Applied Mathematics and Parallel Computing

Herbert Fischer, Bruno Riedmüller
Stefan Schäffler (Editors)

Applied Mathematics and Parallel Computing

Festschrift
for
Klaus Ritter

With 39 Figures

Physica-Verlag
A Springer-Verlag Company

Dr. Herbert Fischer
Dr. Bruno Riedmüller
Priv.-Doz. Dr. Stefan Schäffler
Institut für Angewandte Mathematik und Statistik
Technische Universität München
Arcisstraße 21
D-80333 München
Germany

ISBN 3-7908-0939-X Physica-Verlag Heidelberg

Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Applied mathematics and parallel computing: Festschrift for Klaus Ritter/Herbert Fischer... (ed.). – Heidelberg: Physica-Verl., 1996
ISBN 3-7908-0939-X
NE: Fischer, Herbert [Hrsg.]; Ritter, Klaus: Festschrift

SPIN 10536516 88/2202-5 4 3 2 1 0 – Printed on acid-free paper

Applied Mathematics and Parallel Computing

Preface

The authors of this Festschrift prepared these papers to honour and express their friendship to Klaus Ritter on the occasion of his sixtieth birthday. Because of Ritter's many friends and his international reputation among mathematicians, finding contributors was easy. In fact, constraints on the size of the book required us to limit the number of papers.

Klaus Ritter has done important work in a variety of areas, especially in various applications of linear and nonlinear optimization and also in connection with statistics and parallel computing. For the latter we have to mention Ritter's development of transputer workstation hardware. The wide scope of his research is reflected by the breadth of the contributions in this Festschrift.

After several years of scientific research in the U.S., Klaus Ritter was appointed as full professor at the University of Stuttgart. Since then, his name has become inextricably connected with the regularly scheduled conferences on optimization in Oberwolfach. In 1981 he became full professor of Applied Mathematics and Mathematical Statistics at the Technical University of Munich. In addition to his university teaching duties, he has made the activity of applying mathematical methods to problems of industry to be centrally important.

The editors wish to thank all authors of this Festschrift for their contributions. Our TEX-nicians Margit Stanglmeir and Christian Hertneck were particularly helpful in the final stage of the book. We are also grateful to Dr. Werner A. Müller from Physica-Verlag for his efficient and expeditious production of this book.

All the authors wish Klaus Ritter many more years of fruitful scientific work with friends and colleagues. At the same time, the editors also wish him much time with his family.

Munich, February 1996

Herbert Fischer　　Bruno Riedmüller　　Stefan Schäffler

Contents

Informatics and the Internal Necessity for the Mathematization of the Sciences

N. Apostolatos, Athens

Abstract: In this paper some general thoughts about the significant role of Informatics in the future, the necessity for a strict foundation of Informatics and the Mathematization of Sciences are presented.

1 Mathematization

The question What is Mathematics? is a question of fundamental character. Mathematics has a unique universality. When we abstract the structures which we find in any branch of science we do really only Mathematics. So Mathematics is Queen and Servant of Science in accordance with the title of [9]. My belief that Mathematics is not a simple scientific branch but the unique and decisive means for the formulation and solving of all problems independent of field, is continually strengthened. And so the necessity for the mathematization of the sciences is evident.

Mathematics, if classified in the field of sciences, could be characterised as the science of sciences. However, if we wish to be more precise, we need to classify Mathematics in the field of languages. Mathematics is the most precise language available to people today.

According to Gauß: "*Die Mathematik ist die Königin der Wissenschaften, und die Zahlentheorie ist die Königin der Mathematik.*"

We would say: *Mathematics is the language of all sciences (in a strict sense of the concept of science) as well as the language of all languages.*

A science is considered more advanced when a more precise language is used for the formulation and processing of its concepts. The level of a science is, therefore, determined by the extent of the use of the mathematical language by this science.

Any lack of clearness or inaccuracy or doubt accredited to the definitions and processing of the concepts of various scientific fields, is substantially due to the fact that we have not as yet succeeded in using exclusively the mathematical language at these points.

The lack of mathematization in the various scientific fields constitutes a barrier for the decisive application of modern methods of Informatics. A barrier which will be surpassed only with drastic changes to the structure of these fields, changes that will enable their systematic mathematization.

The 1940s end with the establishment of the field of computers. It would not be an exaggeration to say that our century will close with the name "Century of Computers" or, more appropriately, "Century of Informatics". Therefore, language, with the wider meaning of the term, constitutes the predominant element in our life. The necessity for a strict definition of a language led us to a mathematization of it. Mathematics becames the basic instrument for this purpose. Mathematics, in reality, is a super-language because not only is it a language but also substantially constitutes the only language for the strict definition of the other languages.

The division of Mathematics into sub-fields has been made in such a way as to make it practically impossible to work in one of them continuously without using the others as well. That is, the various sub-fields are not disjoint to each other.

In periods of intensive philosophical thought, such as this of ancient Greece, mathematicians, and not only, have discussed on the nature of Mathematics. In a sense, Mathematics is unique in the intellectual development. We must not forget that Euclid's *Elements* are not only the most used mathematical book in world's history, but also this work has had the greater influence on scientific thought. The strict axiomatic foundation, and so the preparation for the mathematization, is an internal necessity for any scientific branch.

The introduction of infinity in Mathematics was a decisive turning point in the history of Mathematics. And while the finite seems to us attainable, many times in practice it leads us to insuperable impasses. Let us try to conceive the magnitude of the finite number:

$$10^{10^{10^{10}}}$$

In order to, therefore, estimate "how great" this number is, we will attempt to "describe" it by using other magnitudes. This number, of course, is, as we say in Mathematics, finite, it is, nonetheless, inconceivably great. Let us imagine the whole of the universe as a sphere with such a radius that it would take ten billion years for the light to cover it with its unimaginable speed. Let us accept now that we have the ability to register a digit on some material matter which occupies a mass such as that of the smallest atom of matter. Then, within the whole of the universe, it would be impossible to place anything other than a "minute" part of the digits of this number, so minute that we could say that "nearly all" digits of this number have been excluded from the universe.

The enormous development of computers made it possible for us to use enormous amounts of information, however always within a limited space whose power is close to zero compared with the above number. This need to be within accessible finite spaces was what led to the development and foundation of Interval Mathematics. The Interval Arithmetic is a necessity for the computation of safe solutions. Without a relative theory (e.g. Arithmetic of sets) the strict foundation of Numerical Mathematics would be impossible.

2 Mathematical Foundations of Informatics

The subject of Informatics is immense. Mathematicians are regarded as the substantial creators of the current branch of Informatics. In the oncoming years, the mathematical language will play the decisive role for the on steady basis foundation of Informatics.

A significant example supporting the thesis that every founded progress within the field of Informatics cannot but rely on the possibilities offered by the unlimited and continuously expanding mathematical language, is the modern functional programming.

Characteristic, in this case, is the following paragraph taken from the paper of J. Backus: "Can Programming Be Liberated from the Von Neumann Style? A Functional Style and its Algebra of Programs." ([8] p. 614).

"Associated with the functional style of programming is an algebra of programs whose variables range over programs and whose operations are combining forms. This algebra can be used to transform programs and to solve equations whose "uknowns" are programs in much the same way one transforms equations in high school algebra."

It is an undisputable fact that today we live in an exceptional phase of human history. The catalytic effects of modern technology which are characterized by the surprising and unique speed of the development of Informatics, determine the future development and mark the frame in which human beings will move in during the following decades.

For the first time in the human history, the capital, the workforce and the raw materials do not constitute the basic kernel of development. In the years to come, knowledge, the renewal of human activity and the information will constitute the dominant elements for future development. The Informatics, as we have emphasised previously in an other place, will have a decisive effect on the form of the future community, since, prior to this, it will have caused successive, essential changes in the political systems, with the major ones existing today, approaching each other. Thus, new political systems will result which will be able to meet the demands of the dynamic conditions created by the continuously developing of Informatics.

A political system cannot be static; essentially, a correct political system cannot be anything else but a "dynamic" algorithm leading to a solution which, nonetheless, will have to readapt itself continuously to its respective tendencies, thus causing a continuous readaptation of this algorithm.

Since the question arises from time to time, whether or not a computer can think, this question depends on what is meant by "to think". Thus, let us begin with the following

Definition. *The term "to think" means the process of interaction between informations and the rules of processing informations for the creation of new informations and new rules of processing informations.*

Therefore, an electronic computer, just like a person, can think. However, if we were to add to this definition that the above interaction has to be made independently without external influence, then we would have to accept that one electronic computer does not think, and, simultaneously, a person also does not think, because here, too, external factors (although

we do not know how) influence the way a person thinks. That a person has a conscience of its existence is definitely regarded as a matter of fact today, but we cannot say the same for the computer.
According to René Descartes: "*Cogito, ergo sum*", here "thinking" is something that "exist" a priori. So since the copmputer has not the conscience of its "thinking" it's not possible to say: "I am", that, of course can say a person.

The continuously lowering costs of the hardware and software of Informatics, which simultaneously is developing rapidly, opens up avenues for the enrolment of continuously more individuals to the process of learning. And in this way, althoug the process of learning is undergoing a revolutionary redefining, simultaneously a new human force is created which is entering a new form of progressive renaissance.

24 years ago, in our book "Digital Computers" ([2]), we were stating:

> *"In the near future the person who completely ignores some basic topics related to the computer science will have the feeling that he lives in a foreign world with serious effects upon his neuropsychological state and his production within the society."*

In 1980, the President of ACM, P.J. Denning, in an ACM President's Letter "The U.S. Productivity in Crisis" ([11] p. 618), writes:

> *" Computers are now everywhere - in cars, in watches and pens, in light switches and in calculators, in radios and in ovens, in cash registers and in banks, and in personal computers - they have invaded the province of Everyman. The ability to understand this tecnology, and use it wisely, will be a prerequisite of society in 1990 and beyond."*

Today, in the mid 1990s, the above constitutes a reality, which, he who does not take into account due to ignorance, is left outside place and time together with all the frightening results for himself and whatever is influenced by his actions.

That knowledge increases with geometric progression has been extensively emphasised and continues to be so. Simultaneously any biological improvement in a person's ability to think is essentially restricted. How then will a person be able to command this knowledge when such a plethora is offered to him and indeed when some of this knowledge can be used destructively upon himself?

This immense problem cannot be confronted except through a revolutionary reformation of the educational system. The various educational systems lack the capability of a dynamic readaptation, something which is primarily necessary, especially with the rapidly developing technology of our times.

The only language which within its foundations and development has followed logical rules, is Mathematics, and it plays and still plays and will continue to do so, a protagonistic role in future developments. It is necessary, therefore, for conventional languages, before the forth coming developments, to adapt themselves in accordance with the dominant logic. That is to acquire a unified orthological structure with the assistance of the super-language of Mathematics. In the near future, the gradual creation of a logical syntax, equipped with logical-dynamic rules which will allow it to adapt itself to the respective demands, seems to be the natural course of development.

3 Complexity

0.4cm The criterion of performance is the basic one for valueing algorithms. So algorithms that perform significantly quickly a class of problems than any previously algorithm for the same class of problems, is a valuable scientific contribution. Otherwise for classes of problems for which there is not an algorithm to solve them, the only objective is to find a such algorithm.
During the recent years, significant progress has been made in the parallel systems and in the parallel algorithms. This, together with the assistance of modern computers, will increase the speed of processing of problems, significantly. Today we have made it possible to have parallel systems with many processors. Numerical models, such as fractal structure and chaotic behavior, have disclosed noticeable complexity. However, greatly complicated problems cannot be dealt with successfully by depending only on the increase of the speed of computer systems which is derived either from the general improvement or the further use of any parallelism.
On these occasions we need to develop new revolutionary algorithms and this will be attained in two ways:

(a) With the assistance of the existing Mathematics and its normal development.

(*b*) With the assistance of revolutionary development in Mathematics, similar to the creation of Infinitisimal Calculus. With this assistance revolutionary algorithms will be developed that will definitely exploit whatever progress in computer media, but foremostly will have classes more greater speed than the speed of algorithms existing today, for the solution of corresponding problems.

At present, we only have possibility (a). At this stage, I will mention a simple example in order to illustrate what is meant by faster algorithm ([7] p. 375).

Problem

The number 9801 has the following property:

$$(98+01)^2 = 9801$$

We are searching to find all the numbers with an even number of digits:

$$y_1 y_2 \cdots y_i y_{i+1} \cdots y_{2i}$$

so that for all $i = 1(1)n$ hold:

$$(y_1 y_2 \cdots y_i + y_{i+1} y_{i+2} \cdots y_{2i})^2 = y_1 y_2 \cdots y_i y_{i+1} \cdots y_{2i}$$

If we work typically, we will have to examine all the numbers with $2, 4, \ldots,$ $2n$ digits, in other words we will examine:

$$90 + 9000 + 900000 + \ldots + 9 * 10^{2n-1} = 9090\ldots 90 \ldots 90$$

numbers.

With the algorithm we are giving in [7], it will be necessary to examine only:

$$\left[\frac{25 * 10^{2n-2}}{10^n - 1}\right] + 1$$

numbers.

A number which is by far much smaller. So e.g. for $n = 4$, using the typical method 90909090 numbers will need to be examined, whereas with the giving, only 2501.

4 Future Developments

Due to the leaping increase of human knowledge and the limits of the human brain, it is obvious that the way of teaching must aim not on the accumulation of knowledge but on the development of the capacities of learning that will enable, with the relevant methodology, access to the sources of knowledge.
In this way, every member of tomorrow's society, will be involved in the process of learning for life.

Certainly the recent political developments in the various countries of our planet depend closely upon the revolution of Informatics. Along side with whatever rapid reorganisation of the politico-economic and social structures, Informatics will have a decisive influence on the structure of human thought.
It may appear strange if one were to maintain that the great problems preoccupying people today such as environmental pollution, traffic congestion and, above all energy consumption, will find a solution through Informatics. And this because substantially these problems and many others, are created and worsened by the unavoidable increase in the movement of individuals on Earth's restricted space. Movement with the revolutionary conquests of Informatics, not only will become less essential with the passage of time, but will become at one stage and after, negative for the individual's very productivity within society.
This circumstantial "immobility" that will be imposed on individuals will have enormous effects on the very biological and intellectual development of individuals, resulting with also the complete readaptation of the socio-political systems.

And the speedup of the computers today, as well as those of the future, will be too weak to confront the enormous calculating problems, if effective algorithms are not discovered. It's precisely the need for finding a way to overcome limited possibilities of any technological development that will provide the basic motive for a new period in the development of the mathematical language and a systematic utilization of its applications. Such as the revolutionary development of Mathematics as the creation of the Infinitesimal Calculus, which was based on the fundamental work of Newton and Leibnitz.

In a time of rapid increase of human knowledge, the inability of direct estimation causes, in many cases, unnecessary strain on the thinking person. We believe that in the near future a strong attempt will be made for a unified form of human knowledge with the simulataneous removal of the non useful parts. The specific need to confront this acute problem will provide us with new ways of separating and classifying this knowledge, something which will become possible with the suitable development of Informatics.

At this point it is perhaps relevant to refer to something we have put forward at the beginning of our last book ([7]):

> *"The universality, the endurance in time, the uniqueness and, above all, the precision, are basic properties of the mathematical language. Thus, in the attempt for a continuously more unified expression of human knowledge, something which is imposed due to its increase with geometric progress, mathematics will be the field which, itself, continuously unified will constitute the basic instrument for this purpose."*

References

[1] Apostolatos, N., Kulisch, U.: *Grundlagen einer Maschinenintervallarithmetik. Computing 2, 89-104, (1967).*

[2] Apostolatos, N.: *Digital Computers. Book (Greek). Athens 1972.*

[3] Apostolatos, N.: *A proposal for the renaming of the scientific branch of Computer Science. The Computer Journal. Vol. 32, No 3, p. 282, 1989.*

[4] Apostolatos, N.: *The Magic of Mathematics. (Greek). TO BHMA, Athens 25-26.3.1989.*

[5] Apostolatos, N.: *The Informatics determines the Structure of all Sciences (Greek). Mathematical Review (Greek Mathematical Society) 37, 1-10, (1990).*

[6] Apostolatos, N.: *On a unified concept of Mathematics. Computing Suppl. 9, 1-10 (1993).*

[7] Apostolatos, N.: *Programming - Applications. Book (Greek). Athens 1995.*

[8] Backus, J.: *Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs. Comm. of ACM 21, 613-641 (1978).*

[9] Bell, E. T.: *Mathematics, Queen and Servant of Science. McGraw-Hill, New York 1951.*

[10] Courant, R., Robbins, H.: *Was ist Mathematik? Sringer-Verlag, Berlin 1962.*

[11] Denning, P. J.: *U.S. Productivity in Crisis. Comm. of the ACM 23, 617-619 (1980).*

[12] Nash, S. G.: *A History of Scientific Computing. ACM Press - Addison Wesley, 1990.*

[13] Strassen, V.: *Algebraic Complexity Theory. In Handbook of Theoretical Computer Science. Vol. A, Algorithms and Complexity, 633-672. Edided by Jan van Leeuwen. Elsevier Science Publ. B. V. 1990.*

[14] Wilder, R. L.: *Introduction to the Foundations of Mathematics. John Wiley, 1956.*