

STUDIES IN LOGIC
AND
THE FOUNDATIONS OF MATHEMATICS

VOLUME 89

J. BARWISE / D. KAPLAN / H. J. KEISLER / P. SUPPES / A. S. TROELSTRA
EDITORS

*Non-Classical Logics,
Model Theory
and Computability*

Edited by
A. I. ARRUDA / N. C. A. DA COSTA / R. CHUAQUI

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NON-CLASSICAL LOGICS, MODEL THEORY, AND COMPUTABILITY

Proceedings of the Third Latin-American Symposium on
Mathematical Logic, Campinas, Brazil, July 11-17, 1976

Edited by

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PREFACE

This volume constitutes the Proceedings of the Third Latin American Symposium on Mathematical Logic which was held at the State University of Campinas, Campinas, São Paulo, Brazil from July 11 to July 17, 1976. The meeting was sponsored by the State University of Campinas, the Association for Symbolic Logic, and the Division of Logic, Methodology, and Philosophy of Science of the International Union of History and Philosophy of Science. The meeting was organized by an executive committee consisting of A. I. Arruda (Chairman), R. B. Chuaqui, N. C. A. da Costa, and F. Miró Quesada.

The Symposium was divided into three sections: Non-classical Logics, Model Theory, and Computability. The papers which appear in this volume are the texts, at time considerably expanded and revised, of most of the addresses presented by invitees to the meeting. Also included are expanded versions of short communications which the editors thought would give an idea of the present topics of research in Latin America.

Abstracts of all addresses and contributed papers were issued to members at the beginning of the Symposium and appeared in the Journal of Symbolic Logic. The titles of all papers are listed in the end of the introduction to this volume.

The editors would like to acknowledge the financial support given to the meeting by the following institutions: State University of Campinas,

Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Organization of American States, International Union of History and Philosophy of Science, Banco do Estado de São Paulo, and Ministry of Foreign Relations of Brazil.

The editors would also to thank North-Holland Publishing Co. for the inclusion of this volume in the series Studies in Logic and the Foundations of Mathematics.

The Editors.

Departamento de Matemática
Universidade Estadual de Campinas
February 1977.

The editors N. C. A. da Costa and R. B. Chuaqui wish to express their appreciation for the work of A. I. Arruda. Her efforts were the main force behind these Proceedings.

A SHORT HISTORY OF THE LATIN AMERICAN LOGIC SYMPOSIA

We would like to give an idea of part of the development of Mathematical Logic in Latin America through a short history of the three symposia sponsored by the Association for Symbolic Logic. Because it is mainly based on the recollections of the editors, who were participants or organizers of the three symposia, it will not be a completely balanced history.

Since the early sixties there had been the intention of having a meeting of the A.S.L. in South America. The good experience with the European meetings encouraged the idea of their being held in other continents. William Craig, when President of the A.S.L., for instance, talked about it with several people.

The first concrete step, however, was taken in 1967 by A.S.L. President Abraham Robinson. While in attendance at the logic year at the University of California, Los Angeles, Professor Robinson approached Rolando Chuaqui, who was visiting from the University of Chile, with the idea of having a first Latin American meeting in Chile. This was immediately accepted. Robinson then named the first Advisory Committee for Logic in Latin America formed by himself, as A.S.L. President, David Kaplan from UCLA, Antonio Monteiro, from Universidad Nacional del Sur, Bahía Blanca, Argentina, and Rolando Chuaqui (Chairman) from University of Chile, Santiago.

The meeting was first planned for January 1969. Due to internal difficulties in the University of Chile it could not take place at this time. When Chuaqui moved to the Catholic University of Chile (Santiago), in 1969, he found enthusiastic support for the idea from the Rector Fernando Castillo Velasco and other university authorities, and it was decided to hold the First Latin American Symposium on Mathematical Logic (I SLALM) in July 1970.

Abraham Robinson was an indefatigable worker for the success of this

meeting. He sent letters to all possible organizations asking for financial support. We reproduce below Robinson's proposal for I SLALM:

A PROPOSAL FOR THE ORGANIZATION OF A SEMINAR AND COLLO-
QUIUM IN MATHEMATICAL LOGIC, TO BE HELD IN CHILE.

(Revised Version, June 3, 1969)

1. There is, at present, a small but growing group of mathematical logicians in South America. The following proposal is designed to encourage the further development of interest in Mathematical Logic in that region. It is patterned after a format which has been employed repeatedly in recent years in Western Europe and has proved successful in strengthening Mathematical Logic there. The Association for Symbolic Logic (ASL), which has co-sponsored some of these activities, stands ready to give its backing also to the plan outlined below. While the proposal is stated in rather precise terms, modifications would of course be in order.

2. The event is to take place at the Universidad Católica de Chile in July 1970. It is to include:

(i) A seminar of about three weeks' duration, to consist of three courses, at an intermediate level, in Model Theory, in Set Theory, and in Recursion Theory.

(ii) A colloquium of 3-5 days' duration, to be held at the end of the seminar, to include invited addresses and contributed papers. The scientific standards of the colloquium would be strengthened by including the seminar course instructors in the list of invited speakers. The colloquium will be sponsored or co-sponsored by the Association for Symbolic Logic. This will ensure that an account of the colloquium, including abstracts, will be published in the Journal of Symbolic Logic. It is hoped that the Mathematical Society of Chile and the Universidad Católica de Chile will join us in co-sponsoring the meeting.

3. The Association for Symbolic Logic will offer advice and moral support to the organizers of the seminar and colloquium. We are applying to the Pan American Union for financial su-

port. We also hope that it may be possible to obtain financial support from ICSU through International Union of the History and Philosophy of Science (IUHPS). In case such support is forthcoming IUHPS will be invited to co-sponsor the meeting.

4. The Association for Symbolic Logic has created an Advisory Committee on Logic in Latin America. Its members are Professor Rolando Chuaqui, Universidad Católica de Chile (chairman), Professor David Kaplan, University of California at Los Angeles, Professor Antonio Monteiro, Bahía Blanca, Argentina, and Professor Abraham Robinson, Yale University, New Haven, Connecticut. It will be the general function of the committee to encourage the development of Symbolic Logic in Latin America, and will be one of its major task to assist in the organizations of the activities outlined in the present proposal.

Abraham Robinson, President
Association for Symbolic Logic

Department of Mathematics
Yale University
New Haven, Connecticut 06520.

The I SLALM was held at the Catholic University of Chile in Santiago, in July 1970, according to Robinson's proposals: Three weeks of short courses, the Seminar, and one week of invited lectures and contributed papers, the Colloquium.

The courses in the Seminar were three:

A. Lévy (Israel), *Foundations of Set Theory*,
J. R. Shoenfield (U.S.A.), *Degrees of Unsolvability*,
R. Sikorski (Poland), *Boolean Algebras*.

A summary of the Colloquium appeared in the Journal of Symbolic Logic. We reproduce the main features (J.S.L. vol. 36, 576-581):

A Latin American meeting of the Association for Symbolic Logic was held in Santiago, Chile, from July 27 to July 31. The meeting was co-sponsored by the Catholic University of Chile, The Organization of American States, and the International Union for History and Philosophy of Science. The meeting was preceded by

a Seminar on Mathematical Logic at the Catholic University of Chile.

There were thirteen invited one hour lectures and thirteen short communications presented at the meeting. The lectures were the following:

- M. Morley (U.S.A.), *Some applications of topology to model theory I.*
- R. Sikorski (Poland), *On strictly positive measures.*
- S. Kochen (U.S.A.), *Quantum logic.*
- J. Shoenfield (U.S.A.), *Hilbert's tenth problem.*
- N. C. A. da Costa (Brazil), *Inconsistent formal systems.*
- R. Cignoli (Argentina), *Moisil and Post algebras.*
- G. Stahl (Chile), *Questions with numerical and totality requests.*
- M. Morley (U.S.A.), *Some applications of topology to model theory, II.*
- A. Lévy (Israel), *Normal ultrafilters and supercompact cardinals.*
- A. Robinson (U.S.A.), *Model theoretic aspects of algebra.*
- F. Alvim (Brazil), *Logic of quantum mechanics.*
- G. Reyes (Canada), *Pro-discrete spaces.*
- A. Robinson (U.S.A.), *Finite and infinite forcing in model theory.*

Below are reproduced the abstracts of the short communications presented at the meeting (we reproduce only the author and title):

- M. M. Fidel (Argentina), *On the calculi \mathcal{C}_n , $1 \leq n \leq \omega$.*
- M. M. Fidel (Argentina), *An algebraic study of logics with constructible falsity.*
- M. M. Fidel (Argentina), *Moisil algebras and modal logic.*
- I. Bicudo (Brazil), *Dually equivalent structures: an example.*
- L. de Moraes (Brazil), *On discursive predicate calculus.*
- R. Chuaqui (Chile), *A representation theorem for linearly ordered cardinal algebras.*
- N. C. A. da Costa (Brazil), *On the systems T and T^* .*
- A. I. Arruda (Brazil), *On Griss' propositional calculus.*
- E. Nemesszeghy (Chile), *A calculus of "δ" elimination (eliminability).*
- L. P. de Alcantara (Brazil), *On the relative consistency of two systems of set theory.*
- A. J. Engler (Brazil), *Symmetrical lattices.*

Professor Robinson participated actively in the meeting even to the

point of giving one of his lectures in his hotel because he fell ill.

From the list of Latin American participants, it can be seen that they came only from Brazil, Argentina and Chile. From the beginning it was a problem to contact the Latin American logicians. The A.S.L. committee, at first, tried to obtain information directly from the members appearing in the roster of the A.S.L. living in Latin American countries. Only a few responses were received. Most contacts were made through friends. The main groups represented were from Bahía Blanca, Argentina, from the State of São Paulo (Universidade Estadual de Campinas (UNICAMP), and Universidade de São Paulo (USP)), Brazil, and from Santiago, Chile.

Most of the financial support was provided by the Catholic University of Chile, and the Organization of American States (O.A.S.). The O.A.S. treated the Symposium as two meetings and gave double their usual amount.

During the I SLALM logicians from the different countries represented met with A. Robinson. It was decided that the next Symposium would be in Brazil. The new advisory Committee for Logic in Latin America was to be composed of the A.S.L. President (ex-officio), Newton C. A. da Costa (Brazil), Antonio Monteiro (Argentina), and Rolando Chuaqui (Chairman, Chile). (The A.S.L. President disappeared from the Committee in the December 1972 issue of the J.S.L.)

After consultations among Brazilian universities, the Brazilian delegation decided that it was best to hold the II SLALM at the University of Brasília, in July 1972. The last act of A. Robinson as President of the A.S.L. was to send a letter to the Rector of the University of Brasília offering the sponsorship of the A.S.L. for the meeting.

In preparation for the symposium, in January 1972, there was a pre-symposium mainly for Brazilian students, at the Technological Institute of Aeronautics (ITA) in São José dos Campos, São Paulo.

The chairman of the Organizing Committee of the II SLALM was Fausto Alvim. The meeting had a different character than the first as it was not composed of two parts: a Seminar and a Colloquium. It lasted for three weeks and its main activities were short courses of about ten lectures each. There were a few research lectures. The J.S.L. received no account of this meeting, so recollections of its courses and lectures may not be complete.

The following short courses were given:

R. Chuaqui (Chile), *Consistency and independence of the axiom of choice in the impredicative theory of classes.*

- A. Robinson (U.S.A.), *Non Standard analysis*.
 A. Robinson (U.S.A.), *Forcing in model theory*.
 M. Dickmann (Chile), *Back and forth arguments in model theory*.
 R. Cignoli (Argentina), *Algebra of logic*.
 O. Porchat (Brazil), *Elementary logic*.

There were individual lectures by M. Guillaume (France), L. Monteiro (Argentina), P. Suppes (U.S.A.), A. I. Arruda (Brazil), E. Farah (Brazil). N. C. A. da Costa, who was at this time visiting the University of California at Berkeley, presented a paper by title.

The main support for this meeting came from Brazilian sources, especially from the Conselho Nacional de Pesquisas (CNPq), Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), and the University of Brasília. There were a large number of Brazilian participants (most of them attending the course: Elementary Logic) and, as in Santiago, logicians from Chile, Argentina, plus one from Caracas, Venezuela.

It was decided at this meeting that the III SLALM would take place in Bahía Blanca, Argentina, in July 1974. However, due to the difficult situation in that country this was not possible, and there was no Symposium in 1974.

The revival in interest in these symposia came in March 1975. After an extended visit to the Catholic University of Chile, in Santiago, Professor Alfred Tarski visited UNICAMP with R. Chuaqui, during the first two weeks of March. A small logic meeting was organized there by Ayda I. Arruda with the participation of the two Visiting Professors and logicians from USP, UNICAMP, and University of Pernambuco, Brasil. The Proceedings of this meeting were published by the Institute of Mathematics, Statistics, and Computing Science (IMECC) of UNICAMP. Support for this meeting came from UNICAMP.

Due to the success of this meeting and the growing number of Brazilian logicians it was decided that the III SLALM would be held at the IMECC, UNICAMP, in July 1976. The A.S.L., through its President, J. R. Shoenfield, agreed to sponsor the meeting and named a new Advisory Committee on Logic in Latin America, formed, this time, by Newton C. A. da Costa (Brazil), Francisco Miró Quesada (Peru), and Rolando Chuaqui (Chairman, Chile). The incorporation of the Peruvian member signaled the participation of a new group of logicians from the universities in Lima, besides the Chilean, Brazilian and Argentinian groups.

After receiving a letter of J. R. Shoenfield, the Rector of UNICAMP, Dr. Zeferino Vaz, accepting the suggestion of the Director of the IMECC, Dr. Ubiratan D'Ambrosio named the Organizing Committee, formed by: Rolando Chuaqui (Universidad Católica de Chile, Santiago), Newton C. A. da Costa (USP, Brazil), Francisco Miró Quesada (Universidad Cayetano Heredia, Lima, Peru), and Ayda I. Arruda (Chairman, UNICAMP, Brazil).

In anticipation of the meeting there was a logic semester in the Department of Mathematics of IMECC, and the Center of Logic, Epistemology, and History of Science of UNICAMP. Advanced courses and seminars were given by R. Chuaqui (Visiting Professor at UNICAMP), N. C. A. da Costa (USP) and A. R. Raggio (IMECC). There were also advanced seminars from June 28 to July 9, with the participation of J. Kotas (Poland), C. Pinter (U.S.A.), M. Benda (USP), R. Routley (Australia), and R. Fraïssé (France). These courses and seminars were attended by logicians from Campinas and São Paulo as well as by students from UNICAMP, USP, and the Catholic University of Chile.

The Third Latin-American Symposium on Mathematical Logic was divided into three sections: Model Theory, Non-Classical Logics, and Applied Logic; and lasted for a week (July 11 to July 17, 1976). The financial support was given by: UNICAMP, Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), The Organization of American States (O.A.S.), The International Union for History and Philosophy of Science, Division of Logic, Methodology and Philosophy of Science (IUHPS/DLMPS), and the Bank of the State of São Paulo.

The scientific program was the following:

JULY 12.

09:15-09:30 A. M. — Opening session.

09:30-10:20 A. M. — R. Chuaqui (Chile, Brazil), *A semantical definition of probability.*

10:40-11:30 A. M. — N. C. A. da Costa (Brazil), *On Jaśkowski discussive logic.*

2:00 - 2:20 P. M. — A. Loparić (Brazil), *A semantical study of some propositional calculi.*

2:20 - 2:40 P. M. — L. H. Lopez dos Santos (Brazil), *Some remarks on discussive logic.*

2:40 - 3:00 P. M. — E. H. Alves (Brasil), *On paraconsistent logic.*

3:00 - 3:20 P. M. — C. Lungarzo (Brazil), *A paraconsistent infinitary propositional calculus.*

4:00 - 5:00 P. M. — M. Benda (U.S.A., Brazil), *Some directions in model theory.*

JULY 13.

9:00 - 9:40 A. M. — A. I. Arruda (Brazil), *On the imaginary logic of N. A. Vasil'ev.*

9:40-10:10 A. M. — A. R. Raggio (Brazil), *Semi-formal Beth tableaux.*

10:40-11:30 A. M. — F. G. Asenjo (U.S.A.), *Formalizing multiple location.*

2:00 - 2:20 P. M. — L. P. de Alcantara (Brazil), *On the equivalence of some axioms of strong infinity.*

2:20 - 2:40 P. M. — P. A. S. Veloso (Brazil), *Two model theoretic properties of class of representable relation algebras.*

2:40 - 3:00 P. M. — R. Morais (Brazil), *Projective logic and projective Boolean algebras.*

3:00 - 3:20 P. M. — X. Caicedo Ferrer (Colombia), *Independent sets of axioms in $L_{\kappa\alpha}$.*

4:00 - 5:00 P. M. — M. Krasner (France), *Galois theory of relations.*

JULY 14.

9:00-10:00 A. M. — A. Tarski (U.S.A.), *Relation Algebras* (replay of a lecture recorded in videotape at UNICAMP in March, 1975).

10:30-11:20 A. M. — C. Pinter (U.S.A.), *Some theorems on omitting types with applications to model completeness, and related properties.*

2:00 - 2:20 P. M. — J. Simon (Brazil), *Polynomially bounded quantification over higher types and a new hierarchy of the elementary sets.*

2:20 - 2:40 P. M. — M. S. de Gallego (Brazil), *On the abstract family of languages of categorical types.*

2:40 - 3:00 P. M. — H. P. Sankappanavar (Brazil), *On the decision problem of the congruence lattice of pseudocomplemented semilattices.*

3:00 - 3:20 P. M. — I. Mikenberg (Chile), *A logical system for partial algebras.*

4:00 - 5:00 P. M. — R. Solovay (U.S.A.), *On random r. e. sets.*

JULY 15.

9:00 - 9:40 A. M. — M. Dascal (Brazil, Israel), *Conversational relevance.*

- 9:40-10:20 A. M. — F. Miró Quesada (Peru), *Heterodox logics and the problem of the unity of logic.*
- 10:50-11:40 A. M. — R. Routley (Australia), *Ultramodal logic as universal.*
- 2:00 - 2:20 P. M. — L. Piscoya (Peru), *Probability and empirical scientific statements.*
- 2:20 - 2:40 P. M. — D. M. de Souza Filho (Brazil), *Some remarks on meta-language.*
- 2:40 - 3:00 P. M. — R. Lintz (Brasil), *Organic and inorganic logic.*
- 3:00 - 3:20 P. M. — A. M. Sette (Brazil), *Fraïssé's and Robinson's forcing.*
- 4:00 - 5:00 P. M. — R. Fraïssé (France), *Present problems about intervals in relation theory and in logic.*

JULY 16.

- 9:00-10:00 A. M. — E. G. K. López-Escobar (U.S.A.), *Infinite rules in finite systems.*
- 10:30-11:30 A. M. — J. Kotas (Poland), *On some modal systems defined in connexion with Jaśkowski's problem.*
- 2:00 - 3:00 P. M. — J. R. Shoenfield (U.S.A.), *Quantifier elimination in fields.*
- 3:00 - 4:00 P. M. — Closing session.

The following communications were presented by title:

- A. A. Mullin (U.S.A.), *Applications of fixed point theory to number theory.*
- L. F. Monteiro (Argentina), *Algèbres de Post et de Moisil trivalentes monadiques libres.*
- I. M. L. D'Ottaviano (Brazil), *Fuzzy sets in da Costa's systems T and T*.*
- R. Routley (Australia), *Dialectical set theory.*
- R. Routley (Australia), *Choice of logical foundations: ultramodal logic and dialectical foundations.*

There were 69 participants distributed by countries as follows:

Brazil - 43,	U.S.A. - 6,
Chile - 6 ,	France - 2,
Argentina - 3,	Poland - 1,
Peru - 4,	Canada - 1,
Colombia - 2,	Australia - 1.

This is the first time that full Proceedings of a Latin American Symposium are appearing. For the first symposium, North-Holland, after the meeting, offered to publish the invited lectures in full in its series Studies in Logic. However, as the publication was not planned in advance, it was too late to gather enough articles for the volume. Shoenfield's course, *Degrees of Unsolvability*, however, was published by North-Holland. For the second symposium, Springer-Verlag in its series Lecture Notes, agreed to publish three of the courses in one volume. Due to various problems the manuscript was never sent to the publisher.

When we compare the list of papers presented in 1970 with those of this meeting, we have reason to be encouraged: the number and overall quality of the contributions of Latin American logicians have certainly increased. The picture is not uniformly bright, however. In Chile, due mainly to economic difficulties, there has been no marked increase. The group from Bahía Blanca, Argentina, has been scattered, as most of its members have been excluded from the university. This explains the scarcity of papers for this symposium from this group, which made significant contributions to the algebra of logic.

On the bright side, we have the incorporation of the Peruvian group and the significant development of logic in Brazil, where besides the group around da Costa (USP, and UNICAMP) in the State of São Paulo, there are now participants from other sectors of the country. It is worthwhile to note also the support that the growing group of UNICAMP is receiving from the university, specially from the Director of the IMECC.

In 1969, A. Robinson said of the Latin America Logic group, that it was a small but growing group. Perhaps, now it is not so small, but, we hope, still growing.

Ayda I. Arruda,
Newton C. A. da Costa,
Rolando Chuaqui.

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