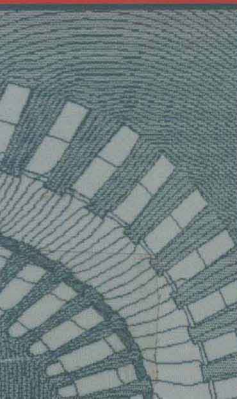
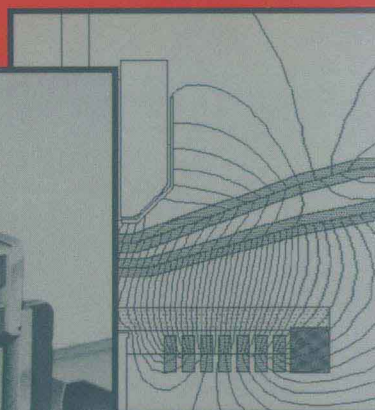
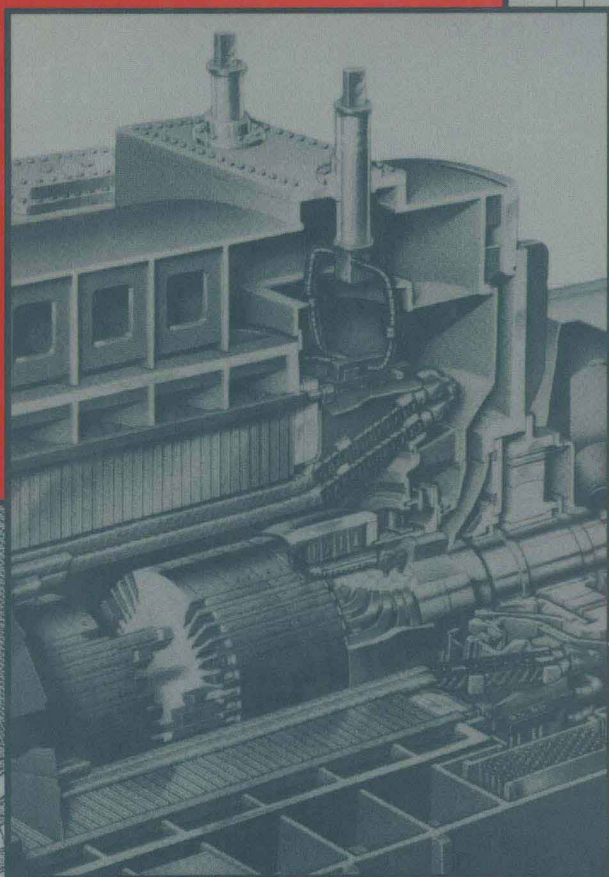


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FINITE ELEMENT METHODS IN ELECTRICAL POWER ENGINEERING

A.B.J. REECE
and T.W. PRESTON



Finite Element Methods in Electrical Power Engineering

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*To Gwen and Pat
for their patience and encouragement
and to our colleagues past and present
whose contributions over the years have made this book possible*

Preface

The appearance of yet another book on finite element computation of electromagnetic fields needs some explanation.

As developers and users of the finite element method in electrical engineering and advocates of the value of the method in design and development work, we have become aware of the difficulties experienced by new users when introduced to the subject. The difficulties appear to arise in two areas, viz., (i) an inadequate understanding of relevant electromagnetics and (ii) an inability to appreciate the tricks and traps of modelling. Whilst there are excellent books which deal with electromagnetics, numerical computation and some aspects of modelling, we believe there is no single comprehensive volume which gives the new user or the interested inquirer the foundation needed. This book is therefore aimed at aiding this group: it is *not* intended to help the reader write finite element software.

We have chosen to divide the book into three parts, believing this will facilitate its use. Part I covers 'Relevant Theory', avoiding the use of mathematics as far as possible and aiming to emphasise the physical significance of the subjects discussed. This means that it may not be sufficiently rigorous to satisfy the purist, but it should provide the background needed to understand the later sections. Although three-dimensional situations are briefly considered, the emphasis is on two-dimensional cases, which are both easier to appreciate physically and, with care, adequate for many practical cases. It also shows how the governing potential differential equations for two-dimensional cases may be represented in finite element terms.

Part II discusses the 'Principles of Modelling'. After describing data generation and post-processing possibilities, it outlines basic hardware and software requirements. It concludes by considering representation of saturation and permanent magnet effects and the determination of derived quantities such as inductances and forces. It also describes additions to basic software to deal with excitation prediction, voltage-fed circuits, machine end fields and time stepping. It concludes with a chapter on the importance of engineering judgement when setting up models.

Part III is entitled 'Case Studies'. The examples described here are either taken from, or based on, investigations carried out by the Electromagnetics Group of ALSTOM Research and Technology Centre. They cover a wide range of studies made on electrical machines, transformers and auxiliary equipment. Some of the examples given date back an appreciable time, but have been included either because they make an important point and/or because test data is available to illustrate accuracy. The aim is to show what is possible, to outline the steps necessary and, in some cases, to indicate what can go wrong. The authors' experience is, in the main, with the ALSTOM package SLIM and the examples given used this package. However, the comments throughout are intended, as far as possible, to be of general applicability.

The cases described and the programs used are the result of work carried out in the Electromagnetics Group at ALSTOM Research and Technology Centre at

Stafford over a period of more than 30 years. The work has been a team effort and we are grateful to past and present colleagues for all their contributions in developing and applying finite element software to aid designers in many product companies. A mark of the team's success is that most of these companies are now regularly using finite element methods as part of the design process.

We wish to thank Lucas Varity (formally Lucas Electrical Industries) for permission to use data relating to their products and ALSTOM product companies (particularly ALSTOM Energy Ltd and ALSTOM T&D Transformers Ltd) for allowing us to publish work carried out on their behalf. Thanks are due to the Institution of Electrical Engineers and the Institute of Electrical and Electronic Engineers for permission to reproduce tables and illustrations from papers published in their proceedings. We also wish to thank the management of ALSTOM Research and Technology Centre, Stafford for encouragement to publish. Finally, we thank Helen Johnson for her efforts in producing the final manuscript from our drafts.

Stafford
January 1999

A.B.J.R.
T.W.P.

Symbols

Many symbols are defined as they occur, but frequently used ones are given below:

<i>A</i>	vector magnetic potential	Wb/m
<i>B</i>	magnetic flux density	T
<i>B_R</i>	remanent flux density	T
<i>C</i>	capacitance	F
<i>C_e</i>	coenergy	J
<i>D</i>	electric flux density	C/m ²
<i>E</i>	electric field	V/m
<i>F</i>	'energy' functional	J
<i>f</i>	force	N
<i>H</i>	magnetising force (field strength)	A/m
<i>H_C</i>	coercive force	A/m
<i>J</i>	current density	A/m ²
<i>L</i>	self inductance	H
<i>M</i>	mutual inductance	H
<i>T</i>	vector electric potential	A/m
<i>V</i>	electric potential	V
<i>W_e</i>	stored magnetic energy	J
<i>W_{elec}</i>	stored electric energy	J
<i>δ</i>	depth of penetration = $(2\rho/(\mu_0\mu_r\omega))^{1/2}$ (penetration constant)	m
<i>ε</i>	absolute permittivity = $\epsilon_0\epsilon_r$	F/m
<i>ε₀</i>	permittivity of free space	8.854×10^{-12} F/m
<i>ε_r</i>	relative permittivity	—
<i>μ</i>	absolute permeability = $\mu_0\mu_r$	H/m
<i>μ₀</i>	permeability of free space	$4\pi \times 10^{-7}$ H/m
<i>μ_r</i>	relative permeability	—
<i>ν</i>	reluctivity = $1/\mu$	m/H
<i>σ</i>	electrical conductivity	S/m
<i>Φ</i>	flux linkage	Wb
<i>ρ</i>	electrical resistivity	ohm-m
<i>Ω</i>	scalar magnetic potential (mmf)	A
<i>ω</i>	angular frequency	rads/s

Suffices

d	direct axis
e	eddy (or induced)

q	quadrature axis
s	source (or applied)

Bold letters indicate vector quantities. However, in two-dimensional problems, ***A***, ***E*** and ***J*** will have one component only, and in this case they are represented by normal characters *A*, *E* and *J*.

Contents

List of symbols	xvii
1 Introduction	01
1.1 Why the need for numerical analysis?	01
1.2 Why finite elements?	02
1.3 A note on coordinate systems	03
1.4 Nomenclature	03

Part I: Relevant Theory

2 Electromagnetics	07
2.1 Magnetostatics	07
2.1.1 Scalar magnetic potential, Ω	07
2.1.2 Vector magnetic potential, A	09
2.1.3 Vector magnetic potentials and flux lines	11
2.1.4 Vector magnetic potentials and flux linkages	12
2.2 Electrostatics and scalar electrical potential V	13
2.3 Magnetodynamics	13
2.3.1 The effects of changing fields	13
2.3.2 Sinusoidally changing fields	14
2.3.3 Flux penetration in solid components	14
2.4 Boundary and continuity conditions	16
2.4.1 Boundary conditions	16
2.4.2 Continuity conditions	19
2.4.3 The surface impedance boundary condition	20
2.4.4 Derivation of basic equation	22
2.4.5 Restrictions imposed by assumptions	23
2.4.6 Surface impedance condition in terms of potentials	25
2.5 Approximate representation of currents in scalar potential formulations	25
2.6 Equations for cylindrically-based systems	28
2.6.1 Scalar magnetic potential	28
2.6.2 Vector magnetic potential	30
References	30

3	Approaches to three-dimensional problems	32
3.1	Introduction	32
3.2	Magnetostatics	32
3.2.1	'Total' scalar magnetic potential formulation	32
3.2.2	'Reduced' scalar magnetic potential formulation	32
3.2.3	Combined total and reduced scalar magnetic potential formulation	33
3.2.4	Combined vector electric potential (T) and scalar magnetic potential (Ω)	33
3.3	Magnetodynamics	34
3.3.1	Combined vector electric potential (T) and scalar magnetic potential (Ω) formulation	34
3.3.2	Vector magnetic potential and scalar electric potential formulation	34
	References	35
4	Formulation of electromagnetic equations in finite element terms	37
4.1	Introduction	37
4.2	Shape functions	37
4.3	Methods of deriving the numerical approximation	40
4.4	The nodal method	40
4.5	The variational or energy method	42
4.5.1	Energy and coenergy	42
4.5.2	Scalar magnetic potential	44
4.5.3	Vector magnetic potential	47
4.6	The Galerkin method	50
4.7	Sample finite element calculation	50
4.7.1	Analytical solution	54
4.7.2	Comparison	55
4.8	Accuracy	55
4.9	Concluding comments	56
	References	57
5	Treatment of non-linear materials	58
5.1	Soft magnetic materials	58
5.1.1	Cyclic adjustment of permeabilities	58
5.1.2	Newton–Raphson iteration	59
5.2	Permanent magnets (hard materials)	62
5.2.1	Some principles	62
5.2.2	Vector potential formulation	64
5.2.3	Scalar potential formulation	65
	References	67

6	Derived quantities	68
6.1	Force and force distributions	68
6.1.1	The Lorentz-force (or <i>Bli</i>) method	68
6.1.2	The Maxwell-stress (surface-integral) method	68
6.1.3	Virtual-work method	70
6.1.4	Concluding comments	71
6.2	Inductance	72
6.2.1	Self-inductance	72
6.2.2	Mutual inductance	73
6.2.3	Capacitance	74
	References	74

Part II: Principles of Modelling

7	Data generation (or pre-processing)	79
7.1	Introduction	79
7.2	Discretisation	79
7.2.1	Boundary selection and conditions	79
7.2.2	Distribution of elements	81
7.2.3	Carrying out the discretisation	81
7.2.4	Sub-models	82
7.2.5	Labelling	82
7.2.6	Geometry file	84
7.3	Control data	84
7.3.1	Material properties	84
7.3.2	Excitation data (magnetic field calculations)	85
7.3.3	Boundary conditions	86
7.3.4	Solution and output control	86
	References	87
8	Post-processing	88
8.1	The purpose of post-processing	88
8.2	Contour plotting	88
8.3	Display of stresses: flux densities, electric fields and loss intensities	91
8.4	Arrow display to give direction and approximate strength of stresses	94
9	Selection of hardware	95
9.1	Introduction	95
9.2	Classes of problem	95
9.3	Required memory and processor speed	96

9.4	Examples of relation between computer performance and solution times	97
9.4.1	No-load characteristic	97
9.4.2	Frequency response characteristic	99
9.5	Computer processor speed	101
10	Extensions for steady-state machine problems	103
10.1	Voltage fed devices: the induction motor	103
10.2	Excitation calculations for synchronous generators	105
10.2.1	Combination of finite element and machine circuit equations	105
10.2.2	Connection of load impedance	106
10.2.3	The semi-graphical approach	106
10.3	Machine end-region fields	108
10.4	Machine impedance—frequency calculations and parameter determination	109
10.4.1	Background	109
10.4.2	Simulation principles	110
10.4.3	Stator iron and current representation	111
10.4.4	Three-dimensional effects	112
10.4.5	Selection of rotor permeability	112
10.4.6	The governing equation	113
10.4.7	Additional possibilities	114
	References	114
11	Time stepping for steady-state and transient problems	115
11.1	The need	115
11.2	Relative motion	115
11.3	Principles of time-stepping with induced currents	117
11.3.1	Periodic conditions	117
11.3.2	Transients	118
	References	119
12	The importance of engineering judgement	120
12.1	Introduction	120
12.2	Relevant questions	120
12.3	Two-dimensional approximations of three-dimensional problems	121
12.3.1	End fringing etc.	121
12.3.2	Unequal axial lengths	121
12.3.3	Effects of ends on induced currents	123
12.3.4	Laminated structures and directional parameters	126
	References	129

Part III: Case Studies

13	Steady-state performance of large turbine generators: open circuit and load excitation and reactances	133
13.1	The open-circuit magnetisation curve	133
13.1.1	Study 1: an early trial	133
13.1.2	Study 2: the established process	139
13.2	Load excitation	141
13.2.1	Representation of stator currents	141
13.2.2	Results	142
13.3	Steady-state reactances	144
13.4	Example of discretisation procedure	146
13.5	Additional post-processing	151
	References	152
14	Turbine-generator end-leakage fields and losses	153
14.1	Introduction	153
14.2	660 MW turbine-generator end region	155
14.3	Modelling of the end region	155
14.4	End air region—Model A	157
14.4.1	Boundary conditions	157
14.4.2	Stator winding representation	159
14.4.3	Rotor winding representation	161
14.4.4	End-region discretisation	161
14.4.5	Calculations and results	162
14.5	Stator core region—Model B	164
14.5.1	Boundary conditions	164
14.5.2	End-region discretisation	167
14.5.3	Calculations and results	167
14.6	Overall comment	175
	References	175
15	Turbine-generator transient parameters by simulated frequency response testing	176
15.1	Introduction	176
15.2	Modelling the turbine generator	176
15.3	Calculation of operational impedances	177
15.4	Calculation of field–stator transfer function	179
15.5	Results for the 350 MW generator	180
15.6	Reactances and time constants	182
15.7	Conclusions	183
	References	184

16	Simulation of turbine-generator short circuit and application to parameter prediction	185
16.1	Introduction	185
16.2	The turbine generator	186
16.3	The finite element model	186
16.4	Time stepping	188
16.5	Induced currents	189
16.6	Results	189
16.7	Application to more complex cases	193
16.8	Concluding comments	194
	References	194
17	Performance calculations on permanent-magnet d.c. motor	195
17.1	Introduction	195
17.2	The trial motor	195
17.3	The model	195
17.4	Results	197
18	Performance prediction for switched reluctance motors (two- and three-dimensional)	201
18.1	Introduction	201
18.2	The experimental s.r.m.	202
18.3	Two-dimensional calculations	203
18.3.1	Results	204
18.4	Three-dimensional calculations	207
18.4.1	Results	208
	References	211
19	Induction-motor steady-state performance prediction	212
19.1	Introduction	212
19.2	Some finite element aids to performance prediction	212
19.3	Improved equivalent circuit	213
19.4	Two-dimensional finite element method with sinusoidal time variation	216
19.4.1	Theory	217
19.4.2	Constant-voltage excitation	218
19.4.3	Rotor-bar conductivity	218
19.4.4	Magnetic non-linearity	218
19.4.5	Representation of the outer frame and shaft	219
19.4.6	Stator end leakage reactance	220
19.4.7	Iron loss	221
19.4.8	Skew	221
19.4.9	Treatment of slip	221
19.4.10	Application	221

19.5	Combined circuit/finite element method	223
19.6	Full two-dimensional time-stepping finite element method	224
19.6.1	Time-stepping method	224
19.6.2	Operation from constant voltage	225
19.6.3	Movement	226
19.6.4	Application	226
	References	230
20	Transformer fields and reactances	232
20.1	Introduction	232
20.2	The axi-symmetric single-phase model	232
20.2.1	Application of axially-symmetric model to full load calculations	234
20.3	The Cartesian three-phase model	236
20.3.1	Application of Cartesian model to a three-phase transformer	237
21	Electric-field analysis in power transformers	242
21.1	Introduction	242
21.2	Design of a stress-control ring on an HV transformer winding	244
21.2.1	General	244
21.2.2	Finite element model	245
21.2.3	Finite element mesh	246
21.2.4	Effect of stress-control rings	248
21.3	Electric-field calculation in the region around the HV lead of a large single-phase transformer	251
21.4	Electric-field distribution in the vicinity of an HV lead clamp	259
21.4.1	Modelling	259
21.4.2	Results	264
22	Electric-stress calculations in bushings and surge arresters	267
22.1	Introduction	267
22.2	Zinc oxide surge arresters	267
22.2.1	The axi-symmetric model	268
22.2.2	Results	270
22.3	High-voltage instrument transformer bushing assembly	271
22.4	High-voltage insulating column	273