Sampling Inspection and Quality Control

G. Barrie Wetherill

SECOND EDITION

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Preface

Since the pioneering work of Dodge and Romig in the 1920's there has grown up a vast literature on sampling inspection and quality control. However, most of the available texts are written for personnel of inspection departments, giving practical details of exactly what should be done to operate various plans. Many of these are excellent books for their purpose and it is not my intention to attempt to replace them, and indeed I would not be qualified to do this.

My intention in this book has rather been to give a broad coverage of the field, with some emphasis on the principles upon which various plans are constructed. I have also given a simple treatment of important background theory. I hope that the book will be suitable for courses in Universities and Technical Colleges.

The lack of a book of this kind is partially responsible for many statisticians and operational research workers finishing their training with only a smattering of knowledge of this important practical field.

Those interested in pursuing the theoretical aspects will find adequate references throughout, and at the end of the book a list of papers for further study.

Exercises are provided at the end of most sections, and some of these which may give difficulty are marked with an asterisk.

I am grateful to a number of colleagues for detailed comments on an earlier draft of this book, and I mention particularly Mr A. F. Bissell, Dr G. E. G. Campling, Professor D. R. Cox, Mr W. D. Ewan, Mr W. A. Hay and Dr D. V. Hinkley.

G. B. W.

Preface to the second edition

The principal changes in this edition are that tables, nomograms, and explanation have been added throughout so that numerical exercises can be set, and the sections on acceptance sampling have been rewritten. In chapter 3 I have included both ($1{\cdot}96\sigma$, $3{\cdot}09\sigma$) and (2σ, 3σ) limits for control charts, and in chapter 4 I have included an explanation of the use of the nomogram for designing CUSUM schemes. In numerous places the text has been brought up to date with current work. I am indebted to Professor K. W. Kemp and the editors of *Applied Statistics* for permission to reproduce the CUSUM nomogram given in Appendix II.

G. B. W.

Contents

1. Introduction

1.1 Examples and definitions

The importance of sampling inspection and quality control procedures is very widely accepted, and there is a long history of applications to various branches of industry. The purpose of this book is to give a brief account of the procedures available, and to outline the principles upon which they are based. Some typical situations are illustrated in the following examples.

Example 1.1 (Griffiths and Rao, 1964). Large batches of electrical components have been purchased for manufacture into parts of a computer. Each batch contains an unknown proportion of defective components which will cause faults at a later stage if passed on to the manufacturing process. It was decided that any batch containing more than a critical proportion p_0 of defectives should be rejected, and that a single sampling plan was to be operated. This plan was to select n items from the batch at random, and reject the batch if the number of defective items found in the sample were greater than some quantity c. □□□

Example 1.2. Morgan *et al.* (1951) have described a sampling procedure used in the grading of milk. Films of milk were prepared on slides and viewed under a microscope. Several microscopic fields were observed on each film, and the number of bacterial clumps counted. The observations were used to estimate the density of bacterial clumps in the milk, and it was this latter quantity which determined the grade of the milk. □□□

Example 1.3. Grant and Leavenworth (1972, pp. 16–27) described in detail two situations of the following type. The output of a production process is a continuous series of items and the most important characteristic of each item can be described by a single measurement, such as length, strength, etc. If the production process is operating correctly the measurements on the items are approximately normally distributed with a certain mean and variance. A sample of five items is drawn from the process every hour and measurements made on each item. From the results it is required to decide whether the process is operating correctly (the term 'in control' is used), or whether some kind of corrective action needs to be taken. Sometimes in such applications it is also required to decide whether the current output should be passed, or whether it needs to be sorted and reprocessed, etc. □ □ □

Examples 1.1 and 1.2 illustrate what we call *sampling inspection*; in these examples it is necessary to decide what to do with a given quantity of material, e.g. to decide whether to accept or reject a batch of goods. We say that we wish to *sentence* batches of goods.

Example 1.3 describes the *quality control* situation, where the interest is more in controlling the production process than sentencing goods. Inevitably there are many situations where the aim is both to control a process and sentence goods, so that it is impossible to draw a clear boundary between sampling inspection and quality control.

Example 1.2 illustrates a case where, in comparison with Example 1.1 or perhaps Example 1.3 the observations are relatively expensive. We shall see that this leads to a rather different sampling plan being appropriate for Example 1.2.

There are certain common features to the three examples. In each case procedures are required by which we decide among a small number of possible courses of action, and in each case the procedures are to use a small sample of observations, and not, for example, inspection of every item. Now in industry it is sometimes necessary to defend inspection by samples against 100% inspection, and to explain why sample procedures are reliable. Clearly there are some situations in which 100% inspection is desired rather than sampling inspection, but such situations are infrequent. The reasons why sample methods are preferred are as follows:

(*i*) We never require absolutely accurate information about a batch or quantity of goods to be sentenced. Thus in Example 1.1 it would be sufficient to estimate the percentage of defective items in the batch to within ½% or so. Complete inspection in Example 1.1 would be an unnecessary waste of time and labour, unless the aim is to *sort* all the items into good and bad. For the purpose of sentencing the batch, an estimate of the percentage defective is quite sufficient.

(*ii*) A point allied to (*i*) is that under the usual assumptions, the standard error of an estimate reduces as the number of observations increases, approximately as the reciprocal of the square root of the number of observations. Therefore in order to halve the standard error we must take four times as many observations. Beyond a certain point it is either impractical or not worth while achieving greater accuracy.

(*iii*) Even if the entire batch is inspected in Example 1.1 say, we still do not have an absolutely accurate estimate of the percentage defective *unless inspection is perfect*. In industrial situations inspection is very rarely perfect and Hill (1962) quotes a probability of 0·9 as being 'not unreasonable' for the probability of recognizing defects by visual inspection. Some experiments have indicated that if inspectors are faced with batches for 100% inspection, then the inspection tends to be less accurate than if sample methods are used.

(*iv*) In some cases, such as in Example 1.2, inspection is very costly and 100% inspection is obviously ruled out. One case of this is *destructive testing*, as in testing of artillery shells. Another case of costly inspection is when complicated laboratory analyses are involved.

One situation where 100% inspection is appropriate is when it can be arranged cheaply by some automatic device. More usually sample methods will be appropriate.

When sample methods are employed we shall usually make the assumption that sampling is *random*. Thus in Example 1.1 a sample should be taken in such a way that every item in the batch is equally likely to be taken. In practice this assumption is rarely satisfied and this has to be taken into account when drawing up a plan.

Sometimes it is possible to stratify the items to be sentenced, and use this to draw up a more efficient sample procedure. For example, in the transport of bottled goods in cartons, the bottles next to the face of the carton are more likely to be damaged than those in the interior. In this case it would be better to define two strata, one being those bottles next to a face of the carton, and the other stratum being the remainder. A procedure which sampled these strata separately would be more efficient than a straight random sample. To the author's knowledge, very little use has been made of this kind of device.

1.2 Where inspection?

In any industrial process there are a number of places where inspection can and should be carried out. Consider an industrial process which produces nominally constant output over long periods of time, pictured diagrammatically in Figure 1.1. The process may be producing continuously, as in nylon spinning, or small components may be produced at a high rate, as in light engineering, or production may be non-continuous, as, say, of petrol engines, pottery, etc. Any such process can be thought of in three parts; the input stage, where the raw materials are accepted for the process, the process itself, and the output stage, where the product is passed on for sale, or for use in the next stage. Sometimes a process can be thought of as

Figure 1.1. *A typical process.*

being composed of several stages, each as described in Figure 1.1, and the output of one stage is the input of the next, and often there are several inputs to a process. For example, car bodies are pressed and made in one factory, engines manufactured in another, tyres in a third, etc., and these are all inputs to the final stage of assembling finished cars.

We can now consider the inspection suitable for each of the three parts of the process featured in Figure 1.1.

INPUT. We may inspect the input to ensure that it is of sufficiently high quality. For example, in weaving cotton garments, yarn of low tensile strength leads to frequent breaks and loom stoppages. Bad material may be returned to the vendor, returned for reprocessing, scrapped, or set aside for a different use. If the quality offered at input is variable, sampling inspection here can save a good deal of trouble and money.

OUTPUT. We may inspect the output to reduce the risk of bad quality being passed on and causing loss of prestige and loss of money if bad quality items must be replaced. If there is a guarantee, the manufacturer will wish to guard against too many claims against this. With items such as packets of detergent, it may be necessary to reduce the risk of prosecution for selling underweight. Sometimes the aim of inspection of the output is to earn some quality seal, such as a British Standards Institution mark. Any output rejected may be scrapped, sold as inferior products, or completely sorted and the defective items rectified or replaced.

PROCESS. It is usually possible to inspect the process itself, sometimes at several points, and check up how it is working. Two different aims might be involved with such inspection. Firstly it may be possible to use the information to adjust the process and so reduce the amount of bad production. Secondly, it may be desirable to sort out the bad production and sort or return articles for reprocessing before further processing costs are incurred. When the main aim is to control the process we have the quality control situation. Frequently inspection of the process has both process control and product sorting in view. For example, in production of chocolate bars, inspection of the process, before wrapping, may lead both to adjustments to the process and also to sorting out underweight production for melting down and reprocessing.

When planning any particular inspection plan, it is important to bear in mind the various possibilities for inspection. Sometimes inspection effort is more worth while at one place than another. The type of inspection plan which is appropriate depends on the particular situation, and the aims in view.

Exercises 1.2

1. Decribe a production process with which you are familiar. Detail the places in which inspection plans could be operated, and describe the action taken on inspection results.

1.3 Classification of inspection plans

Any system of classifying inspection plans is unsatisfactory in that borderline categories exist. Nevertheless it will be found useful to have some classification system. We shall first list different inspection situations and then give alternative sampling plans.

(*a*) *Inspection situations*

(*i*) *Batch inspection* or *continuous production inspection*. Batch inspection occurs when we have items presented in, say, boxes, and it is desired to pass sentence on each box of items together, and not on each individual item. If on the other hand we have continuous nylon thread, or a production line of continuously produced small items such as chocolate bars and items are *not* treated in batches for sentencing, then we have continuous production inspection. The essential distinction is whether items are batched for inspection purposes or not; often with a continuous production process, items are batched for inspection purposes. With batch inspection there is no need for any order in the batches presented, although sometimes there is an order, and this information can be used, see below. Example 1.4 illustrates one of the earliest types of continuous sampling plans (CSP); batch inspection plans are illustrated later in this section.

Example 1.4. *Dodge plan.* At the outset inspect every item until i successive items are found free of defects. Then inspect every nth item until a defect is found when 100% inspection is restored. □ □ □

(*ii*) *Rectifying inspection* or *acceptance inspection.* If, say, batches of items are presented for sentencing, and the possible decisions are, say, accept or reject, or accept or sell at a reduced price, etc., we have acceptance inspection. Rectifying inspection occurs when one of the possible decisions is to sort out the bad items from a batch and adjust or rectify them, or else replace them. That is, with rectifying inspection, the proportion of defective items may be changed.

(*iii*) *Inspection by attributes* or *inspection by variables.* Inspection by attributes occurs when items are classified simply as effective or defective, or when mechanical parts are checked by go–not-go gauges. The opposite of this is inspection by variables when the result of inspection is a measurement of length, the voltage at which a voltage regulator works, etc. An intermediate classification between these is when items are graded. There is frequently a choice between inspection by attributes or by variables, and also a choice of the number of such variables inspected. The choice between these depends on the costs of inspection, the type of labour employed, and also on the assumptions which can be made about the probability distribution of the measured quantities.

(*b*) *Alternative sampling plans*

We shall be mainly concerned here with batch inspection plans. Example 1.4 illustrates a continuous production inspection plan, and other such plans will be described later. An intermediate situation occurs when items are batched in order from a production process. It is then possible to operate *serial sampling plans* or *deferred sentencing sampling plans*, in which the sentence on a batch depends not only on the results on the batch itself, but also on results from neighbouring or following batches. The plans described below all treat each batch independently; the effect of operating such plans as serial sampling plans would be to modify the sentencing rules depending on the results of inspection on neighbouring batches.

(*i*) *Single sampling plan.* Suppose we have batches of items presented, and the items are to be classified merely as effective or defective. A single sampling plan consists of selecting a fixed random sample of n items from each batch for inspection, and then sentencing each batch depending upon the results. If the sentence is to be either accept or reject the batch, then each batch would be accepted if the number of defectives r found in the n items were less than or equal to the *acceptance number*, c. We summarize as follows:

Single sampling plan:

$$\left.\begin{array}{l}\text{select } n \text{ items,}\\ \text{accept batch if no. of defectives} \leq c,\\ \text{reject batch if no. of defectives} \geq c+1\end{array}\right\} \qquad (1.1)$$

For inspection by variables we have a similar sentencing rule. There is no need for the restriction to two terminal decisions and we could have, for example, accept, reject, or sell at a reduced price.

Example 1.5. For the problem of sampling electrical components, Example 1.1, a suitable sampling plan might be to use a single sampling plan with $n = 30$, $c = 2$. □□□

(*ii*) *Double sampling plan.* In this plan a first sample of n_1 items is drawn, as a result of which we may either accept the batch, reject it, or else take a further sample of n_2 items. If the second sample is taken, a decision to accept or reject the batch is taken upon the combined results.

Example 1.6. A double sampling plan for the electrical component sampling problem might be as follows. Select 12 items from the batch and

accept the batch if there are no defectives,
reject the batch if there are 3 or more defectives,
select another sample of 24 items if there are 1 or 2 defectives.

When the second sample is drawn, we count the number of defectives in the combined sample of 36 items and

accept the batch if no. of defectives ≤ 2,
reject the batch if no. of defectives ≥ 3. □□□

A natural extension of double sampling plans is to have multiple sampling plans, with many stages. It is difficult to see how double or multiple sampling plans would be used when there are more than two terminal decisions, unless more than one attribute (or variable) is measured and a much more complex sentencing rule introduced.

(*iii*) *Sequential sampling plan.* A further extension of the multiple sampling idea is the full sequential sampling plan. In this plan, items are drawn from each batch one by one, and after each item a decision is taken as to whether to accept the batch, reject the batch, or sample another item. A simple method of designing sequential sampling plans was discovered by Professors G. A. Barnard and A. Wald during the 1939–45 war. An essential point is that the sample size is not fixed in advance, but it depends on the way the results turn out.

Sequential sampling plans can save a substantial amount of inspection effort, although the overall gain in efficiency is often not great unless inspection is expensive, as is the case in Example 1.2, concerning grading of milk. Another characteristic of plans where sequential sampling can give great gain in efficiency is when the incoming quality is very variable. Again, Example 1.2 provides just such a situation, as the milk being examined comes from many farms over a wide area, and is of very variable quality.

The theory of sequential sampling plans is discussed by Wald (1947) and Wetherill (1975), and will not be discussed further in this book.

(c) *Discussion*

We have described many different types of inspection situations and inspection plans, and a number of questions arise. What are the relative merits of different types of plan? How should the sample sizes and acceptance numbers be chosen, and upon what principles? In attempting to answer these questions we should consider carefully the aims for which the inspection plan was instituted. For this reason we discuss the inspection situation in greater detail in the next section. In succeeding chapters we shall discuss the rival theories which have been proposed for the design of sampling plans.

1.4 Flow chart for acceptance inspection

In any realistic assessment of alternative sampling inspection plans, the mechanics of the actual situation into which a sampling plan fits must be considered in some detail. In many papers we find that important – even drastic – assumptions are made, both implicitly and explicitly, as to the manner in which a plan works. In this section we do not attempt to give a complete catalogue of inspection situ-

Figure 1.2. *An inspection situation.*

ations, but we aim to give sufficient to form a basis on which to judge the remainder of the book.

Consider the following situation. Batches of approximately N items reach an inspection station through one of I streams. For a consumer, these streams might be different suppliers, while for a producer, the streams might be different production lines; it is possible that the most common case is $I = 1$. The quality of batches in the streams may or may not be correlated with the quality of other neighbouring batches in the same stream or in other streams. It is also possible that these input streams may have different states; for example, a production process may be either in control or out of control. It seems obvious that when several states exist in the input streams, the inspection plan should be specially designed to deal with this.

At the inspection station a sample of items is selected from some or all of the batches and the samples are inspected. Each batch is then sentenced, and placed in one of the J output streams.

If there are only two output streams, these are usually referred to as the *accepted* and the *rejected* batches. For final inspection by a producer, the accepted batches are those passed on for sale to customers. There are many possibilities for the rejected lots, and some of these are set out in Figure 1.3, some of which is taken from Hald (1960). However, this diagram is really appropriate when items are simply classified as effective or defective. More frequently there might be different types of defective, and different action taken on each type.

In some applications of inspection plans there may be more than two output streams. For example, there may be two grades of accepted batches, for different uses, or for sale at different prices. Similarly there could be two grades of rejected batches. However, such plans would often be considered unduly complicated, and liable

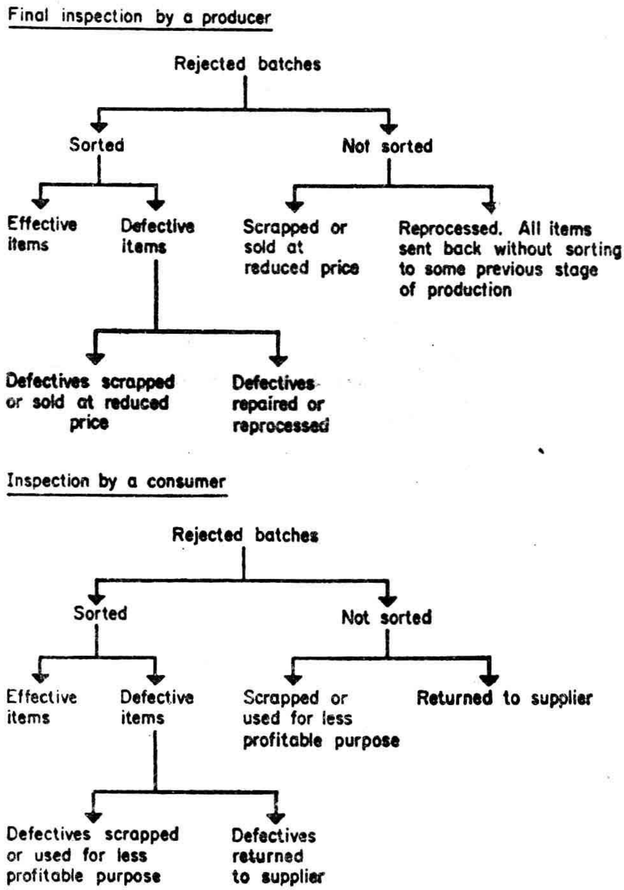

Figure 1.3. *Some possible courses of action on rejected batches.*

to lead to gross errors on the part of the inspector. Here we consider two output streams and call them accepted and rejected batches.

Another point with regard to the flow chart, Figure 1.2, is to specify which parts of this chart work at a given rate, and which parts can work at varying rates. For final inspection by a producer, the input streams are fixed, but for inspection by a consumer, the quantity usually fixed is the number of acceptable batches passed.

In addition to either of these possibilities, the labour and resources available at the inspection station will usually be fixed, and variable only in a long-term sense.

The purpose for which inspection is being applied also needs to be considered in some detail. For a producer, some possible aims are:

(*a*) To satisfy some requirement for the British Standards Institution, etc.
(*b*) To grade batches for sale.
(*c*) To prevent bad batches being passed on to customers.
(*d*) To provide information from which a quality control plan can be operated.

The aims for a consumer might be:

(*e*) To confirm that the quality of goods supplied is up to standard.
(*f*) To prevent bad batches being passed on to a production process.
(*g*) To grade batches for different uses.
(*h*) To encourage the producer to provide the quality desired (Hill, 1960). This purpose can only be achieved if the consumer uses a substantial part of the supplier's output.

It is probable that in many situations in which sampling inspection plans are applied, the aims are not easy to define precisely.

We can see throughout this discussion that inspection by a producer is in general very different from that by a consumer.

An extended discussion of some case studies of quality control practices arising in industry is given by Chiu and Wetherill (1975).