

FEEDBACK CONTROL SYSTEMS

SECOND EDITION



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JOHN VAN DE VEGTE

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University of Toronto*



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To
Maria

and to
Joyce
John
David
Michael

PREFACE

This book, like its first edition, is intended to serve as a text for a first course in control systems to third- or fourth-year engineering students. Material in later chapters beyond what can be covered in a first course is used by the author in an elective second course at the senior undergraduate/graduate level. The book has been written to be suitable also for students with a more remote or less complete educational background, and for self-study.

The text has grown out of many years of experience in teaching the subject to students in mechanical engineering, industrial engineering, engineering science, correspondence courses, and to students from industry in night courses. However, to show the generality and power of the subject, a first course should not be directed toward a particular department, and the book has been written to be suitable in all branches of engineering.

It is a pleasure to acknowledge the numerous suggestions for improvement received from readers and users of the first edition. Also, one pleasing aspect of a second edition for an author is the chance it gives him to change those features he dislikes himself. Numerous rearrangements of material between and inside chapters have been made, and many changes to improve clarity by modifying or adding words or sentences, in particular where the style in the explanation of theory or examples was a little too efficient. More examples are given in Chapter 3 on the modeling of feedback systems, and serve for additional examples of root locus design and frequency response design in Chapters 6 and 8. Chapter 7 provides an expanded discussion of the correlation between frequency response and pole-zero positions. Chapters 9 and 10, on digital control systems, were changed considerably to improve clarity. This applies also to Chapters 11 and 12, on state space methods.

In Chapter 12 much more detail is provided, the linkage to the earlier design techniques is given careful attention, the treatment of optimal control is greatly expanded, and dynamic observers are included. Chapter 13, on multivariable systems in the frequency domain, has been modified extensively. Certain material, including inverse Nyquist plots and arrays, has been omitted, and other sections have been rewritten, in part to accommodate new developments.

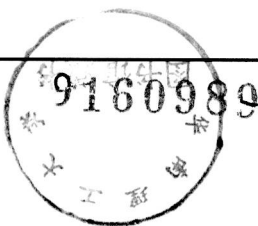
It is emphasized, however, that the fundamental approach of the book is unchanged. To develop insight, concepts are explained in the simplest possible mathematical framework, and concepts of design are developed in parallel with those of analysis. Thus Chapter 1 immediately identifies the two questions to be answered, that is, how dynamic systems behave and how this behavior may be changed by the use of feedback. This chapter also introduces the basic compromise between stability and accuracy which underlies all feedback system design. And Chapter 5 allows a focus on physical explanation of the basic actions of dynamic controllers, unencumbered by the intricacies of the root locus and frequency response techniques discussed in Chapters 6 through 8.

A text must accommodate a wide spectrum of preferences on the extent to which computer aids are incorporated in a course. To achieve this, the text and the 485 problems have been made independent of such aids, and programs for interactive computer-aided analysis and design with graphics are collected in Appendix B, with examples of their use. Reference to these aids is made where appropriate. The author is pleased to acknowledge the work of graduate student Philip W. P. Cheng, who developed these programs for use in the Department of Mechanical Engineering of the University of Toronto.

Finally, the author expresses his gratitude to his wife, who typed the manuscript of the first edition and survived with good grace this new assault on her husband's free time.

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Toronto, Canada

FEEDBACK CONTROL SYSTEMS



CONTENTS

PREFACE

xiii

1 INTRODUCTION AND LINEARIZED DYNAMIC MODELS

1

- 1.1 Introduction 1
- 1.2 Examples and Classifications of Control Systems 1
- 1.3 Open-Loop Control and Closed-Loop Control 2
- 1.4 Control System Analysis and Design 5
- 1.5 Linearized Dynamic Models 6
- 1.6 Laplace Transforms 9
- 1.7 Transfer Functions and Block Diagram Reduction 13
- 1.8 Transient Response and Inverse Laplace Transformation 18
- 1.9 Conclusion 24
- Problems 24

2 TRANSFER FUNCTION MODELS OF PHYSICAL SYSTEMS

32

- 2.1 Introduction 32
- 2.2 Mechanical Systems 32
- 2.3 Electrical Systems: Circuits 37
- 2.4 Electromechanical Systems: Motors and Generators 41

2.5	Thermal Systems	44	
2.6	Fluid Systems	48	
2.7	Conclusion	53	
	Problems	53	
3	MODELING OF FEEDBACK SYSTEMS AND CONTROLLERS		61
3.1	Introduction	61	
3.2	Feedback System Model Examples	62	
3.3	Direct Block Diagram Modeling of Feedback Systems	68	
3.4	Fluid Power Control	71	
3.5	Pneumatic PID Controllers	75	
3.6	Electronic Controllers and System Simulation Using Operational Amplifiers	80	
3.7	Conclusion. Higher-Order Systems	85	
	Problems	86	
4	TRANSIENT PERFORMANCE AND THE S-PLANE		95
4.1	Introduction	95	
4.2	The s -Plane, Pole-Zero Patterns, and Residue Calculation	95	
4.3	Transient Response and System Stability	99	
4.4	Simple Lag and Quadratic Lag	105	
4.5	Transient Response Performance Criteria	109	
4.6	Effect of System Zeros	113	
4.7	Routh-Hurwitz Stability Criterion	118	
4.8	Conclusion. Computer-Aided Analysis and Design	121	
	Problems	121	
5	THE PERFORMANCE AND DYNAMIC COMPENSATION OF FEEDBACK SYSTEMS		127
5.1	Introduction	127	
5.2	Effect of Feedback on Sensitivity and Disturbance Response	128	
5.3	Steady-State Errors in Feedback Systems	132	
5.4	Transient Response versus Steady-State Errors	135	
5.5	Dynamic Compensation: Velocity Feedback	138	
5.6	Series Compensation Using PID Controllers	141	

5.7	Conclusion	145	
	Problems	146	
6	THE ROOT LOCUS METHOD		156
6.1	Introduction	156	
6.2	Root Loci	158	
6.3	Rules for Root Locus Plotting	160	
6.4	Root Locus Examples: Plotting and Sketching	163	
6.5	Root Loci and System Design	168	
6.6	Root Loci for Phase-Lead and Phase-Lag Compensation	176	
6.7	Other Uses of Root Loci and Conclusion	178	
	Problems	179	
7	FREQUENCY RESPONSE ANALYSIS		187
7.1	Introduction	187	
7.2	Frequency Response Functions and Plots	188	
7.3	Nyquist Stability Criterion	191	
7.4	Polar Plots and Nyquist Diagrams	193	
7.5	Bode Plots	197	
7.6	Relative Stability: Gain Margin and Phase Margin	202	
7.7	Examples of Bode Plots	204	
7.8	Closed-Loop Frequency Response and M Circles	207	
7.9	Conclusion	209	
	Problems	209	
8	FREQUENCY RESPONSE DESIGN		215
8.1	Introduction	215	
8.2	Dynamic Compensation	215	
8.3	Frequency Response Performance Criteria	217	
8.4	Performance Specifications on the Bode Plot	219	
8.5	Closed-Loop Frequency Response and the Nichols Chart	223	
8.6	Design of Phase-Lag Compensation	226	
8.7	Design of Phase-Lead Compensation	230	
8.8	Open-Loop Unstable or Nonminimum-Phase Plants	237	
8.9	Systems with Transport Lag	239	

8.10	Feedforward Control	240
8.11	Conclusion	241
	Problems	241
9	DIGITAL CONTROL SYSTEMS	248
9.1	Introduction	248
9.2	Components in a Process Control Configuration	249
9.3	Features and Configurations of Computer Control	251
9.4	Control Practice: A Level Control Example	253
9.5	Sampling Characteristics and Signal Reconstruction	256
9.6	Control Algorithms and Finite Differences	260
9.7	Z Transforms	264
9.8	Z Transfer Functions	266
9.9	System Configuration for Analysis and Design	269
9.10	Conclusion	271
	Problems	272
10	DIGITAL CONTROL SYSTEM ANALYSIS AND DESIGN	275
10.1	Introduction	275
10.2	Closed-Loop Transfer Functions and Block Diagram Reduction	275
10.3	Transient Response of Digital Control Systems	278
10.4	The z -Plane and Root Loci	282
10.5	Digital Controllers by Continuous System Design	286
10.6	Direct Design of Digital Controllers	291
10.7	Frequency Response and the w and Bilinear Forms	295
10.8	Conclusion	299
	Problems	299
11	STATE SPACE ANALYSIS	307
11.1	Introduction	307
11.2	State Space Models	307
11.3	State Space Models for Physical Systems	310
11.4	Transfer Function Matrices and Stability	313
11.5	Solution of the State Equation $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$	315
11.6	Eigenvalues, Eigenvectors, and Modes	320

11.7	Controllability, Observability, and Stabilizability	325
11.8	State Space Methods for Digital Simulation and Control	327
11.9	Conclusion	329
	Problems	329
12	INTRODUCTION TO STATE SPACE DESIGN	338
12.1	Introduction	338
12.2	State Feedback and Pole Assignment	338
12.3	Optimal Control and the Optimal Regulator Problem	344
12.4	Modal Control for Pole Assignment Using State Feedback	350
12.5	Multivariable Integral Control	354
12.6	Feedforward Control for Measurable Disturbances	359
12.7	Dynamic Observers and Output Feedback Design	362
12.8	Conclusion	366
	Problems	366
13	MULTIVARIABLE SYSTEMS IN THE FREQUENCY DOMAIN	372
13.1	Introduction	372
13.2	System Configuration and Equations	373
13.3	Interaction and Decoupling	375
13.4	Basic Stability Theorem and Nyquist Array Techniques	377
13.5	Design of \mathbf{K} by a Vector Diagram Technique	382
13.6	Design of the Diagonal Matrix \mathbf{K}_d	385
13.7	Design Examples for \mathbf{K}_d with Small and Severe Interactions	389
13.8	Conclusion	393
	Problems	394
14	NONLINEAR CONTROL SYSTEMS	399
14.1	Introduction	399
14.2	Nonlinear Behavior and Common Nonlinearities	400
14.3	Phase-Plane Method	402
14.4	Describing Function	406
14.5	Stability Analysis Using Describing Functions	411
14.6	Second, or Direct, Method of Liapunov	413
14.7	Popov and Circle Criteria for Stability	417

14.8 Conclusion	420
Problems	420
APPENDIX A: VECTORS, MATRICES, AND DETERMINANTS	426
A.1 Vectors and Matrices	426
A.2 Vector and Matrix Operations	427
A.3 Determinants, the Inverse, and the Rank of a Matrix	429
A.4 Matrix Calculus	430
APPENDIX B: COMPUTER AIDS FOR ANALYSIS AND DESIGN	431
B.1 Introduction	431
B.2 Roots of a Polynomial	433
B.3 Step Response of a Transfer Function Model	434
B.4 Root Locus Plots	435
B.5 Polar Plots	437
B.6 Bode Plots	438
B.7 Subroutines	440
B.8 References	442
REFERENCES	443
General	443
Fluid Power Control	445
Nonlinear Control System	445
Digital Control Systems	445
Advanced Techniques and Multivariable Systems	446
INDEX	451

Introduction and Linearized Dynamic Models

1.1 INTRODUCTION

In the first part of this chapter, after a general introduction, the concepts of open-loop and closed-loop control are discussed in the context of a water level control system. This example is then used to introduce fundamental considerations in control system analysis and design.

In the second part of the chapter, Laplace transforms are discussed and used to define the transfer function of a system. This is a linearized model of the dynamic behavior of the system that will serve as the basis for system analysis and design in most of this book. Such transfer functions will be derived in the next chapter for a variety of physical systems. In part to motivate this work, transfer functions are used in this chapter to calculate transient responses for a number of systems, including feedback control systems. Block diagram reduction is used to obtain the transfer function of a system consisting of interconnected subsystems from those of the subsystems.

1.2 EXAMPLES AND CLASSIFICATIONS OF CONTROL SYSTEMS

Control systems exist in a virtually infinite variety, both in type of application and level of sophistication. The heating system and the water heater in a house are systems in which only the sign of the difference between desired and actual temperatures is used for control. If the temperature drops below a set value, a constant

heat source is switched on, to be switched off again when the temperature rises above a set maximum. Variations of such relay or on-off control systems, sometimes quite sophisticated, are very common in practice because of their relatively low cost.

In the nature of such control systems, the controlled variable will oscillate continuously between maximum and minimum limits. For many applications this control is not sufficiently smooth or accurate. In the power steering of a car, the controlled variable or system output is the angle of the front wheels. It must follow the system input, the angle of the steering wheel, as closely as possible but at a much higher power level.

In the process industries, including refineries and chemical plants, there are many temperatures and levels to be held to usually constant values in the presence of various disturbances. Of an electrical power generation plant, controlled values of voltage and frequency are outputs, but inside such a plant there are again many temperatures, levels, pressures, and other variables to be controlled.

In aerospace, the control of aircraft, missiles, and satellites is an area of often very advanced systems.

One classification of control systems is the following:

1. *Process control or regulator systems*: The controlled variable, or output, must be held as close as possible to a usually constant desired value, or input, despite any disturbances.
2. *Servomechanisms*: The input varies and the output must be made to follow it as closely as possible.

Power steering is one example of the second class, equivalent to systems for positioning control surfaces on aircraft. Automated manufacturing machinery, such as numerically controlled machine tools, uses servos extensively for the control of positions or speeds.

This last example brings to mind the distinction between continuous and discrete systems. The latter are inherent in the use of digital computers for control.

The classification into linear and nonlinear control systems should also be mentioned at this point. Analysis and design are in general much simpler for the former, to which most of this book is devoted. Yet most systems become nonlinear if the variables move over wide enough ranges. The importance in practice of linear techniques relies on linearization based on the assumption that the variables stay close enough to a given *operating point*.

1.3 OPEN-LOOP CONTROL AND CLOSED-LOOP CONTROL

To introduce the subject, it is useful to consider an example. In Fig. 1.1, let it be desired to maintain the actual water level c in the tank as close as possible to a desired level r . The desired level will be called the system *input*, and the actual level the *controlled variable* or system *output*. Water flows from the tank via a valve V_o and

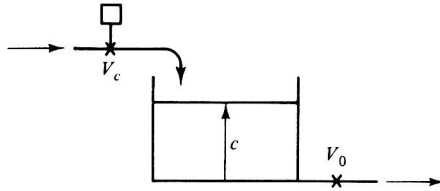


Figure 1.1 Water level control.

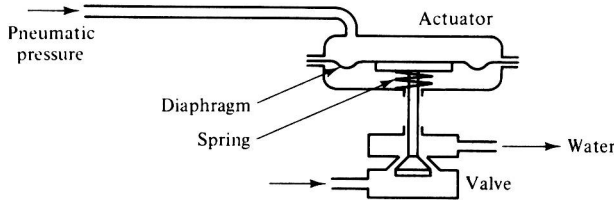


Figure 1.2 Pneumatically actuated valve.

enters the tank from a supply via a *control valve* V_c . The control valve is adjustable, either manually or by some type of *actuator*. This may be an electric motor or a hydraulic or pneumatic cylinder. Very often it would be a pneumatic diaphragm actuator, indicated in Fig. 1.2. Increasing the pneumatic pressure above the diaphragm pushes it down against a spring and increases valve opening.

Open-Loop Control

In this form of control, the valve is adjusted to make output c equal to input r , but not readjusted continually to keep the two equal. Open-loop control, with certain safeguards added, is very common, for example, in the context of sequence control, that is, guiding a process through a sequence of predetermined steps. However, for systems such as the one at hand, this form of control will normally not yield high performance. A difference between input and output, a system *error* $e = r - c$ would be expected to develop, due to two major effects:

1. *Disturbances* acting on the system
2. *Parameter variations* of the system

These are prime motivations for the use of feedback control. For the example, pressure variations upstream of V_c and downstream of V_o can be important disturbances affecting inflow and outflow, and hence level. In a steel rolling mill, very large disturbance torques act on the drive motors of the rolls when steel slabs enter or leave, and these may affect speeds.

For the water level example, a sudden or gradual change of flow resistance of the valves due to foreign matter or valve deposits represents a system parameter variation. In a broader context, not only are the values of the parameters of a process often not precisely known, but they may also change greatly with operating condition.

For an aircraft or a rocket, the effectiveness of control surfaces changes rapidly as the device rises through the atmosphere. In an electrical power plant, parameter values are different at 20% and 100% of full power. In a valve, the relation between

pressure drop and flow rate is often nonlinear, and as a result the resistance parameter of the valve changes with flow rate. Even if all parameter variations were known precisely, it would be complex, say in the case of the level example, to schedule the valve opening to follow time-varying desired levels.

Closed-Loop Control or Feedback Control

To improve performance, the operator could continuously readjust the valve based on observation of the system error e . A *feedback control system* in effect automates this action, as follows:

The output c is measured continuously and fed back to be compared with the input r . The error $e = r - c$ is used to adjust the control valve by means of an actuator.

The feedback loop causes the system to take corrective action if output c (actual level) deviates from input r (desired level), whatever the reason.

A broad class of systems can be represented by the block diagram shown in Fig. 1.3. The *sensor* in Fig. 1.3 measures the output c and, depending on type, represents it by an electrical, pneumatic, or mechanical signal. The input r is represented by a signal in the same form. The *summing junction* or *error junction* is a device that combines the inputs to it according to the signs associated with the arrows: $e = r - c$.

It is important to recognize that if the control system is any good, the error e will usually be small, ideally zero. Therefore, it is quite inadequate to operate an actuator. A task of the *controller* is to amplify the error signal. The controller output, however, will still be at a low power level. That is, voltage or pressure have been raised but current or airflow are still small. The *power amplifier* raises power to the levels needed for the actuator.

The *plant or process* in the level control example includes the valve characteristics as well as the tank. In part this is related to the identification of a *disturbance* d in Fig. 1.3 as an additional input to the block diagram. For the level control, d could represent supply pressure variations upstream of the control valve.

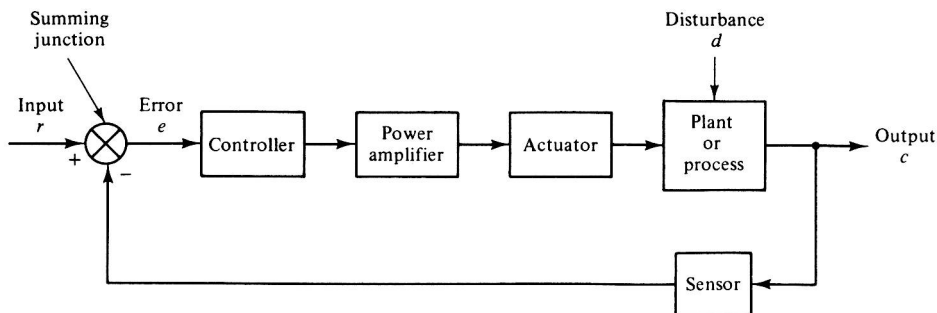


Figure 1.3 System block diagram.