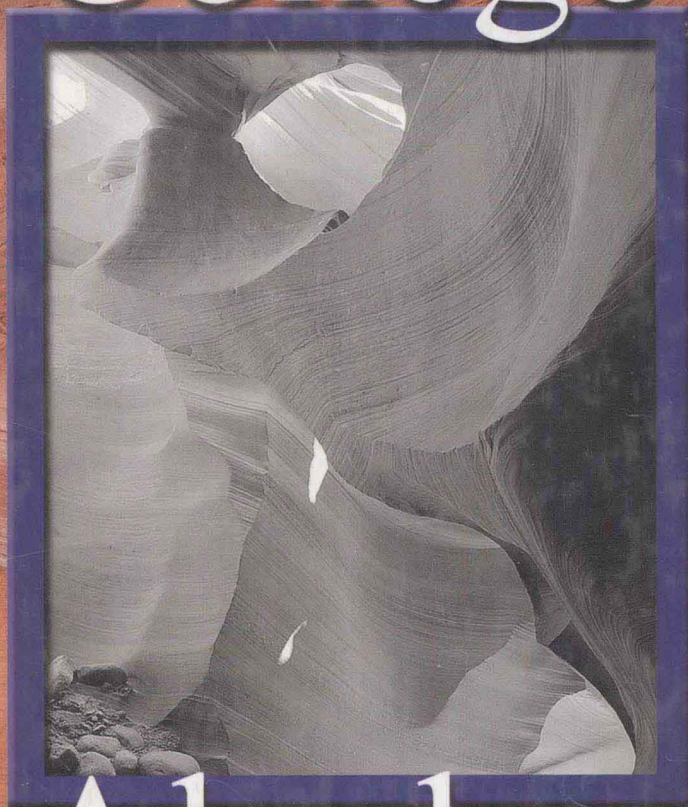# College

Algebra

Beecher ~ Penna ~ Bittinger

College Algebra

Judith A. Beecher
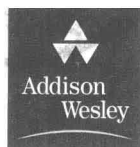
Indiana University Purdue University Indianapolis

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Addison
Wesley

Photo Credits

For our fathers

Floyd W. Swank

J. Keith Melvin

William Sterl Bittinger

Preface

Note to Students

Our challenge, and our goal, when writing this textbook was to do everything possible to help you learn the concepts and skills contained between its covers. Every feature we have included was put here with this in mind. We realize that your time is both valuable and limited, so we communicate in a highly visual way that allows you to focus easily and learn quickly and efficiently. Take advantage of the side-by-side algebraic solutions and their visualizations, the Connecting the Concepts features, and the Study Tips. We included them to enable you to make the most of your study time and to be successful in this course.

Best wishes for a positive learning experience,

Judy Beecher
Judy Penna
Marv Bittinger

Content Features

College Algebra covers college-level algebra and is appropriate for a one-term course in precalculus mathematics. We introduce functions earlier than most college algebra texts, and our approach is more visual as well. Although a course in intermediate algebra is a prerequisite for using this text, Chapter R, Basic Concepts of Algebra, provides sufficient review to unify the diverse mathematical backgrounds of most students.

- **Function Emphasis** Functions are the core of this course and should be presented as a thread that runs throughout the course rather than as an isolated topic. We introduce functions in Chapter 1, whereas most traditional college algebra textbooks cover equation-solving in Chapter 1 (see pp. 56, 64–66, 137, and 153). Our approach introduces students to a relatively new concept at the beginning of the course rather than requiring them to begin with a review of what was previously covered in intermediate algebra. The concept of a function can be challenging for

xi

students. By repeatedly exposing them to the language, notation, and use of functions, demonstrating visually how functions relate to equations and graphs, and also showing how functions can be used to model real data, we hope that students will not only become comfortable with functions but also come to understand and appreciate them.

- **Zeros, Solutions, and x-Intercepts Theme**   We find that when students understand the connections among the real zeros of a function, the solutions of its associated equation, and the first coordinates of the x-intercepts of its graph, a door opens to a new level of mathematical comprehension that increases the probability of success in this course. We emphasize zeros, solutions, and x-intercepts throughout the text by using consistent, precise terminology and including exceptional graphics. (See pp. 155 and 165.)

- **Visual Emphasis**   Our early introduction of functions allows graphs to be used to provide a visual aspect to solving equations and inequalities. For example, we are able to show the students both algebraically and visually that the solutions of a quadratic equation $ax^2 + bx + c = 0$ are the zeros of the quadratic function $f(x) = ax^2 + bx + c$ as well as the x-intercepts of the graph of that function. This makes it possible for students, particularly visual learners, to gain a quick understanding of these concepts. (See pp. 162 and 240.)

- **Side-by-Side Features**   Many examples are presented in a side-by-side, two-column format in which the algebraic solution of an equation appears in the left column and a graphical interpretation of the solution appears in the right column (see pp. 155 and 314). This enables students to visualize and comprehend the connections among the solutions of an equation, the zeros of a function, and the x-intercepts of the graph of a function.

- **Optional Review Chapter**   Chapter R, Basic Concepts of Algebra, provides an optional review of intermediate algebra. Some or all of the topics in this chapter can be taught at the beginning of the course, or it can be used as a convenient source of information throughout the term for students who need a quick review of particular topics.

- **Connecting the Concepts**   This feature highlights the importance of connecting concepts. When students are presented with concepts in a visual form—using graphs, an outline, or a chart—rather than merely in paragraphs of text, comprehension is streamlined and retention is maximized. The visual aspect of this feature invites students to stop and check their understanding of how concepts work together in one section or in several sections. This concept check in turn enhances student performance on homework assignments and exams. (See pp. 139 and 280.)

- **Real-Data Applications**   We encourage students to see and interpret the mathematics that appears every day in the world around them. Throughout the writing process, we conducted an energetic search for real-data applications, and the result is a variety of examples and exercises that connect the mathematical content with everyday life. Most of

these applications feature source lines and frequently include charts and graphs. Many are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and travel. (See pp. 76, 277, and 329.)

• Study Tips   Appearing in the text margin, each Study Tip provides helpful study hints throughout the text and also briefly reminds students to use the electronic and print supplements that accompany the text. (See pp. 94 and 210.)

• Review Icons   Placed next to the concept that a student is currently studying, a review icon references one or more sections of the text in which the student can find and review the topics on which the current concept is built. (See pp. 153 and 226.)

• Technology Connections   This feature appears throughout the text to demonstrate how a graphing calculator can be used to solve problems. The technology is set apart from the traditional exposition, so that it does not intrude if no technology is desired. Although students might not be using graphing calculators, the graphing calculator windows that appear in the Technology Connection features enhance the visual element of the text, providing graphical interpretations of solutions of equations, zeros of functions, and x-intercepts of graphs of functions (see pp. 136, 162, and 311). Exercises that are designed to be worked using a graphing calculator are grouped together in the exercise sets under the heading Technology Connection so that they can be easily identified. A graphing calculator manual, providing keystroke-level instruction for six models of graphing calculators, is also available. (See Supplements for the Student.)

Pedagogical Features

• Chapter Openers   Each chapter opens with an application relevant to the content of the chapter. Also included is a table of contents for the chapter, listing section titles. (See pp. 43 and 193.)

• Section Objectives   Content objectives are listed at the beginning of each section. Together with subheadings throughout the section, these objectives provide a useful outline of the section for both instructors and students. (See pp. 56 and 153.)

• Annotated Examples   Realizing that students quickly become discouraged and frustrated when a textbook does not prepare them adequately for the exercises in the exercise sets, we have included over 520 examples designed to prepare the student fully to do the exercises. Learning is carefully guided with the use of numerous color-coded art pieces and step-by-step annotations. Substitutions and annotations are highlighted in red for emphasis. (See pp. 157 and 305.)

• Use of Color   The text uses full color in an extremely functional way, as seen in the design elements and numerous pieces of art. The use of color has been carefully thought out so that it carries a consistent meaning that enhances students' ability to read and comprehend the exposition. (See pp. 120 and 266.)

• Art Package   The text contains over 800 art pieces including photographs, situational art, and statistical graphs that not only highlight the abundance of real-world applications but also help students visualize the mathematics being discussed (see pp. 44, 235, and 326). In particular, photorealistic art, in which mathematics is superimposed on a photograph, encourages students to see mathematics in familiar settings (see pp. 102 and 173).

• Five-Step Problem-Solving Process   The basis for problem solving is a distinctive five-step process established early in the text (Section 2.1) to help students learn strategic ways to approach and solve applied problems. This process is then used consistently throughout the text to give students a consistent framework for problem solving. (See pp. 139 and 163.)

• Variety of Exercises   There are over 4100 exercises in this text. The exercise sets are enhanced with real-data applications and source lines, detailed art pieces, tables, and graphs. In addition to the exercises that provide students practice with the concepts presented in the section, the exercise sets feature the following elements.

Collaborative Discussion and Writing Exercises can be used in small groups or by the class as a whole to encourage students to talk and write about the key mathematical concepts in each section. (See pp. 167 and 212.)

Skill Maintenance Exercises provide an ongoing review of concepts previously presented in the course, enhancing students' retention of these concepts. Answers to *all* Skill Maintenance exercises appear in the answer section at the back of the book along with a section reference that directs students quickly and efficiently to the appropriate section of the text if they need help with an exercise. (See pp. 124 and 224.)

Synthesis Exercises appear at the end of each exercise set and encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the general exercises. (See pp. 213 and 276.)

Technology Exercises are to be done using a graphing calculator. They are set apart from the non-calculator exercises, making it convenient for the instructor to avoid assigning them if graphing calculators are not being used in the course. (See pp. 69 and 316.)

• Highlighted Information   Important definitions, properties, and rules are displayed in screened boxes, and summaries and procedures are

listed in boxes outlined in color. This organization and presentation provides for efficient learning and review. (See pp. 218 and 233.)

- **Summary and Review**   The Summary and Review at the end of each chapter contains an extensive set of review exercises along with a list of important properties and formulas covered in that chapter. This feature provides excellent preparation for chapter tests and the final examination. Answers to all review exercises appear in the answer section at the back of the book, along with corresponding section references. (See pp. 255 and 331.)

- **Chapter Tests**   The test at the end of each chapter allows students to test themselves and target areas that need further study before taking the in-class test. Answers to all Chapter Test questions appear in the answer section at the back of the book, along with corresponding section references. (See pp. 130 and 334.)

Supplements for the Instructor

For more information on these and other helpful instructor supplements, please contact your Addison-Wesley sales representative.

Instructor's Edition *(ISBN 0-201-75037-6)*
This specially bound version of the student edition contains answers to both even- and odd-numbered exercises at the back of the text.

Instructor's Solutions Manual *(ISBN 0-201-75952-1)*
The *Instructor's Solutions Manual* by Judith A. Penna contains worked-out solutions to all exercises in the exercise sets, including the Collaborative Discussion and Writing exercises.

Printed Test Bank/Instructor's Resource Guide *(ISBN 0-201-74953-X)*
The *Printed Test Bank/Instructor's Resource Guide*, prepared by Laurie Hurley, contains the following:

- 4 free-response test forms for each chapter
- 2 multiple-choice test forms for each chapter
- 6 forms of a final examination, 4 with free-response questions and 2 with multiple-choice questions
- Index to the videotapes that accompany the text

TestGen-EQ with QuizMaster-EQ *(ISBN 0-201-74955-6)*
Available on a dual-platform Windows/Macintosh CD-ROM, this fully networkable software enables instructors to build, edit, print, and administer tests using a computerized test bank of questions organized according to the chapter content of the text. Tests can be printed or saved for on-line testing via a network or the Web, and the software can generate a variety of grading reports for tests and quizzes.

Supplements for the Student

For more information on these and other helpful student supplements, please contact your bookstore.

Student's Solutions Manual *(ISBN 0-201-74951-3)*

The *Student's Solutions Manual* by Judith A. Penna contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the text, with the exception of the Collaborative Discussion and Writing exercises.

InterAct Math Tutorial CD-ROM *(ISBN 0-201-74954-8, stand-alone)*

This interactive tutorial software provides algorithmically generated practice exercises that correlate at the objective level to the odd-numbered exercises in the text. Each practice exercise is accompanied by an example and guided solution designed to involve students in the solution process. The software recognizes common student errors and provides appropriate feedback.

MyMathLab.com   www.mymathlab.com

A complete on-line course that integrates interactive multimedia instruction correlated to the textbook content, MyMathLab is easily customized to suit the needs of students and instructors and provides a comprehensive and efficient on-line course-management system that allows for diagnosis, assessment, and tracking of students' progress. Free when bundled with a new Addison-Wesley text.

AW Math Tutor Center *(ISBN 0-201-72170-8, stand-alone)*
www.aw.com/tutorcenter

The AW Math Tutor Center is staffed by qualified mathematics instructors who provide students with tutoring on examples and exercises from the textbook. Tutoring is available via toll-free telephone, fax, or e-mail five days a week, seven hours a day.

Videotapes *(ISBN 0-201-74957-2)*

Developed and produced especially for this text, these videotapes feature an engaging team of instructors presenting material and concepts in a format that stresses student interaction. There is a videotaped lesson for each section of the text. The lecturers' presentations include examples and exercises from the text and support an approach that emphasizes visualization and problem solving.

Digital Video Tutor *(ISBN 0-201-74996-3, stand-alone)*

The videos for this text are available on CD-ROM, making it easy, convenient, and affordable for students to watch video segments from a computer at home or on campus. The portability of this complete digitized video set makes it ideal for supplemental instruction or distance learning.

InterAct MathXL *(12-month registration ISBN 0-201-71630-5, stand-alone)*
www.mathxl.com

This Web-based diagnostic testing and tutorial system allows students to take practice tests correlated to the textbook and receive customized study plans based on their results. Each time a student takes a practice test, the resulting study plan identifies areas for improvement and links to appropriate practice exercises and tutorials generated by InterAct Math. A course-management feature allows instructors to view students' test results, study plans, and practice work.

Graphing Calculator Manual *(ISBN 0-201-74956-4)*

The *Graphing Calculator Manual* by Judith A. Penna, with the assistance of Daphne A. Bell, contains keystroke-level instruction for the Texas Instruments TI-82®, TI-83®, TI-83 Plus®, TI-85®, TI-86®, and TI-89® graphing calculators. This manual uses examples and exercises from the text to teach students to use the graphing calculator. The order of topics mirrors that of the text, providing a just-in-time mode of instruction.

Acknowledgments

Feature Walkthrough

Functions, Equations, and Inequalities 2

*I*n this chapter, we examine functions and equations, placing particular emphasis on visualizing the connections among the zeros of a function, the *x*-intercepts of the graph of that function, and the solutions of the corresponding equation. We study linear, quadratic, rational, and radical equations as well as equations that are reducible to quadratic and equations with absolute value. We also study linear and quadratic functions and models. We end our work in this chapter by solving linear inequalities.
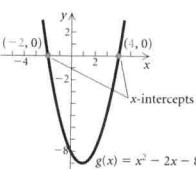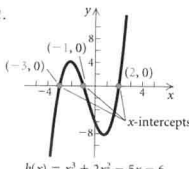
APPLICATION

In a recent study of 68 metropolitan areas, it was determined that in 1999 drivers in Indianapolis were delayed in traffic an average of 37 hr annually. This was 7 hr less than the annual delay in the mid-1990s. (*Source:* Texas Transportation Institute Urban Mobility Study) What was the annual delay in the mid-1990s?

This problem appears as Exercise 50 in Section 2.1.

Applied Chapter Openers
Each chapter begins with an application showing how chapter concepts are used in the real world. The chapter openers are not only motivating to students but also help them develop their critical-thinking skills.

ZEROS, SOLUTIONS, AND INTERCEPTS

FUNCTION	REAL ZEROS OF THE FUNCTION; SOLUTIONS OF THE EQUATION	x-INTERCEPTS OF THE GRAPH

Quadratic Polynomial
$$g(x) = x^2 - 2x - 8$$
$$= (x + 2)(x - 4),$$
or
$$y = (x + 2)(x - 4)$$

To find the **zeros** of $g(x)$, we solve $g(x) = 0$:

$$x^2 - 2x - 8 = 0$$
$$(x + 2)(x - 4) = 0$$
$$x + 2 = 0 \quad or \quad x - 4 = 0$$
$$x = -2 \quad or \quad x = 4.$$

The **solutions** of $x^2 - 2x - 8 = 0$ are -2 and 4. They are the zeros of the function $g(x)$. That is,

$$g(-2) = 0 \quad \text{and} \quad g(4) = 0.$$

The zeros of $g(x)$ are the x-coordinates of the **x-intercepts** of the graph of $y = g(x)$.

Cubic Polynomial
$$h(x)$$
$$= x^3 + 2x^2 - 5x - 6$$
$$= (x + 3)(x + 1)(x - 2),$$
or
$$y = (x + 3)(x + 1)(x - 2)$$

To find the **zeros** of $h(x)$, we solve $h(x) = 0$:

$$x^3 + 2x^2 - 5x - 6 = 0$$
$$(x + 3)(x + 1)(x - 2) = 0$$
$$x + 3 = 0 \quad or \quad x + 1 = 0 \quad or \quad x - 2 = 0$$
$$x = -3 \quad or \quad x = -1 \quad or \quad x = 2.$$

The **solutions** of $x^3 + 2x^2 - 5x - 6 = 0$ are -3, -1, and 2. They are the zeros of the function $h(x)$. That is,

$$h(-3) = 0,$$
$$h(-1) = 0, \quad \text{and}$$
$$h(2) = 0.$$

The zeros of $h(x)$ are the x-coordinates of the **x-intercepts** of the graph of $y = h(x)$.

The connection between the zeros of a function and the x-intercepts of the graph of the function is easily seen in the examples above. If c is a real zero of a function (that is, $f(c) = 0$), then $(c, 0)$ is an x-intercept of the graph of the function.

Zeros, Solutions, and x-Intercepts Theme

There is a strong emphasis on visualizing and connecting the following concepts:

- the real zeros of a function
- the solutions of the associated equation
- the x-coordinates of the x-intercepts of the graph of the function.

Seeing the connections among these concepts increases student understanding. This theme is reinforced visually throughout the text.

Study Tips

Appearing in the text margins, each Study Tip provides helpful study hints throughout the text and briefly reminds students to use the electronic and print supplements that accompany the text.

Let $M = \log_a x$. Then $a^M = x$. Substituting $\log_a x$ for M, we obtain $a^{\log_a x} = x$. This also follows from the definition of a logarithm: $\log_a x$ is the power to which a is raised in order to get x.

A Base to a Logarithmic Power

For any base a and any positive real number x,

$$a^{\log_a x} = x.$$

(The number a raised to the power $\log_a x$ is x.)

EXAMPLE 10   Simplify each of the following.

a) $4^{\log_4 k}$ **b)** $e^{\ln 5}$ **c)** $10^{\log 7t}$

Solution

a) $4^{\log_4 k} = k$

b) $e^{\ln 5} = e^{\log_e 5} = 5$

c) $10^{\log 7t} = 10^{\log_{10} 7t} = 7t$

Side-by-Side Features

Many examples are presented in a side-by-side, two-column format in which the algebraic solution of an equation appears in the left column and a graphical interpretation of the solution appears in the right column. This enables students to visualize and comprehend the connections among the solutions of an equation, the zeros of a function, and the x-intercepts of the graph of the function.

EXAMPLE 6 Solve: $\log x + \log (x + 3) = 1$.

Algebraic Solution

In this case, we have common logarithms. Writing the base of 10 will help us understand the problem:

$$\log_{10} x + \log_{10} (x + 3) = 1$$
$$\log_{10} [x(x + 3)] = 1 \qquad \text{Using the product rule to obtain a single logarithm}$$
$$x(x + 3) = 10^1 \qquad \text{Writing an equivalent exponential equation}$$
$$x^2 + 3x = 10$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0 \qquad \text{Factoring}$$
$$x - 2 = 0 \quad or \quad x + 5 = 0$$
$$x = 2 \quad or \quad x = -5.$$

CHECK: For 2:

$$\log x + \log (x + 3) = 1$$
$$\overline{\log 2 + \log (2 + 3)} \ ? \ 1$$
$$\log 2 + \log 5$$
$$\log (2 \cdot 5)$$
$$\log 10$$
$$1 \ \mid \ 1 \quad \text{TRUE}$$

For -5:

$$\log x + \log (x + 3) = 1$$
$$\overline{\log (-5) + \log (-5 + 3)} \ ? \ 1 \quad \text{FALSE}$$

The number -5 is not a solution because negative numbers do not have real-number logarithms. The solution is 2.

Visualizing the Solution
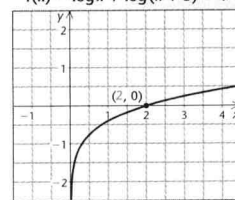
The solution of the equation

$$\log x + \log (x + 3) = 1$$

is the zero of the function

$$f(x) = \log x + \log (x + 3) - 1.$$

The solution is also the first coordinate of the x-intercept of the graph of the function.
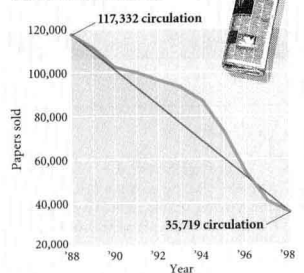
$$f(x) = \log x + \log (x + 3) - 1$$

The solution of the equation is 2. From the graph, we can easily see that there is only one solution.

Real-Data Applications

We encourage students to see and interpret the mathematics that appears every day in the world around them. Throughout the writing process, we conducted an energetic search for real-data applications, and the result is a variety of examples and exercises that connect the mathematical content with the real world. Most of these applications feature source lines and frequently include charts and graphs.

CIRCULATION DROPS
Circulation of the afternoon News fell 70 percent in 10 years.

117,332 circulation

35,719 circulation

Papers sold — Year

The Star and *The News* merged newsgathering staffs in Sept. 1995.
Staff Graphic/Greg Nichols

EXAMPLE 4 *Newspaper Circulation.* On September 30, 1999, the last edition of *The Indianapolis News*, an afternoon daily paper, rolled off the press. Circulation of the paper had dropped approximately 70% in the 10 preceding years, as shown in the graph at left. Find the average rate of change in the circulation from 1988 to 1998.

Solution We determine the coordinates of two points on the graph. In this case, we will use (1988, 117,332) and (1998, 35,719). Then we compute the slope, or average rate of change, as follows:

$$\text{Slope} = \text{Average rate of change} = \frac{\text{Change in } y}{\text{Change in } x}$$

$$= \frac{35,719 - 117,332}{1998 - 1988} = \frac{-81,613}{10} = -8161.3.$$

This result tells us that, on average, each year the circulation of the paper decreased by approximately 8161 copies. The *average rate of change* over the 10-year period was -8161 copies per year.

Connecting the Concepts

Comprehension is streamlined and retention is maximized when concepts are presented in a visual form. Combining design (usually in outline or chart form) and art, this feature emphasizes the importance of connecting concepts. Its visual aspect invites students to stop and check their understanding of how concepts work together within one or throughout several sections. The increased understanding gained from this feature enhances student performance on homework assignments.
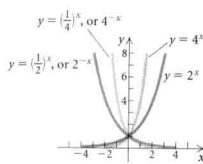
CONNECTING THE CONCEPTS

PROPERTIES OF EXPONENTIAL FUNCTIONS

Let's list and compare some characteristics of exponential functions, keeping in mind that the definition of an exponential function, $f(x) = a^x$, requires that a be positive and different from 1.

$f(x) = a^x$ for $0 < a < 1$ or
$f(x) = a^{-x}$, $a > 1$:

Continuous
One-to-one
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Decreasing
Horizontal asymptote is x-axis
y-intercept: $(0, 1)$

$y = \left(\tfrac{1}{4}\right)^x$, or 4^{-x}
$y = \left(\tfrac{1}{2}\right)^x$, or 2^{-x}
$y = 4^x$
$y = 2^x$

$f(x) = a^x$, $a > 1$:

Continuous
One-to-one
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Increasing
Horizontal asymptote is x-axis
y-intercept: $(0, 1)$

The Domain of a Rational Function

DOMAINS OF FUNCTIONS
REVIEW SECTION 1.2.

EXAMPLE 1 Consider

$$f(x) = \frac{1}{x - 3}.$$

Find the domain and graph f.

Review Icons

These icons prompt students to review topics necessary to understand a concept at hand, and the resulting review enhances students' comprehension. For easy navigation, the section number of the relevant review topic is given in the icon.
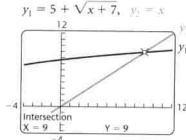
Technology Connections

This optional feature appears throughout the text to demonstrate how a graphing calculator can be used to solve problems. The technology is set apart from the traditional exposition, so that it can be easily omitted, if desired. Although students may not be using graphing calculators, the graphing calculator windows that appear in the Technology Connection features enhance the visual aspect of the text.

Technology Connection

We can use a graphing calculator to solve the equation in Example 3. We can graph $y_1 = 5 + \sqrt{x + 7}$ and $y_2 = x$. Using the INTERSECT feature, we see in the window on the left below that the solution is 9.
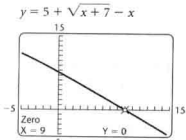
We can also use the ZERO feature to get this result as shown in the window on the right. To do so, we first write the equivalent equation $5 + \sqrt{x + 7} - x = 0$. The zero of the function is 9, so the solution of the original equation is 9.

$y_1 = 5 + \sqrt{x + 7}$, $y_2 = x$

Intersection
X = 9   Y = 9

$y = 5 + \sqrt{x + 7} - x$

Zero
X = 9   Y = 0