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Ron Kimmel
Nir Sochen
Joachim Weickert (Eds.)

Scale Space and PDE Methods in Computer Vision

5th International Conference, Scale-Space 2005
Hofgeismar, Germany, April 2005
Proceedings

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Preface

Welcome to the proceedings of the 5th International Conference on Scale-Space and PDE Methods in Computer Vision.

The scale-space concept was introduced by Iijima more than 40 years ago and became popular later on through the works of Witkin and Koenderink. It is at the junction of three major schools of thought in image processing and computer vision: the design of filters, axiomatic approaches based on partial differential equations (PDEs), and variational methods for image regularization. Scale-space ideas belong to the mathematically best-understood approaches in image analysis. They have entered numerous successful applications in medical imaging and a number of other fields where they often give results of very high quality.

This conference followed biennial meetings held in Utrecht, Corfu, Vancouver and Skye. It took place in a little castle (Schlösschen Schönburg) near the small town of Hofgeismar, Germany. Inspired by the very successful previous meeting at Skye, we kept the style of gathering people in a slightly remote and scenic place in order to encourage many fruitful discussions during the day and in the evening.

We received 79 full paper submissions of a high standard that is characteristic for the scale-space conferences. Each paper was reviewed by three experts from the Program Committee, sometimes helped by additional reviewers. Based on the results of these reviews, 53 papers were accepted. We selected 24 manuscripts for oral presentation and 29 for poster presentation.

It is a tradition at scale-space conferences to invite keynote speakers who can provide valuable additional inspirations beyond the mainstream topics in scale-space analysis. Also this time it was our pleasure to thank three leading experts for accepting our invitation for a keynote lecture: Prof. Achi Brandt of The Weizmann Institute of Science (Rehovot, Israel), Prof. Michael Unser of the Swiss Federal Institute of Technology (Lausanne, Switzerland), and Prof. Carl-Fredrik Westin of the Harvard Medical School (Boston, USA).

We thank all authors for their excellent contributions, and the referees for their time and valuable comments. Regarding local arrangements, we are indebted to the staff at Schlösschen Schönburg, as well as to Bernhard Burgeth, Martin Welk, and Uta Merkle of Saarland University. We also thank Micha Feigin, Julia Getslev and Lori Sochen for their help with the website and Yana Katz for her help with the proceedings. Finally we are grateful to the German Pattern Recognition Society (DAGM) for sponsorship.

We wish you an exciting journey through the latest results on scale-space ideas in image analysis.

April 2005 Ron Kimmel, Nir Sochen, and Joachim Weickert

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Relativistic Scale-Spaces

Bernhard Burgeth, Stephan Didas, and Joachim Weickert

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Abstract. In this paper we extend the notion of Poisson scale-space. We propose a generalisation inspired by the linear parabolic pseudodifferential operator $\sqrt{-\Delta + m^2} - m$, $0 \le m$, connected with models of relativistic kinetic energy from quantum mechanics. This leads to a new family of operators $\{Q_t^m \mid 0 \le m, t\}$ which we call relativistic scale-spaces. They provide us with a continuous transition from the Poisson scale-space $\{P_t \mid t \ge 0\}$ (for $m = 0$) to the identity operator I (for $m \longrightarrow +\infty$). For any fixed $t_0 > 0$ the family $\{Q_{t_0}^m \mid m \ge 0\}$ constitutes a scale-space connecting I and P_{t_0}. In contrast to the α-scale-spaces the integral kernels for Q_t^m can be given in explicit form for any $m, t \ge 0$ enabling us to make precise statements about smoothness and boundary behaviour of the solutions. Numerical experiments on 1D and 2D data demonstrate the potential of the new scale-space setting.

Keywords: Kinetic energy, Poisson scale-space, semigroup, pseudodifferential operator.

1 Introduction

The pioneering work of Taizo Iijima [16] in the late fifties, though unrecognised in the western scientific world for decades, marks the actual beginning of modern scale-space theory. Since then the vivid research on scale-space methodologies has brought forward many valuable techniques in image processing and computer vision, as it is documented in numerous articles and books, see [24, 11, 31, 21, 28, 33] and the literature cited there. The Gaussian scale-space is the prototype of a linear scale-space. Its connection to linear diffusion processes was first pointed out by Iijima [17]. However, the field of non-linear diffusion, instigated by the influential work of Perona and Malik [25] also exhibits scale-space properties. These non-linear theories encompass anisotropic diffusion processes [33, 26], morphological operations [32, 6, 18] as well as the evolution of level curves [2, 23, 27, 19]. Non-linear differential equations are the mathematical language to describe these theories [31, 33, 14, 3, 12, 7].

Nevertheless, the exploration of the axiomatic principles of the various scale-space approaches [4, 33, 11, 22, 24, 34] usually emanates from the assumption of

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linearity, that is to say, the validity of the superposition principle. In this linear setting the Gaussian scale-space basically had played the leading role in a one man show until the Poisson scale-space from potential theory has been made popular in image processing by Felsberg and Sommer [10].

Soon after the so-called α-scale-spaces with $\alpha \in [\frac{1}{2}, 1]$ have been advocated to bridge the gap between those two prominent representatives since they are ruled by the pseudodifferential equations $\partial_t u = (-\Delta)^\alpha u$ with initial condition $u(x,0) = f(x)$, (for more details and a histortic overview consult the very comprehensive article [8] by Duits et. al. and the literature cited therein). In this setting $\alpha = 0$ produces the family of identity operators I, $\alpha = \frac{1}{2}$ corresponds to the Poissonian, while $\alpha = 1$ delivers the Gaussian version of a linear scale-space. For the later two cases explicit integral representation formulas are known utilising the Poisson and the Gaussian kernel.

The primary tool for the investigation of the α-scale-spaces are Fourier methods since, unfortunately, no explicit integral kernel can be determined. In our paper, however, we propose a counterpart to α-scale-spaces that admits explicit kernel representations. We generalise the Poisson scale-space to a novel scale-space by exploiting the properties of a pseudodifferential operator known from Schrödinger operators in relativistic quantum mechanics [20]. The pseudodifferential operators in question read

$$\sqrt{-\Delta + m^2} - m,$$

and represent the kinetic energy operators in relativistic systems with $m > 0$ denoting mass. Therefore we will refer to these novel scale-spaces as *relativistic scale-spaces* in the sequel. Though heavily taking advantage of spectral methods during the theoretical investigation of this family of operators (indexed by m) we emphasise that the associated integral kernels can be computed explicitly. The knowledge of these kernels enables us to employ techniques from analysis to prove regularity and a maximum-minimum-principle for the solutions of the associated evolution equation.

In the sequel $\mathcal{F}(f)$ will denote the Fourier transform of a function $f \in L^2(\mathbb{R}^n)$ given by

$$\mathcal{F}(f)(k) = \int_{\mathbb{R}^n} e^{-2\pi i k\cdot x} f(x)\, dx \,.$$

The structure of our paper is as follows: After a very brief motivating account of some basic facts about Poisson and Gaussian scale-space we introduce and study the relativistic scale-spaces. Section 3 reports on experiments displaying the potential and limitations of the novel scale-spaces while a summary and an outlook for future research in Section 4 conclude the paper.

2 Relativistic Scale-Spaces

We recall [9, 20] that the action of the Laplace operator $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ on functions in the Fourier domain is multiplication by $-4\pi^2|k|^2$, i.e.

$$\mathcal{F}(\Delta f) = -4\pi^2|k|^2\mathcal{F}(f)$$

while the convolution with the heat or Gaussian kernel $G(x,t,y)$ means multiplication with $e^{-t\ 4\pi^2|k|^2}$, $\mathcal{F}(G * f) = e^{-t\ 4\pi^2|k|^2}\mathcal{F}(f)$ providing solutions of the heat equation $\partial_t u = \Delta u$.

Furthermore, the action of the pseudodifferential operator $\sqrt{-\Delta}$ is multiplication by $-2\pi|k|$, while convolution with the Poisson kernel $P(\cdot,t)$ means multiplication with $e^{-t\ 2\pi|k|}$ in the Fourier domain. The Poisson kernel appears as the inverse Fourier transform $\mathcal{F}^{-1}$ of $e^{-t\ 2\pi|k|}$:

$$P(x-y,t) = \mathcal{F}^{-1}\big(e^{-t\ 2\pi|\cdot|}\big) = \int_{\mathbb{R}^n} e^{-t\,2\pi|k|+2\pi ik\cdot(x-y)}\,dk\,.$$

This integral can be evaluated in every dimension n yielding the well-known explicit formula for the Poisson kernel [29]

$$P(x-y,t) = \Gamma\left(\frac{n+1}{2}\right)\frac{1}{\pi^{\frac{n+1}{2}}}\frac{t}{(t^2+|x-y|^2)^{\frac{n+1}{2}}}\,. \tag{1}$$

The kernel itself and all convolutions $P(\cdot,t) * f$ with suitable functions f solve in a certain sense the pseudodifferential equation $\partial_t u = \sqrt{-\Delta}u$. The heat and the Poisson kernel generate the Gaussian, resp., the Poisson scale-space.

This can be generalised as follows: In quantum mechanics the pseudodifferential operator $L := \sqrt{-\Delta+m^2} - m$ describes the relativistic kinetic energy of a particle with mass $m \geq 0$ [20] seemingly extending the Poisson operator. In Fourier space this operator acts on function by multiplication with $\sqrt{|2\pi k|^2+m^2} - m$ as a straightforward computation shows. According to standard spectral methods the corresponding integral operator in Fourier space reads

$$e^{-t\ (\sqrt{|2\pi k|^2+m^2}-m)}\,.$$

The inverse Fourier transform of this exponential

$$T_m(x-y,t) := \mathcal{F}^{-1}\left(e^{-t\ (\sqrt{4\pi^2|\cdot|^2+m^2}-m)}\right)(x,y)$$

can be calculated explicitly yielding the expression

$$T_m(x-y,t) := 2\left(\frac{m}{2\pi}\right)^{\frac{n+1}{2}} e^{tm}\frac{t}{(t^2+|x-y|^2)^{\frac{n+1}{4}}}K_{\frac{n+1}{2}}(m\sqrt{t^2+|x-y|^2}) \tag{2}$$

for $(x-y,t) \in R^n \times]0,+\infty[$. Here K_ν stands for the modified Bessel function of the third kind [1, 13]. We briefly sketch the computational steps by pointing out the formulas

$$\int_{S^{n-1}} e^{i\langle\omega,x\rangle}\,d\omega = (2\pi)^{\frac{n}{2}}\,|x|^{1-\frac{n}{2}}\,J_{\frac{n}{2}-1}(|x|)$$

and

$$\int_{[0,+\infty[} x^{\nu+1}J_\nu(xs)\,e^{-\alpha\sqrt{x^2+\beta^2}}dx = \sqrt{\frac{2}{\pi}}\alpha\beta^{\nu+\frac{3}{2}}(s^2+\alpha^2)^{-\frac{\alpha}{2}-\frac{3}{4}}s^\nu K_{\nu+\frac{3}{2}}(\beta\sqrt{s^2+\alpha^2})\,,$$

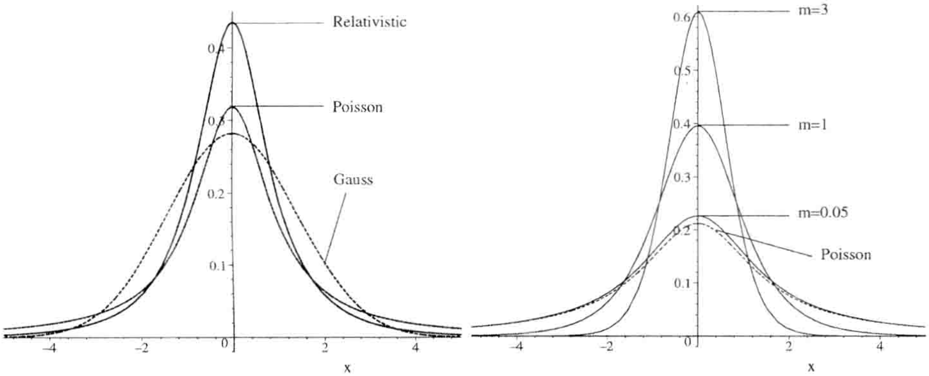

Fig. 1. *Left:* Comparison between different kernels including Poisson, eqn. (1) and relativistic kernel, eqn. (2) in 1D for $y = 0$ and $t = 1$. *Right:* Examples of the relativistic kernel (2) with $m = 3,\ 1,\ ,0.05$ in comparison with the Poisson kernel (1) for $y = 0$ and $t = 1.5$

where J_ν denotes the ν-th order Bessel function. For later use we define the operator Q_t^m on $L^2(\mathbb{R}^n)$ via the convolution

$$Q_t^m f(x) := T_m(\cdot, t) * f\ (x) = \int_{\mathbb{R}^n} T_m(x - y, t) f(y)\, dy\,. \tag{3}$$

2.1 Comparison with the Poisson Kernel

For $m \downarrow 0$ we regain the Poisson kernel which follows from

$$\mathcal{F}(Q_t^m)(k) = e^{-t\ (\sqrt{|2\pi k|^2 + m^2} - m)} \longrightarrow e^{-t\ 2\pi|k|} \quad \text{if } m \downarrow 0 \tag{4}$$

for any complex number k together with the continuity of the (inverse) Fourier transform (according to a theorem of P. Levy) [5]. Furthermore, since

$$\mathcal{F}(Q_t^m)(k) = e^{-t\ (\sqrt{|2\pi k|^2 + m^2} - m)} \longrightarrow 1 \quad \text{if } m \to +\infty\,,$$

a similar reasoning proves that Q_t^m approximates therefor the identity operator I if m is large. Remarkably, despite the approximation property (4), we learn from the theory of Bessel functions [1, 13] that $K_\nu(x)$ for any $\nu \geq 0$, and hence T_m as a function of x (or of y) decreases exponentially to 0 for x tending to infinity, $|x| \to +\infty$. Figure 1 displays the relativistic kernel for various values of m and also its comparison with a Poisson and a Gaussian kernel.

The relation between Poisson scale-space and the relativistic scale-spaces is sketched in the diagram to the right. $\{Q_t^m \mid 0 \leq t, m\}$ is positioned between The Poisson scale-space $\{P_t \mid 0 \leq t\}$ and $\{I \mid 0 \leq t\}$ including them as limiting cases.

$$\begin{array}{cccc}
+\infty & I_0 & \longrightarrow & I_0 \\
\uparrow & \uparrow & & \uparrow \\
m & Q_0^m = I_0 & \longrightarrow & Q_t^m \\
\uparrow & \uparrow & & \uparrow \\
0 & P_0 = I_0 & \longrightarrow & P_t \\
 & & & \\
 & 0 & \longrightarrow & t
\end{array}$$

2.2 Further Properties of the Relativistic Scale-Spaces

From the theory of contraction semigroups [15] we learn that the operator Q_t^m determines a contraction semigroup on $L^2(\mathbb{R}^n)$. Indeed, in view of Plancherel's theorem, it is enough to verify that the Fourier transforms $\mathcal{F}(Q_t^m) = e^{-t(\sqrt{|2\pi k|^2+m^2}-m)}$ of the family $\{Q_t^m\}$ satisfy the conditions

1. $\mathcal{F}(Q_{s+t}^m)\mathcal{F}(f) = \mathcal{F}(Q)_s^m\mathcal{F}(Q_t^m)\mathcal{F}(f) = \mathcal{F}(Q_t^m)\mathcal{F}(Q_s^m)\mathcal{F}(f)$ for all $s, t \geq 0$.
2. $\|\mathcal{F}(Q_t^m)\mathcal{F}(f) - \mathcal{F}(Q_s^m)\mathcal{F}(f)\|_2 \longrightarrow 0$ for $t \longrightarrow s$.
3. $\mathcal{F}(Q_0^m) = 1$, expressing the fact that $Q_0^m = I$, the identity.
4. $\|\mathcal{F}(Q_t^m)\mathcal{F}(f)\|_2 \leq \|\mathcal{F}(f)\|_2$, the contraction property.

Due to the properties of the exponentials e^{-ct} with $c > 0$ it is not difficult to check that the operator Q_t^m indeed meets these conditions. The associated *generator* is the pseudodifferential operator $L = \sqrt{-\Delta + m^2} - m$ with the Sobolev space $H^1(\mathbb{R}^n)$ as its domain $D(L)$. Here we followed [30] in the definition of the Sobolev spaces

$$H^s(\mathbb{R}^n) := \left\{u \in L^2(\mathbb{R}^n) \mid \left(1 + |k|^2\right)^{\frac{s}{2}} \mathcal{F}(u) \in L^2(\mathbb{R}^n)\right\} \tag{5}$$

of all functions in $L^2(\mathbb{R}^n)$ and $s \in \mathbb{R}$.

Next we are going to study in some detail the properties of the function $F_m(x,t)$ defined for $f \in L^2(\mathbb{R}^n)$ by

$$F_m(x,t) := Q_t^m f(x) = \int_{\mathbb{R}^n} T_m(x-y,t) f(y)\, dy$$

with $x \in \mathbb{R}^n$ and $t > 0$. Since the Bessel functions $K_\nu(x)$ are analytic for $0 < x$, the following result is not surprising.

Proposition 2.1. *F_m is analytic in $\mathbb{R}^n \times]0, \infty[$ for any function $f \in L^2(\mathbb{R}^n)$.*

Proof: Thanks to the analyticity of K_ν the function T can be expanded locally in a multivariate power series to the effect that the exchange of integration and summation yields a corresponding expansion for F_m.

Having the explicit integral kernel at our disposal will enable us to study the boundary behaviour of $F_m(x,t)$ as $t \downarrow 0$. To this end we need the next lemma.

Lemma 2.2. *For any $z \geq 0$ and $\nu \geq -\frac{1}{2}$ the following estimate holds:*

$$K_\nu(z) \leq \frac{\Gamma(\nu)}{2}\left(\frac{2}{z}\right)^\nu. \tag{6}$$