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Global Optimization and Constraint Satisfaction

Second International Workshop, COCOS 2003
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Preface

The formulation of many practical problems naturally involves constraints on the variables entering the mathematical model of a real-life situation to be analyzed. It is of great interest to find the possible scenarios satisfying all constraints, and, if there are many of them, either to find the best solution, or to obtain a compact, explicit representation of the whole feasible set.

The 2nd Workshop on Global Constrained Optimization and Constraint Satisfaction, COCOS 2003, which took place during November 18–21, 2003 in Lausanne, Switzerland, was dedicated to theoretical, algorithmic, and application oriented advances in answering these questions. Here global optimization refers to finding the absolutely best feasible point, while constraint satisfaction refers to finding all possible feasible points. As in COCOS 2002, the first such workshop (see the proceeedings [1]), the emphasis was on complete solving techniques for problems involving continuous variables that provide all solutions with full rigor, and on applications which, however, were allowed to have relaxed standards of rigor.

The participants used the opportunity to meet experts from global optimization, mathematical programming, constraint programming, and applications, and to present and discuss ongoing work and new directions in the field. Four invited lectures and 20 contributed talks were presented at the workshop. The invited lectures were given by John Hooker (Logic-Based Methods for Global Optimization), Jean-Pierre Merlet (Usual and Unusual Applications of Interval Analysis), Hermann Schichl (The COCONUT Optimization Environment), and Jorge Moré (Global Optimization Computational Servers).

This volume contains the text of Hooker's invited lecture and of 12 contributed talks. Copies of the slides for most presentations can be found at [2].

Constraint satisfaction problems. Three papers focus on algorithmic aspects of constraint satisfaction problems.

The paper *Efficient Pruning Technique Based on Linear Relaxations* by Lebbah, Michel and Rueher describes a very successful combination of constraint propagation, linear programming techniques and safe rounding procedures to obtain an efficient global solver for nonlinear systems of equations and inequalities with isolated solutions only, providing mathematically guaranteed performance.

The paper *Inter-block Backtracking: Exploiting the Structure in Continuous CSPs* by Jermann, Neveu and Trombettoni shows how the sparsity structure often present in constraint satisfaction problems can be exploited to some extent by decomposing the full problem into a number of subsystems. By judiciously distributing the work into (a) searching solutions for individual subsystems and (b) combining solutions of the subsystems, one can often gain speed, sometimes orders of magnitude.

The paper *Accelerating Consistency Techniques for Parameter Estimation of Exponential Sums* by Garloff, Granvilliers and Smith discusses constraint satisfaction techniques for the estimation of parameters in time series modeled as exponential sums, given uncertainty intervals for measured time series.

Global optimization. Five papers deal with improvements in global optimization methods.

The paper *Convex Programming Methods for Global Optimization* by Hooker describes how to reduce global optimization problems to convex nonlinear programming in case the problem becomes convex when selected discrete variables are fixed. The techniques discussed include disjunctive programming with convex relaxations, logic-based outer approximation, logic-based Benders decomposition, and branch-and-bound using convex quasi-relaxations.

The paper *A Method for Global Optimization of Large Systems of Quadratic Constraints* by Lamba, Dietz, Johnson and Boddy presents a new algorithm for the global optimization of quadratically constrained quadratic programs, which is shown to be efficient for large problems arising in the scheduling of refineries, involving many thousands of variables and constraints.

The paper *A Comparison of Methods for the Computation of Affine Lower Bound Functions for Polynomials* by Garloff and Smith shows how to exploit Bernstein expansions to find efficient rigorous affine lower bounds for multivariate polynomials, needed in global optimization algorithms.

The paper *Using a Cooperative Solving Approach to Global Optimization Problems* by Kleymenov and Semenov presents SIBCALC, a cooperative solver for global optimization problems.

The paper *Global Optimization of Convex Multiplicative Programs by Duality Theory* by Oliveira and Ferreira shows how to use outer approximation together with branch and bound to minimize a product of positive convex functions subject to convex constraints. This arises naturally in convex multiobjective programming.

Applications. The paper *High-Fidelity Models in Global Optimization* by Peri and Campana applies global optimization to large problems in ship design. An important ingredient of their methodology is the ability to use models of different fidelity, so that the most expensive computations on high-fidelity models need to be done with lowest frequency.

The paper *Incremental Construction of the Robot's Environmental Map Using Interval Analysis* by Drocourt, Delahoche, Brassart and Cauchois uses constraint propagation based algorithms for building maps of the environment of a moving robot.

The paper *Nonlinear Predictive Control Using Constraints Satisfaction* by Lydoire and Poignet discusses the design of nonlinear model predictive controllers satisfying given constraints, using constraint satisfaction techniques.

The paper *Gas Turbine Model-Based Robust Fault Detection Using a Forward-Backward Test* by Stancu, Puig and Quevedo presents a new, constraint propagation based method for fault detection in nonlinear, discrete dynamical systems with parameter uncertainties which avoids the wrapping effect that spoils most computations involving dynamical systems.

The paper *Benchmarking on Approaches to Interval Observation Applied to Robust Fault Detection* by Stancu, Puig, Cugueró and Quevedo applies interval techniques to the uncertainty analysis in model-based fault detection.

This volume of contributions to global optimization and constraint satisfaction thus reflects the trend both towards more powerful algorithms that allow us to tackle larger and larger problems, and towards more-demanding real-life applications.

January 2005

Christophe Jermann
Arnold Neumaier
Djamila Sam

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2. COCOS 2003 – Global Constrained Optimization and Constraint Satisfaction, Web site (2003), `http://liawww.epfl.ch/Events/Cocos03`

Organization

The COCOS 2003 workshop was organized by the partners of the COCONUT project (IST-2000-26063) with financial support from the European Commission and the Swiss Federal Education and Science Office (OFES).

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Efficient Pruning Technique Based on Linear Relaxations

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Abstract. This paper extends the Quad-filtering algorithm for handling general nonlinear systems. This extended algorithm is based on the RLT (Reformulation-Linearization Technique) schema. In the reformulation phase, tight convex and concave approximations of nonlinear terms are generated, that's to say for bilinear terms, product of variables, power and univariate terms. New variables are introduced to linearize the initial constraint system. A linear programming solver is called to prune the domains. A combination of this filtering technique with Box-consistency filtering algorithm has been investigated. Experimental results on difficult problems show that a solver based on this combination outperforms classical CSP solvers.

1 Introduction

Numerical constraint systems are widely used to model problems in numerous application areas ranging from robotics to chemistry. Solvers of nonlinear constraint systems over the real numbers are based upon partial consistencies and searching techniques.

The drawback of classical local consistencies (e.g. 2B-consistency [13] and Box-consistency [3]) comes from the fact that the constraints are handled independently and in a blind way. 3B-consistency [13] and kB-consistency [13] are partial consistencies that can achieve a better pruning since they are "less local" [10]. However, they require numerous splitting steps to find the solutions of a system of nonlinear constraints; so, they may become rather slow.

For instance, classical local consistencies do not exploit the semantic of quadratic terms; that's to say, these approaches do not take advantage of the very specific semantic of quadratic constraints to reduce the domains of the variables. Linear programming techniques [1, 25, 2] do capture most of the semantic of quadratic terms (e.g., convex and concave envelopes of these particular terms).

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That's why we have introduced in [11] a global filtering algorithm (named Quad) for handling systems of quadratic equations and inequalities over the real numbers. The Quad-algorithm computes convex and concave envelopes of bilinear terms xy as well as concave envelopes and convex underestimations for square terms x^2.

In this paper, we extend the Quad-framework for tackling general nonlinear system. More precisely, since every nonlinear term can be rewritten as sums of products of univariate terms, we introduce relaxations for handling the following terms:

- power term x^n
- product of variables $x_1x_2...x_n$
- univariate term $f(x)$

The Quad-algorithm is used as a global filtering algorithm in a branch and prune approach [29]. Branch and prune is a search-tree algorithm where filtering techniques are applied at each node. Quad-algorithm uses Box-consistency and 2B-consistency filtering algorithms. In addition, linear and nonlinear relaxations of non-convex constraints are used for range reduction in the branch-and-reduce algorithm [19]. More precisely, the Quad-algorithm works on the relaxations of the nonlinear terms of the constraint system whereas Box-consistency algorithm works on the initial constraint system.

Yamamura et. al. [31] have first used the simplex algorithm on quasi-linear equations for excluding interval vectors (boxes) containing no solution. They replace each nonlinear term by a new variable but they do not take into account the semantic of nonlinear terms[1]. Thus, their approach is rather inefficient for systems with many nonlinear terms.

The paper is organised as follows. Notations and classical consistencies are introduced in section 2. Section 3 introduces and extends the Quad pruning algorithm. Experimental results are reported in section 4 whereas related works are discussed in section 5.

2 Notation and Basics on Classical Continuous Consistencies

This paper focuses on CSPs where the domains are intervals and the constraints are continuous. A n-ary continuous constraint $C_j(x_1, \ldots, x_n)$ is a relation over the reals. $\mathcal{C}$ stands for the set of constraints.

D_x denotes the domain of variable x, that's to say, the interval $[\underline{x}, \overline{x}]$ of allowed values for x. $\mathcal{D}$ stands for the set of domains of all the variables of the considered constraint system.

We use the "reformulation-linearization technique" notations introduced in [25, 2] with some modifications. Let E be some nonlinear expression, $[E]_L$ denotes the set of linear terms coming from a linearization process of E.

[1] They introduce only some weak approximation for convex and monotone functions.

We also use two local consistencies derived from Arc-consistency [14]: 2B-consistency and Box-consistency.

2B-consistency [13] states a local property on the bounds of the domains of a variable at a single constraint level. Roughly speaking, a constraint c is 2B-consistent if, for any variable x, there exists values in the domains of all other variables which satisfy c when x is fixed to $\underline{x}$ or $\overline{x}$.

Box-consistency [3] is a coarser relaxation of Arc-consistency than 2B-consistency. It mainly consists of replacing every existentially quantified variables but one with its interval in the definition of 2B-consistency. Box-consistency [3] is the most successful adaptation of arc-consistency [14] to constraints over the real numbers. Furthermore, the narrowing operator for the Box-consistency has been extended [29] to prove the unicity of a solution in some cases.

The success of 2B-consistency and Box-consistency depends on the precision of enforcing local consistency of each constraint on each variable lower and upper bounds. Thus they are very local and do not exploit any specific semantic of the constraints.

3B-consistency and kB-consistency are partial consistencies that can achieve a better pruning since they are "less local" [10]. However, they require numerous splitting steps to find the solutions of a system of nonlinear constraints; so, they may become rather slow.

3 Using Linear Relaxations to Prune the Domains

In this section, we introduce the filtering procedure we propose for handling general constraints. The Quad filtering algorithm (see Algorithm 1.1) consists of three main steps: reformulation, linearization and pruning.

The reformulation step generates $[\mathcal{C}]_R$, the set of implied linear constraints. More precisely, $[\mathcal{C}]_R$ contains linear inequalities that approximate the semantic of nonlinear terms of $[\mathcal{C}]$.

The linearization process first decomposes each non linear term E in sums and products of univariate terms. Then, it replaces nonlinear terms with their associated new variables. For example, consider $E = \{x_2x_3x_4^2(x_6+x_7)+\sin(x_1)(x_2x_6-x_3) = 0\}$, a simple linearization transformation may yield the following sets:

- $[E]_L = \{y_1 + y_3 = 0, y_2 = x_6 + x_7, y_4 = y_5 - x_3\}$
- $[E]_{LI} = \{y_1 = x_2x_3x_4^2y_2, y_3 = \sin(x_1)y_4, y_5 = x_2x_6\}$.

$[E]_{LI}$ denotes the set of equalities that keep the link between the new variables and the nonlinear terms.

Finally, the linearization step computes the set of final linear inequalities and equalities $LR = [\mathcal{C}]_L \cup [\mathcal{C}]_R$, the linear relaxation of the original constraints $\mathcal{C}$.

The pruning step is just a fixed point algorithm that calls a linear programming solver iteratively to reduce the upper and the lower bound of each initial variable. The algorithm terminates when the maximum achieved reduction is smaller than a non-null predetermined threshold ϵ.

Function Quad_filtering(IN: $\mathcal{X}$, $\mathcal{D}$, $\mathcal{C}$, ϵ) **return** $\mathcal{D}'$
% $\mathcal{X}$: initial variables ; $\mathcal{D}$: input domains; $\mathcal{C}$: constraints; ϵ: minimal reduction, $\mathcal{D}'$: output domains

1. *Reformulation*: generation of linear inequalities $[\mathcal{C}]_R$ for the nonlinear terms in $\mathcal{C}$.
2. *Linearization*: linearization of the whole system $[\mathcal{C}]_L$.
   We obtain a linear system $LR = [\mathcal{C}]_L \cup [\mathcal{C}]_R$.
3. $\mathcal{D}' := \mathcal{D}$
4. *Pruning* :
   While the reduction amount of some bound is greater than ϵ **and** $\emptyset \notin \mathcal{D}'$ **Do**
   (a) Update the coefficients of the linearizations $[\mathcal{C}]_R$ according to the domain $\mathcal{D}'$
   (b) Reduce the lower and upper bounds $\underline{D'_i}$ and $\overline{D'_i}$ of each *initial* variable $x_i \in \mathcal{X}$ by computing *min* and *max* of x_i subject to LR with a linear programming solver.

Algorithm 1.1. The Quad algorithm

Now, we are in position to introduce the reformulation of nonlinear terms. Section 3.1 recalls the relaxations for the simplest case of bilinear term xy, the product of two distinct variables. Relaxations for the power term are given in section 3.2. The process for approximating general product terms is given in section 3.3. Finally, in section 3.4, we introduce a procedure to relax some univariate terms.

3.1 Bilinear Terms

In the case of bilinear terms xy, Al-Khayal and Falk [1] showed that convex and concave envelopes of xy over the box $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$ can be approximated by the following relations:

$$[xy]_R = \begin{cases} BIL1 \equiv [(x-\underline{x})(y-\underline{y}) \geq 0]_L \\ BIL2 \equiv [(x-\underline{x})(\overline{y}-y) \geq 0]_L \\ BIL3 \equiv [(\overline{x}-x)(y-\underline{y}) \geq 0]_L \\ BIL4 \equiv [(\overline{x}-x)(\overline{y}-y) \geq 0]_L \end{cases} \tag{1}$$

BIL1 and BIL3 define a convex envelope of xy whereas BIL2 and BIL4 define a concave envelope of xy over the box $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$. Thus, these relaxations are the optimal convex/concave outer-estimations of xy.

3.2 Power Terms

First let us consider square terms. The term x^2 with $\underline{x} \leq x \leq \overline{x}$ is approximated by the following relations:

$$L1(\alpha) \equiv [(x-\alpha)^2 \geq 0]_L \text{ where } \alpha \in [\underline{x}, \overline{x}] \tag{2}$$

$$L2 \equiv [(\underline{x}+\overline{x})x - y - \underline{x}\overline{x} \geq 0]_L \tag{3}$$

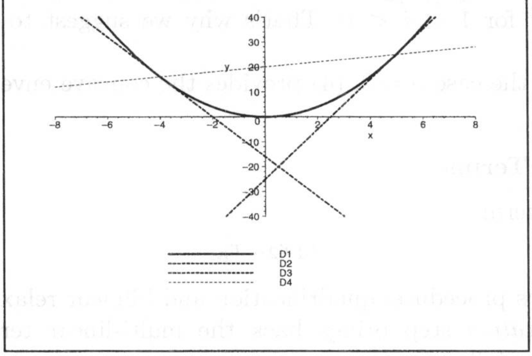

Fig. 1. Approximation of x^2

Note that $[(x-\alpha)^2 = 0]_L$ generates the tangent line to the curve $y = x^2$ at the point $x = \alpha$. Actually, `Quad` computes only $L1(\overline{x})$ and $L1(\underline{x})$. Consider for instance the quadratic term x^2 with $x \in [-4, 5]$. Figure 1 displays the initial curve (i.e., D_1), and the lines corresponding to the equations generated by the relaxations: D_2 for $L1(-4) \equiv y + 8x + 16 \geq 0$, D_3 for $L1(5) \equiv y - 10x + 25 \geq 0$, and D_4 for $L2 \equiv -y + x + 20 \geq 0$.

We may note that $L1(\overline{x})$ and $L1(\underline{x})$ are underestimations of x^2 whereas $L2$ is an overestimation. $L2$ is also the concave envelope, which means that it is the optimal concave overestimation.

More generally, a power term of the form x^n can be approximated by $n+1$ inequalities with a procedure proposed by Sherali and Tuncbilek [27], called "bound-factor product RLT constraints". It is defined by the following formula:

$$[x^n]_R \;=\; \{[(x-\underline{x})^i(\overline{x}-x)^{n-i} \geq 0]_L, i = 0...n\} \qquad (4)$$

The essential observation is that this relaxation generates tight relations between variables on their upper and lower bounds. More precisely, suppose that some original variable takes a value equal to either of its bounds. Then, all the corresponding new RLT linearization variables that involve this original variable take a relative value that conform with actually fixing this original variable at each of its particular bound in the nonlinear expressions represented by these new RLT variables [27].

Note that relaxations (4) of the power term x^n are expressed with x^i for all $i \leq n$, and thus provide a fruitful relationship on problems containing many power terms involving the same variable.

The univariate term x^n is convex when n is even, or when n is odd and the value of x is negative; it is concave when n is odd and the value of x is positive. Section 3.4 details the process for handling such convex and concave univariate term. Sahinidis and Twarmalani [21] have introduced the convex and concave envelopes when n is odd by taking the point where the power term x^n and its under-estimator have the same slope. These convex/concave relaxations on x^n