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832

Representation Theory II

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Proceedings of the Second International Conference on Representations of Algebras Ottawa, Carleton University, August 13 – 25, 1979

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PREFACE

The First International Conference on Representations of Algebras was held at Carleton University, Ottawa, on September 3-7, 1974; the Proceedings of the Conference appeared as Springer Lecture Notes #488 the following year.

Since then, the interest in the representation theory grew rapidly, and significant advances and contributions to the theory continued to take place. These were to be reflected in the meeting organized at Carleton University on August 13-25, 1979. The Organizing Committee consisted of Professors J. Alperin, M. Auslander, V. Dlab, P. Gabriel, I. Reiner, C.M. Ringel, A.V. Rojter and H. Tachikawa.

The developments of the five-year period preceding the meeting were the subject of five series of lectures in the WORKSHOP ON THE PRESENT TRENDS IN REPRESENTATION THEORY (August 13-18, 1979) given by J. Alperin (Block theory), P. Gabriel (Trends in representation theory), J.E. Humphreys (Highest weight modules for semi simple Lie algebras), C.M. Ringel (Algorithms for solving vector space problems) and V.A. Rojter (Differential graded categories). The first volume of these Proceedings contains reports from the WORKSHOP.

Recent advances in the representation theory were reported in 42 lectures during the second part of the meeting: THE SECOND INTERNATIONAL CONFERENCE ON REPRESENTATIONS OF ALGEBRAS; the lectures are listed on page VIII. Not all contributions to the CONFERENCE appear in these Proceedings; on the other hand, some papers which were not reported, are included. All

published papers appear in the form submitted by the author; only very few technical alterations have been made. It should be pointed out that some of the contributions to these Proceedings have resulted out of discussions and conversations during the meeting. We should like to thank all referees for their assistance.

It has been suggested that the Proceedings contain a list of publications in the field of representation theory for the past 10 years. An attempt in this direction has been made; we wish to thank Mr. Ibrahim Assem for his assistance in preparing the Bibliography.

We should like to acknowledge financial assistance of the Natural Sciences and Engineering Research Council Canada and of Carleton University. The assistance of other national research bodies to support participants of the meeting has been also greatly appreciated. In particular, we wish to thank Carleton University for the generous support of the first part of the meeting; without its assistance the organization of the WORKSHOP would not have been possible. And, we wish to extend our thanks to the Secretary, Professor Luis Ribes for his unlimited help in the organization of the meeting and to Ms. Alejandra Leon for her efficient secretarial assistance.

Ottawa - Zurich, July 1980 Vlastimil Dlab and Peter Gabriel

LIST OF LECTURES

J.L. ALPERIN	Complexity of modules
M. AUSLANDER- I. REITEN*	Uniserial functors
M. AUSLANDER*- S.O. SMALØ	Subcategories of mod Λ over which mod Λ is functorially finite
M. AUSLANDER - S.O. SMALØ*	Preprojective modules
R. BAUTISTA	Sections in Auslander components
R.E. BLOCK	The algebraically irreducible representations of the Lie algebra $s\ell$ 2
K. BONGARTZ	Algebras of finite representation type without cycles
S. BRENNER	Some co-rank 2 quivers with relations and their null roots
D.W. BURRY	A module-oriented theory of blocks
M.C.R. BUTLER	Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors
J.F. CARLSON	The dimension of modules and their restrictions over modular group algebras
Ch.W. CURTIS	Homology representations of finite groups
E.C. DADE	Algebraically stable modules
K. ERDMANN	On blocks whose defect groups are elementary abelian of order p^2
H.K. FARAHAT	Young and James ideals in a group algebra

C.M. RINGEL	Indecomposable representations of wild quivers
K.W. ROGGENKAMP	Blocks of cyclic defect
A.V. ROJTER	Gelfand-Ponomarev algebra of a quiver
F.J. SERVEDIO	GL(V)-submodules of $V^{\otimes m}$ fixed by subgroups of S_m; forms of degree t in forms of degree r on V
D. SIMSON	Species and hereditary rings of finite representation type
H. TACHIKAWA	Representations of algebras of trivial extension
G. TODOROV**	Almost split sequences for TrD-periodic modules M, with no projectives in the class [M]
J. WASCHBÜSCH	Quasi-Frobenius algebras of finite representation type
P.J. WEBB	Distinguishing non-isomorphic relation modules
K. YAMAGATA	On artinian rings of local representation type

* denotes the speaker

** lecture delivered by I.M. Platzeck

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TABLE OF CONTENTS

UNISERIAL FUNCTORS

Maurice Auslander*) and Idun Reiten

Introduction

Throughout this paper modΛ denotes the category of finitely generated Λ-modules over an artin algebra Λ. We say that a covariant or contravariant functor F from modΛ to abelian groups is uniserial if the subfunctors of F are totally ordered by inclusion, i.e. if F_1 and F_2 are subfunctors of F, then either $F_1 \subset F_2$ or $F_2 \subset F_1$. If F is of finite length, then, as in the case of modules, F is uniserial if and only if the only subfunctors of F are the elements of the radical series of F, or equivalently, the elements of the socle series for F. The main purpose of this paper is to initiate a study of the connections between the existence and structure of various uniserial functors and the representation theory of Λ.

The first uniserial functors to be studied in a systematic way were the simple functors. This study [4] led to right and left almost split morphisms as well as almost split sequences, notions which are playing an increasingly important role in the representation theory of artin algebras. We recall that associated with each nonprojective indecomposable module C is a unique non-split exact sequence $0 \to A \xrightarrow{g} B \xrightarrow{f} C \to 0$ called an almost split sequence which is an invariant of the module C [5]. In particular, if $B = \coprod_{i=1}^{n} B_i$ is a decomposition of B into indecomposable modules, then $\alpha(C) = n$ and $\beta(C) = n$ minus

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*) Written while a Guggenheim Fellow with the partial support of NSF MCS 77 04 951.

the number of projective B_i, are invariants of C which seem to have something to do with the complexity of the morphisms to C. The connection between these invariants and uniserial functors is illustrated by the following result (see Theorem 3.1): Suppose C is an indecomposable module over an artin algebra of finite representation type. If $(,\underline{C})$, the representable functor $(,C)$ modulo projectives, is nonzero uniserial, then $\beta(C) \leq 1$ and $\beta(C') \leq 2$ for all indecomposable C' such that $(\underline{C}',\underline{C}) \neq 0$.

While the interplay between the invariants $\alpha(C)$ and $\beta(C)$ for indecomposable nonprojective C and the uniseriality of certain types of functors is interesting in its own right, it is hoped that these results will help in describing and perhaps classifying certain types of artin algebras, especially those of finite representation type. For example Tachikawa in [17,18] studied artin algebras with the property that each indecomposable module has a simple socle. Here we show that such algebras can be characterized by a) $(,S)$ is a uniserial functor for all simple modules S or b) $\alpha(C) \leq 2$ for all indecomposable modules C, $\underline{r}$ P is indecomposable for all indecomposable projectives P where $\underline{r}$ is the radical of P, and Λ is of finite type.

We give a brief outline of the contents of each section. Section 1 is devoted to a preliminary investigation of the connection between uniseriality of modules and functors. In section 2 we study the following general problem. Given a minimal projective presentation $(,B) \to (,C) \to G \to 0$ of a finitely presented functor G and a subfunctor F with projective cover $(,E) \to F$, we construct projective presentations for F and for G/F and give conditions under which they are minimal. Specialized to semisimple subfactors of G, we get in certain cases a construction of sums of almost split sequences. These results are applied in section 3 to get criteria for uniseriality of functors in terms of the invariants α and β.