

Introduction to
MECHANICS,
MATTER, AND WAVES

by

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and

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PREFACE

This book has grown out of the authors' experience in teaching one year of a two-year sequence of courses in general physics at the Massachusetts Institute of Technology. More material than we have covered in any one year has been included, and it is anticipated that the book may find some use as a text for a second course in mechanics and the related subjects.

Mechanics can be and frequently is taught as a highly deductive subject. The foundations laid down by Galileo and Newton 300 years ago can be taken as postulates and the subject can be developed logically from there into a concise mathematical scheme. A presentation of this kind, although valuable and essential at some point in a student's education, tends to obscure the importance of experiments and can give the beginning student a distorted picture of physics as it really is. A totally analytic approach often overemphasizes the "given the forces—find the motion" situation at the expense of the "given the motion or behavior—what are the model and forces from which the phenomena can be understood" type of situation. We have in the present approach emphasized the study of interactions through observations of motion and have emphasized further that mechanics deals with motion as influenced by *all* the different types of interactions in nature: electric, magnetic, nuclear, etc. as well as the traditional contact, spring, and gravitational forces.

We are led naturally to first investigate two-body collisions* and so can begin the course with inertial mass, momentum and its conservation, and center-of-mass motion—all fundamental and interesting aspects of the physics of motion which require no calculus. The concept of force then emerges as the rate of momentum transfer from one body to another during an interaction and, by this time, the calculus course which the students take concurrently is well into differentiation.

The conservation laws—conservation of mass, momentum, energy, and angular momentum—have been given more than the usual emphasis. In

* The apparatus for the crucial collision experiments as described in the text is relatively simple, and we have found that *quantitative* lecture experiments are well worth the effort and class time expended. Stroboscopic flash pictures have been taken with a Land camera, developed, and then projected and analyzed in front of the students.

fact, collision experiments and momentum conservation not only serve as the starting point but as a central theme through much of the book.

Chapters 5, 8, 10, 12, and 13 discuss, as their titles indicate, special important examples of forces and motion, and have been chosen and placed so as to illustrate the basic concepts as they are developed. Chapter 8, for example, which is on oscillations, follows directly the introduction of potential energy, and Chapter 10 which is on orbits in the gravitational field and alpha-particle scattering follows the introductory chapter on angular momentum. The discussion of planetary motion and the formulation of the universal law of gravitation from Kepler's laws are quite thorough.

The chapter on moving coordinate systems and inertial forces serves both as a summary and as a re-examination of the discussion of the basic laws of motion in the first part of the book. Here we have brought out the important role that the coordinate system plays in the description of motion, and the distinction between interaction forces and inertial forces is emphasized. There *is* a force tending to push passengers toward the rear of an accelerating car, and there *is* such a thing as a centrifugal force. We feel the time is well spent in having a student see why and from what viewpoint his preconceived notions are right.

The transition from the study of the gross motion of bodies to the internal motion in matter and the associated macroscopic properties of matter is accomplished in a chapter on the temperature concept and thermal interactions and a chapter on the elementary theory of the atomic structure of matter. The following chapters on kinetic theory, thermodynamics, and properties of matter have been woven together, and the atomic-molecular interpretation of concepts such as internal energy and entropy and properties of matter have been kept in the foreground.

The discussion of the kinetic theory of gases is comparatively extensive both in terms of basic concepts and discussion of experiments. In the analysis of specific heat the failure of the classical theory is emphasized and a qualitative quantum-mechanical interpretation of experimental results is given.

The role of intermolecular forces in the interpretation of the equation of state of real gases is studied in some detail, together with molecular interpretations of other gross mechanical properties of gases and liquids.

The mechanics of deformable bodies and waves is introduced through a study of what happens when a deformable body is given an impulse by an external force. Through a combination of experiments and application of conservation of momentum we obtain quickly the behavior of

wave pulses, their wavespeed and energy content, and examples are given involving both transverse and longitudinal waves in different media. From the principle of superposition, waves of arbitrary shape are then constructed and the properties of harmonic waves are studied. Similarly, the problem of the reflection of waves at the interface between two elastic media is first solved for a wave pulse by application of the conservation laws.

The authors wish to acknowledge with thanks the helpful criticism of many of their students and colleagues. Our wives, Doris Ingard and Margaret Kraushaar, have contributed to the book and its completion in many ways, including heroic displays of patience.

June 1960

U.I.
W.K.

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CHAPTER 1

INTRODUCTION

Summary. Physics is vitally concerned with motion and its causes. In this chapter some of the early ideas of motion and the development of physics are discussed. There follows a review of the basic concepts of length and time and their ranges and measurement, and a brief discussion of the mathematical description of motion.

1-1 The scope of physics. The scope of physics is very broad indeed—so broad, in fact, that we cannot say precisely where the boundaries lie. If we regard geology as the study of the planet earth, or biology as the study of life, we must regard physics as something like the study of matter, motion, and energy. But this is not very helpful in defining the boundaries of physics, because matter, motion, and energy exclude so very little. What in all of science and technology is not, in some sense, concerned with one or more of these three very general and nebulous ideas?

The very early investigators of nature were called natural philosophers, and they were concerned with such diverse matters as the motion of the heavenly bodies, navigation, projectile motion, the ultimate nature of matter and space, strength of construction materials, the weather, surveying, respiration of animal and plant life, and even alchemy and astrology. Natural philosophy, in other words, was what we now call science, and it included the applied aspects as well as the very basic questions about life, matter, and motion and its causes.

Progress in science, if measured in discoveries and number of profound ideas, became marked during and after the 17th century. Later, natural philosophers tended to specialize and the various divisions of science—biology, chemistry, physics, meteorology, astronomy, etc.—came into being. As specific aspects of applied science became important to society and the number of specialists in these fields increased, engineering divisions were established. But physics, astronomy, chemistry, and biology retained positions that corresponded most closely to the old natural philosophy. These are the sciences upon which the others and the vast engineering applications are based.

Present-day physicists are concerned with two broad categories of problems. The first type, which might be called particle and field physics, concerns the nature of the interactions or forces which exist between the constituents of matter. The second type concerns the gross behavior of matter in bulk.

Gravitational, electromagnetic, and nuclear forces are perhaps the most familiar interaction forces, and a great variety of experimental techniques has been developed for their study. In the case of gravitational interaction the sky has been the best laboratory, and the motion of the planets about the sun has provided the best clue to the basic properties of this type of interaction. The study of electromagnetic interactions, on the other hand, can be carried out in the laboratory with relatively simple experimental arrangements. However, as we probe into the forces between the particles in the nucleus of an atom, experimental techniques and apparatus of great complexity (and size) are required, as exemplified by the high-energy accelerators in which particles are hurled against each other at speeds which often are close to the speed of light.

The problem of interaction forces is considered solved when laws of motion (or preferably a single law of motion) can be formulated which can explain the experimental observations and from which the outcome of new experiments can be predicted. It is clear that such a law must involve (man-made) concepts which cannot be defined or explained but must be accepted as basic building blocks in the theory. Such quantities and concepts are, for example, length, time, and electric charge, about which questions like "what is time," "what is length," and "what is electric charge" are not answerable in terms of more fundamental quantities.

Gross matter, or matter in bulk, we now know is made up of atoms and molecules. The basic problem here is to try to determine the bulk properties of matter in terms of the fundamental forces between the atoms and molecules. For example: How can the thermal expansion of a body be explained and calculated in terms of the intermolecular forces? How are the intermolecular forces related to the speed of sound in matter? What is the microscopic nature of the electrical resistance of a body and what properties of the molecules and their arrangement make the difference between an electrical conductor and an insulator? There is an endless number of questions of this type which we wish to answer and understand in terms of molecular motion and forces. Although great advances have been made in this field in the past few decades, our understanding of matter is far from complete. Therefore in many cases one has to be satisfied with "phenomenological" descriptions of the behavior of matter in bulk. These descriptions, which involve such concepts as electrical resistance, modulus of elasticity, thermal conductivity, reflectivity, etc., are of great practical importance in engineering applications.

Physics is very much concerned with motion. Motion is evident all about us. We move, airplanes move, trees move, clouds move, waves in water move. But these motions are often complicated and, as we shall see presently, one of our first tasks in physics is to explore the simple motion of simple bodies. Then we shall investigate the manner in which the

motions of bodies change—how the presence of one body may affect the motion of another. There is also “hidden” motion. Upon closer examination we shall find that even if a body as a whole is at rest, its constituents are in constant (thermal) motion. The molecules in a gas fly back and forth and collide with one another and with the surrounding walls, and the molecules in a solid vibrate about their equilibrium positions. On an even smaller scale we find “motion” of the electrons in an atom and of neutrons and protons in the nucleus. In this sense it can freely be said that the world is “rest-less,”* and it is therefore natural that the study of physics should start with the study of motion. Most properties of matter can be related to motion, at least on an atomic or subatomic scale. Heat and temperature can be directly described in terms of molecular motion, and electromagnetic waves (radio, heat, and light waves, and x-rays and γ -rays) can be related to the motion of charged particles. Traditionally, the study of motion and its causes has come to be called “mechanics”; other branches of physics are electromagnetism, optics, atomic physics, nuclear physics, etc. All these aspects of physics are concerned with motion too, but what sets mechanics apart from the rest is that mechanics is concerned with motion and its causes *per se*, and with the formulation of general laws and properties of motion that are independent of the type of interaction (gravitational, electromagnetic, nuclear, etc.) involved.

1-2 Motion: brief historical introduction. The ideas about the motion of bodies that come naturally to a casual observer are for the most part the same as the views recorded by Aristotle (384–322 B.C.). Stripped of the philosophical system of which they were a part, the Aristotelian views of motion were as follows: All accessible matter is composed of Earth, Water, Fire, and Air, and these four elements have natural states—Fire and Air above Earth and Water. From this it follows that smoke and steam rise, and stones and water fall. Since big objects contain more Earth than small objects, big objects fall faster than small ones. The time of free fall is inversely proportional to weight. The natural state of bodies is one of rest, so it is necessary to do something to a body to make it move or to keep it moving. The heavenly bodies are not within the scheme and are not subject to the ordinary laws of physics. They, including the sun and the moon, revolve about the earth in circular paths, and the earth does not move.

These are not unreasonable ideas. The “deductions” and statements are in accord with qualitative observation. It should be pointed out that the Aristotelian views of motion were a part of a much larger whole—a

* For a lucid survey of physics, see Max Born, *The Restless Universe*, Dover Publications, Inc., New York, 1951.

philosophy which engulfed religion as well as nature, and in which the completeness, beauty, and symmetry of the philosophy itself took precedence over "minor" discrepancies.

The authority of the Aristotelian doctrine became firmly established and was not seriously questioned for many centuries. There were some grumblings, to be sure, but not until the European Renaissance did anyone offer contrary arguments and compelling contrary evidence. One point of controversy concerned celestial motion, the apparent motion of the sun, moon, planets, and stars; another concerned the motion of bodies on the earth.

Celestial motion. Less than a century after Aristotle, Aristarchus put forth the suggestion that the stars were fixed and that the earth and planets rotated about the sun. There was apparently no general acceptance of his idea. It did offer a reasonable explanation for the variations in the brightness of the planets (the earth-planet distances would of course vary) but it seemed to have no other advantage over the Aristotelian geocentric view. According to the heliocentric view, there should be an annual parallax, i.e., a variation in the apparent position of a star as the earth circled the sun. This parallax went unobserved, and the heliocentric idea found little favor among the astronomers of the time, especially since it contradicted Aristotle's teachings.

The geocentric view was greatly refined by Ptolemy (70–147 A.D.). By this time astronomical observations of the planets, sun, moon, and fixed stars were sufficiently accurate and complete to indicate that not all celestial objects could be circling the earth in perfectly circular earth-centered paths. The rigid geocentric assumptions were therefore relaxed somewhat. The earth was kept at rest at the center of the celestial sphere and the sun, moon, planets, and stars still moved around the earth. The motions, however, were not along perfectly circular earth-centered paths. Ptolemy found it necessary to invoke an increasingly complicated set of additional assumptions. First there were epicycles, shown in Fig. 1–2. These were needed to account for the retrograde motion or apparent reversals in the paths of planets. Then the earth was placed slightly off the center of rotation, as shown in Fig. 1–3. This was needed to explain the observed changes in the brightness of planets. Finally, it was found necessary to assume that while the point P , shown in Figs. 1–2 and 1–3, moved in a circular path about some point O (not the earth), it moved with a varying angular velocity about that point. The angular velocity was constant about some other point in space, say A in Fig. 1–3. With this elaborate mechanism, Ptolemy successfully explained most of the features of celestial motion. It was a gigantic and skillful accomplishment, and for the first time the positions of celestial bodies could be predicted in advance with reasonable accuracy.