by

$$v_{I_{CM}} = \frac{v_{I1} + v_{I2}}{2} \quad (1-1)$$

If $v_{I1} = v_{I2}$, then $v_{I_{CM}} = v_{I1} = v_{I2}$.

The voltage, common to both outputs, is labeled *common-mode output voltage* ($v_{O_{CM}}$). If v_{O1} is the voltage on collector 1 and v_{O2} is the voltage on collector 2, the common-mode output voltage is the average of these two voltages at any instant in time, as expressed by

$$v_{O_{CM}} = \frac{v_{O1} + v_{O2}}{2} \quad (1-2)$$

With the two inputs connected together, v_{O1} should be equal to v_{O2} , presuming matched components, but that is seldom the case for actual circuits. Example 1-1 will help to clarify the use of Eqs. (1-1) and (1-2).

EXAMPLE 1-1

A differential amplifier has the characteristics that cause the common-mode output voltage to be +5.5 V when the input common-mode voltage is zero and to have an output common-mode voltage of +5.0 V when the input common-mode voltage is +1 V. Determine the input and output voltages for the following set of voltages:

Case	v_{I_1}	v_{I_2}	v_{O_1}	v_{O_2}	Find
1	0	0	+5.5		v_{O_2}
2	+1.1	+0.9	+2.0	+8.0	$v_{I_{CM}}, v_{O_{CM}}$
3	+0.01	-0.01		+6.1	$v_{I_{CM}}, v_{O_1}$

SOLUTION

Case 1

$$v_{I_{CM}} = \frac{0 + 0}{2} = 0 \quad \text{thus} \quad v_{O_{CM}} = +5.5 \text{ V}$$

$$+5.5 = \frac{+5.5 + v_{O_2}}{2} \quad v_{O_2} = +5.5 \text{ V}$$

Case 2

$$v_{I_{CM}} = \frac{+1.1 + 0.9}{2} = +1.0$$

$$v_{O_{CM}} = \frac{2.0 + 8.0}{2} = +5.0$$

Case 3

$$v_{I_{CM}} = \frac{+0.01 + (-0.01)}{2} = 0 \text{ V}$$

$$v_{O_{CM}} = +5.5 = \frac{v_{O_1} + 6.1}{2} \quad v_{O_1} = +4.9 \text{ V}$$

In the most frequently encountered case for common-mode voltages, the two input voltages are varying at different rates, thus producing a changing dc or an ac common-mode voltage. In this situation Eq. (1-1), between two instants in time, becomes

$$\Delta v_{I_{CM}} = \frac{\Delta v_{I_1} + \Delta v_{I_2}}{2} \quad (1-3)$$

and Eq. (1-2), between two instants in time becomes

$$\Delta v_{O_{CM}} = \frac{\Delta v_{O_1} + \Delta v_{O_2}}{2} \quad (1-4)$$

where the individual v_{I_i} 's or v_{O_i} 's must be established at two separate instants of time to determine the variation over the time interval.

EXAMPLE 1-2

A differential amplifier has v_{I1} grounded while v_{I2} is connected to an ac voltage of $v_{I2} = 6 \text{ mV} \sin(2\pi \cdot 0.1t)$. The output common-mode voltage is $+5.0 \text{ V}$ when $v_{I_{CM}} = 0$, $+5.001 \text{ V}$ when $v_{I_{CM}} = -3 \text{ mV}$, and $+4.999 \text{ V}$ when $v_{I_{CM}} = +3 \text{ mV}$. Determine the input and output common-mode voltages.

SOLUTION

$$\Delta v_{I_{CM}} = \frac{0 + 6 \text{ mV} \sin(2\pi \cdot 0.1t)}{2} = 3 \text{ mV} \sin(2\pi \cdot 0.1t)$$

which is a sine wave with peak instantaneous voltages of $\pm 3 \text{ mV}$. The collector voltage decreases as the base voltage increases, which results in a phase reversal on the output. Thus $\Delta v_{O_{CM}} = +5 \text{ V} - 1 \text{ mV} \sin(2\pi \cdot 0.1t)$. [Note: The $(-)$ sign indicates the phase reversal.]

EXAMPLE 1-3

A differential amplifier has the following input and output voltages at different instants of time:

Instant	v_{I1}	v_{I2}	v_{O1}	v_{O2}
t_1	-3.4 V	-3.6 V	$+4.2 \text{ V}$	$+4.4 \text{ V}$
t_2	$+1.8 \text{ V}$	$+1.2 \text{ V}$	$+2.6 \text{ V}$	$+3.2 \text{ V}$

Determine the change in the input and output common-mode voltage.

SOLUTION

Δv_{I1} is v_{I1} at t_2 minus v_{I1} at t_1 ; Δv_{I2} is v_{I2} at t_2 minus v_{I2} at t_1 . The same is true for the output voltages.

$$\begin{aligned}\Delta v_{I_{CM}} &= \frac{[1.8 - (-3.4)] + [1.2 - (-3.6)]}{2} = \frac{5.2 + 4.8}{2} \\ &= +5.0 \text{ V} \\ \Delta v_{O_{CM}} &= \frac{(2.6 - 4.2) + (3.2 - 4.4)}{2} = \frac{-1.6 + (-1.2)}{2} \\ &= -1.4 \text{ V}\end{aligned}$$

which illustrates that, as the input common-mode voltage increases by 5 V , the output common-mode voltage decreases by 1.4 V .

The *common-mode gain* is defined as the change in common-mode output voltage divided by the change of input common-mode voltage, or

$$A_{CM} = + \frac{\Delta v_{O_{CM}}}{\Delta v_{I_{CM}}} \quad (1-5)$$

where A_{CM} becomes negative when numerical values are applied.

EXAMPLE 1-4

Using the values determined in Example 1-3, find A_{CM} .

SOLUTION

$$A_{CM} = \frac{-1.4 \text{ V}}{+5.0 \text{ V}} = -0.28$$

Common-mode gains are typically less than unity (1). The most desirable case is for the common-mode gain to be zero, and, as will be seen in subsequent sections, this is the motivation for particular circuit selections.

1.4.2 Differential Voltages

The differential voltage must be considered for two cases: (1) where the deviation voltage is swinging around a zero common-mode voltage, and (2) where the deviation voltage is swinging around a nonzero common-mode voltage. The first case should only be considered for the inputs, as the collector voltages of a differential amplifier generally swing around a nonzero common-mode voltage.

The differential voltage swing about any common-mode voltage, including zero volts, is called the *deviation voltage*. The input deviation voltage,* at any instant in time, is given by one-half the difference voltage at the input or

$$v_{I_{DEV}} = \pm \frac{v_{I_1} - v_{I_2}}{2} \quad (1-6)$$

where $(v_{I_1} - v_{I_2})$ is called the *differential input voltage* and is given an individual label (v_{I_d}); thus

$$v_{I_d} = v_{I_1} - v_{I_2} \quad (1-7)$$

and

$$v_{I_d} = \pm 2v_{I_{DEV}} \quad (1-8)$$

The output deviation voltage, at any instant in time, is given by

$$v_{O_{DEV}} = \pm \frac{v_{O_1} - v_{O_2}}{2} \quad (1-9)$$

which is the voltage deviation around the common-mode output voltage at any instant in time on each collector.

EXAMPLE 1-5

A differential amplifier has zero input and output common-mode voltages (not physically possible). The input and output voltages are:

v_{I_1}	v_{I_2}	v_{O_1}	v_{O_2}
- 0.01	+ 0.01	+ 1.4	- 1.4

Determine the input and output deviation voltages and the differential input voltage.

*The (\pm) sign indicates that the deviation voltage swings to each side of the common-mode voltage. This (\pm) indicator is dropped when discussing v_{I_d} , and this seems to be a mathematical error, but the differential voltage has only one algebraic sign, because it is a single voltage value.