

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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Representation Theory II

Proceedings, Ottawa, Carleton University, 1979

Edited by V. Dlab and P. Gabriel



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Proceedings of the Second International
Conference on Representations of Algebras
Ottawa, Carleton University, August 13 – 25,
1979

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Editors

Vlastimil Dlab
Department of Mathematics
Carleton University
Ottawa K15 5B6
Canada

Peter Gabriel
Mathematisches Institut
Universität Zürich
Freiestrasse 36
8032 Zürich
Switzerland

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PREFACE

The First International Conference on Representations of Algebras was held at Carleton University, Ottawa, on September 3-7, 1974; the Proceedings of the Conference appeared as Springer Lecture Notes #488 the following year.

Since then, the interest in the representation theory grew rapidly, and significant advances and contributions to the theory continued to take place. These were to be reflected in the meeting organized at Carleton University on August 13-25, 1979. The Organizing Committee consisted of Professors J. Alperin, M. Auslander, V. Dlab, P. Gabriel, I. Reiner, C.M. Ringel, A.V. Rojter and H. Tachikawa.

The developments of the five-year period preceding the meeting were the subject of five series of lectures in the WORKSHOP ON THE PRESENT TRENDS IN REPRESENTATION THEORY (August 13-18, 1979) given by J. Alperin (Block theory), P. Gabriel (Trends in representation theory), J.E. Humphreys (Highest weight modules for semi simple Lie algebras), C.M. Ringel (Algorithms for solving vector space problems) and V.A. Rojter (Differential graded categories). The first volume of these Proceedings contains reports from the WORKSHOP.

Recent advances in the representation theory were reported in 42 lectures during the second part of the meeting: THE SECOND INTERNATIONAL CONFERENCE ON REPRESENTATIONS OF ALGEBRAS; the lectures are listed on page VIII. Not all contributions to the CONFERENCE appear in these Proceedings; on the other hand, some papers which were not reported, are included. All

published papers appear in the form submitted by the author; only very few technical alterations have been made. It should be pointed out that some of the contributions to these Proceedings have resulted out of discussions and conversations during the meeting. We should like to thank all referees for their assistance.

It has been suggested that the Proceedings contain a list of publications in the field of representation theory for the past 10 years. An attempt in this direction has been made; we wish to thank Mr. Ibrahim Assem for his assistance in preparing the Bibliography.

We should like to acknowledge financial assistance of the Natural Sciences and Engineering Research Council Canada and of Carleton University. The assistance of other national research bodies to support participants of the meeting has been also greatly appreciated. In particular, we wish to thank Carleton University for the generous support of the first part of the meeting; without its assistance the organization of the WORKSHOP would not have been possible. And, we wish to extend our thanks to the Secretary, Professor Luis Ribes for his unlimited help in the organization of the meeting and to Ms. Alejandra Leon for her efficient secretarial assistance.

Ottawa - Zurich, July 1980

Vlastimil Dlab and Peter Gabriel

LIST OF LECTURES

J.L. ALPERIN	Complexity of modules
M. AUSLANDER- I. REITEN*	Uniserial functors
M. AUSLANDER*- S.O. SMALØ	Subcategories of $\text{mod } \Lambda$ over which $\text{mod } \Lambda$ is functorially finite
M. AUSLANDER - S.O. SMALØ*	Preprojective modules
R. BAUTISTA	Sections in Auslander components
R.E. BLOCK	The algebraically irreducible representations of the Lie algebra $sl\ 2$
K. BONGARTZ	Algebras of finite representation type without cycles
S. BRENNER	Some co-rank 2 quivers with relations and their null roots
D.W. BURRY	A module-oriented theory of blocks
M.C.R. BUTLER	Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors
J.F. CARLSON	The dimension of modules and their restrictions over modular group algebras
Ch.W. CURTIS	Homology representations of finite groups
E.C. DADE	Algebraically stable modules
K. ERDMANN	On blocks whose defect groups are elementary abelian of order p^2
H.K. FARAHAT	Young and James ideals in a group algebra

E.L. GREEN	Remarks on projective resolutions
M. HAZEWINKEL	On the representations of the wild quiver $\rightarrow \circlearrowright$
J.E. HUMPHREYS	Cartan invariants and decomposition numbers for Chevalley groups
Y. IWANAGA	Trivial extension of some artin algebras
H. JACOBINSKI	Hereditary covers and blocks
J.C. JANIZEN	Representations of semisimple groups and their Frobenius kernels
V.G. KAC	Infinite root systems, representations of quivers and invariant theory
H. KUPISCH	Symmetric algebras of finite representation type
P. LANDROCK	Some remarks on Loewy lengths of projective modules of a symmetric algebra
L. LEVY	Mixed modules over $\mathbb{Z}G$, G cyclic of prime order, and over related Dedekind pullbacks
R. MARTINEZ-VILLIA	Algebras stably equivalent to ℓ -hereditary
G.O. MICHLER	On blocks of finite groups with abelian defect groups
R.V. MOODY	Hyperbolic Lie algebras and singularities
F. OKOH	A hereditary finite dimensional algebra that is not pure-hereditary
W. PLESKEN	Projective lattices over group orders as amalgamations of irreducible lattices
I. REINER	Solomon's conjecture and the functional equation for zeta function of orders
Ch. RIEDTMANN	Selfinjective algebras of class A_n and D_n

C.M. RINGEL	Indecomposable representations of wild quivers
K.W. ROGGENKAMP	Blocks of cyclic defect
A.V. ROJTER	Gelfand-Ponomarev algebra of a quiver
F.J. SERVEDIO	$GL(V)$ -submodules of $V^{\otimes m}$ fixed by subgroups of S_m ; forms of degree t in forms of degree r on V
D. SIMSON	Species and hereditary rings of finite representation type
H. TACHIKAWA	Representations of algebras of trivial extension
G. TODOROV **	Almost split sequences for TrD-periodic modules M , with no projectives in the class $[M]$
J. WASCHBUSCH	Quasi-Frobenius algebras of finite representation type
P.J. WEBB	Distinguishing non-isomorphic relation modules
K. YAMAGATA	On artinian rings of local representation type

* denotes the speaker

** lecture delivered by I.M. Platzeck

LIST OF REGISTERED PARTICIPANTS

ABRAMS Gene, University of Oregon, Eugene, Oregon, U.S.A.
ALPERIN Jonathan L., University of Chicago, Chicago, Illinois, U.S.A.
AMDAL Ivar, University of Trondheim, Norway.
ASSEM Ibrahim, Carleton University, Ottawa, Ontario, Canada.
AUSLANDER Maurice, Brandeis University, Waltham, Mass., U.S.A.
BAUTISTA Raymundo, Universidad Nac. Autónoma de México, Mexico.
BECKER Helmut, Hochschule der Bundeswehr, München, West Germany.
BINGEN Franz, Vrije Universiteit Brussel, Belgium.
BLOCK Richard E., University of California, Riverside, Calif., U.S.A.
BONGARTZ Klaus, Universität Zürich, Switzerland.
BRENNER Sheila, University of Liverpool, United Kingdom.
BRITTEN Dan, University of Windsor, Ontario, Canada.
BURRY David, Yale University, New Haven, Connecticut, U.S.A.
BUTLER M.C.R., University of Liverpool, United Kingdom.
BUTSAN George, Math. Institute, AN USSR, Kiev, U.S.S.R.
CARLSON J.F., University of Georgia, Athens, U.S.A.
CIBILS Claude, Universidad Autónoma Metropolitana, Mexico City, Mexico.
CLIFF Gerald, University of Alberta, Edmonton, Alberta, Canada.
CURTIS Charles W., University of Oregon, Eugene, Oregon, U.S.A.
DADE Everett C., University of Illinois, Urbana, Illinois, U.S.A.
DAVIS Richard A., Columbia University, New York, U.S.A.
DIETERICH Ernst, Universität Bielefeld, West Germany.
DIPPER Richard, Universität Essen, West Germany.
DIXON John, Carleton University, Ottawa, Ontario, Canada.
DLAB Vlastimil, Carleton University, Ottawa, Ontario, Canada.
ERDMANN Karin, Universität Essen, West Germany.
FARAHAT H.K., University of Calgary, Calgary, Alberta, Canada.
FORD Charles, St. Louis University, Missouri, U.S.A.
FULLER Kent, R., University of Hawaii, Manoa, Hawaii, U.S.A.

- GABRIEL Peter, Universität Zürich, Switzerland.
- GODFREY Colin M., University of Massachusetts, Boston, Mass., U.S.A.
- GREEN Edward L., Virginia Polytechnic Institute, Blacksburg, U.S.A.
- GRIESS Robert L. Jr., University of Michigan, Ann Arbor, U.S.A./
Institute for Advanced Studies, Princeton, N.J., U.S.A.
- GUSTAFSON William H., Texas Tech. University, Lubbock, Texas, U.S.A.
- HAPPEL Dieter, Universität Bielefeld, West Germany.
- HAZEWINKEL Michiel, Erasmus Univ. Rotterdam, Holland.
- HUGHES David, University of Liverpool, United Kingdom.
- HUMPHREYS J. E., University of Massachusetts, Amherst, Mass., U.S.A.
- IWANAGA Yasuo, University of Tsukuba, Ibaraki, Japan/Carleton University,
Ottawa, Ontario, Canada.
- JACOBINSKI H., Chalmers University of Technology, Göteborg, Sweden.
- JANSEN Willem G., McMaster University, Hamilton, Ontario, Canada.
- JANTZEN Jens C., Universität Bonn, West Germany
- JANUSZ Gerald J., University of Illinois, Urbana, Ill., U.S.A.
- JONES Alfredo, Universidade de São Paulo, Brasil.
- KAC Victor, M.I.T., Cambridge, Mass., U.S.A.
- KANEDA Masaharu, University of Oregon, Eugene, Oregon, U.S.A.
- KLEINER Mark, New York, U.S.A.
- KLEISLI Heinrich, Université de Fribourg, Switzerland.
- KNODLE Stephen, State University of New York, Potsdam, N.Y., U.S.A.
- KNÖRR Reinhard, University of Illinois, Urbana, Ill., U.S.A.
- KOVÁCS László G., Australian National University, Canberra, Australia.
- KRAEMER Julius, Universität München, West Germany.
- KUBO Fujio, Southern Illinois University, Edwardsville, Ill., U.S.A.
- KUPISCH H., Freie Universität, West Berlin.
- LADY Lee E., University of Hawaii, Honolulu, Hawaii, U.S.A.
- LANDROCK Peter, Aarhus University, Denmark.
- LARRION Francisco, Universidad Nac. Autónoma de México, Mexico.
- LEON Alejandra, Carleton University, Ottawa, Ontario, Canada.
- LEUNG Tat-Wing, Queen's University, Kingston, Ontario, Canada.
- LEVY Lawrence S., University of Wisconsin, Madison, Wisconsin, U.S.A.
- MARMARIDIS Nikolaos, University of Crete, Greece.

- MARTINEZ-VILA Roberto, Universidad Nac. Autónoma de México, Mexico.
- MERKLEN Hector A., Universidade de São Paulo, Brasil.
- MICHLER Gerhard O., Universität Essen, West Germany.
- MOLLIN Richard A., McMaster University, Hamilton, Ontario, Canada.
- MOODY Robert V., University of Saskatchewan, Saskatoon, Canada.
- MORTIMER Brian, Carleton University, Ottawa, Ontario, Canada.
- OKOH Frank, York University, Downsview, Ontario, Canada.
- OLSSON Jörn B., Universität Dortmund, West Germany.
- O'NEILL John D., University of Detroit, Michigan, U.S.A.
- OTAL Javier, Universidad de Zaragoza, Spain.
- PLATZECK Maria Ines, Universidad Nacional del Sur, Bahía Blanca,
Argentina.
- PLESKEN Wilhelm, RWTH Aachen, West Germany.
- PUTTASWAMAIAH B., Carleton University, Ottawa, Ontario, Canada.
- REIDNER Irving, University of Illinois, Urbana, Ill., U.S.A.
- REITEN Idun, University of Trondheim, Norway.
- RIEDTMANN Christine, Universität Basel, Switzerland.
- RIBES Luis, Carleton University, Ottawa, Ontario, Canada.
- RINGEL Claus Michael, Universität Bielefeld, West Germany.
- ROGGENKAMP Klaus W., Universität Stuttgart, West Germany.
- ROJTER A.V., Math. Inst., AN USSR Kiev, U.S.S.R.
- ROLDAN Oscar E., Carleton University, Ottawa, Ontario, Canada.
- RUMP Wolfgang, Ohio State University, Columbus, Ohio, U.S.A.
- SALMERON Leonardo, Universidad Nac. Autónoma de México, Mexico.
- SANTHAROUBANE Louis-Joseph, Université de Paris, France.
- SCHNEIDER Gerhard, Universität of Essen, West Germany.
- SERVEDIO Frank J., William Paterson College, Wayne, New Jersey, U.S.A.
- SHIAO Long-Shung, Carleton University, Ottawa, Ontario, Canada.
- SIBLEY David, Pennsylvania State University, University Park, U.S.A.
- SIMSON Daniel, University of Torun, Poland.
- SMALØ Sverre O., University of Trondheim, Norway.
- STAMBACH Urs, Eidg. Technische Hochschule, Zürich, Switzerland.
- STOLTZFUS Neal W., Université de Genève, Switzerland/Louisiana State
University, Baton Rouge, Louisiana, U.S.A.

TACHIKAWA H., University of Tsukuba, Ibaraki, Japan.

TINBERG Nalsey B., Southern Illinois University, Edwardsville, U.S.A.

WASCHBUSCH Josef, Freie Universität, West Berlin.

WEBB P.J., Queen Mary College, London, United Kingdom.

WIEDEMANN Alfred, Universität Stuttgart, West Germany.

YAMAGATA Kunio, University of Tsukuba, Ibaraki, Japan.

YOKONUMA Takeo, University of Saskatchewan, Saskatoon, Sask., Canada/

Sophia University, Tokyo, Japan.

TABLE OF CONTENTS

List of lectures	VIII
List of registered participants	XI
 <u>Volume I</u> <u>WORKSHOP</u>	
PETER GABRIEL	
Auslander-Reiten sequences and representation-finite algebras	1
JAMES E. HUMPHREYS	
Highest weight modules for semisimple Lie algebras	72
CLAUS MICHAEL RINGEL	
Report on the Brauer-Thrall conjectures	104
CLAUS MICHAEL RINGEL	
Tame algebras	137
V.A. ROUTER	
Matrix problems and representations of BCS's	288
Bibliography 1969 - 1979	325
 <u>Volume II</u> <u>CONFERENCE</u>	
M. AUSLANDER and IDUN REITEN	
Uniserial functors	1
M. AUSLANDER and S.O. SMALØ	
Preprojective modules: An introduction and some applications	48
R. BAUTISTA	
Sections in Auslander-Reiten quivers	74
K. BONGARTZ	
Zykellose Algebren sind nicht zügellos	97
SHEILA BRENNER and M.C.R. BUTLER	
Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors	103

H. BRUNE	
On finite representation type and a theorem of Kulikov	170
CHARLES W. CURTIS	
Homology representations of finite groups	177
EVERETT C. DADE	
Algebraically rigid modules	195
VLASTIMIL DIAB and CLAUS MICHAEL RINGEL	
The preprojective algebra of a modulated graph	216
P. DOWBOR, C.M. RINGEL and D. SIMSON	
Hereditary artinian rings of finite representation type	232
JU. A. DROZD	
Tame and wild matrix problems	242
EDWARD L. GREEN	
Remarks on projective resolutions	259
DIETER HAPPEL, UDO PREISER and CLAUS MICHAEL RINGEL	
Vinberg's characterization of Dynkin diagrams using subadditive functions with application to DTr-periodic modules	280
YASUO IWANAGA and TAKAYOSHI WAKAMATSU	
Trivial extension of artin algebras	295
C.U. JENSEN and H. LENZING	
Model theory and representations of algebras	302
VICTOR G. KAČ	
Some remarks on representations of quivers and infinite root systems	311
HERBERT KUPISCH and EBERHARD SCHERZLER	
Symmetric algebras of finite representation type	328
PETER LANDROCK	
Some remarks on Loewy lengths of projective modules	369
NIKOLAOS MARMARIDIS	
Reflection functors	382

ROBERTO MARTINEZ-VILLA	
Algebras stably equivalent to ℓ -hereditary	396
FRANK OKOH	
Hereditary algebras that are not pure-hereditary	432
WILHELM PLESKEN	
Projective lattices over group orders as amalgamations of irreducible lattices	438
CHRISTINE RIEDTMANN	
Representation-finite selfinjective algebras of class A_n	449
K.W. ROGGENKAMP	
Representation theory of blocks of defect 1	521
EBERHARD SCHERZLER AND JOSEF WASCHBÜSCH	
A class of self-injective algebras of finite representation type	545
DANIEL SIMSON	
Right pure semisimple hereditary rings	573
HIROYUKI TACHIKAWA	
Representations of trivial extensions of hereditary algebras	579
GORDANA TODOROV	
Almost split sequences for TrD-periodic modules	600
JOSEF WASCHBÜSCH	
A class of self-injective algebras and their indecomposable modules	632
KUNIO YAMAGATA	
Hereditary artinian rings of right local representation type	648

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Maurice Auslander^{*)} and Idun Reiten

Introduction

Throughout this paper $\text{mod } \Lambda$ denotes the category of finitely generated Λ -modules over an artin algebra Λ . We say that a covariant or contravariant functor F from $\text{mod } \Lambda$ to abelian groups is uniserial if the subfunctors of F are totally ordered by inclusion, i.e. if F_1 and F_2 are subfunctors of F , then either $F_1 \subset F_2$ or $F_2 \subset F_1$. If F is of finite length, then, as in the case of modules, F is uniserial if and only if the only subfunctors of F are the elements of the radical series of F , or equivalently, the elements of the socle series for F . The main purpose of this paper is to initiate a study of the connections between the existence and structure of various uniserial functors and the representation theory of Λ .

The first uniserial functors to be studied in a systematic way were the simple functors. This study [4] led to right and left almost split morphisms as well as almost split sequences, notions which are playing an increasingly important role in the representation theory of artin algebras. We recall that associated with each nonprojective indecomposable module C is a unique non-split exact sequence $0 \rightarrow A \xrightarrow{g} B \xrightarrow{f} C \rightarrow 0$ called an almost split sequence which is an invariant of the module C [5]. In particular, if $B = \bigoplus_{i=1}^n B_i$ is a decomposition of B into indecomposable modules, then $\alpha(C) = n$ and $\beta(C) = n$ minus

^{*)} Written while a Guggenheim Fellow with the partial support of NSF MCS 77 04 951.

the number of projective B_i , are invariants of C which seem to have something to do with the complexity of the morphisms to C . The connection between these invariants and uniserial functors is illustrated by the following result (see Theorem 3.1): Suppose C is an indecomposable module over an artin algebra of finite representation type. If $(\ , \underline{C})$, the representable functor $(\ , C)$ modulo projectives, is nonzero uniserial, then $\beta(C) \leq 1$ and $\beta(C') \leq 2$ for all indecomposable C' such that $(\underline{C'}, \underline{C}) \neq 0$.

While the interplay between the invariants $\alpha(C)$ and $\beta(C)$ for indecomposable nonprojective C and the uniseriality of certain types of functors is interesting in its own right, it is hoped that these results will help in describing and perhaps classifying certain types of artin algebras, especially those of finite representation type. For example Tachikawa in [17,18] studied artin algebras with the property that each indecomposable module has a simple socle. Here we show that such algebras can be characterized by a) $(\ , S)$ is a uniserial functor for all simple modules S or b) $\alpha(C) \leq 2$ for all indecomposable modules C , \underline{r} P is indecomposable for all indecomposable projectives P where \underline{r} is the radical of P , and Λ is of finite type.

We give a brief outline of the contents of each section. Section 1 is devoted to a preliminary investigation of the connection between uniseriality of modules and functors. In section 2 we study the following general problem. Given a minimal projective presentation $(\ , B) \rightarrow (\ , C) \rightarrow G \rightarrow 0$ of a finitely presented functor G and a subfunctor F with projective cover $(\ , E) \rightarrow F$, we construct projective presentations for F and for G/F and give conditions under which they are minimal. Specialized to semisimple subfactors of G , we get in certain cases a construction of sums of almost split sequences. These results are applied in section 3 to get criteria for uniseriality of functors in terms of the invariants α and β .