Introductory Mathematics for Economists

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Preface

This book introduces to students with a limited mathematical background the essential mathematics needed for a study of economics. It is not intended to replace the more formal mathematics texts and does not include proofs of all the formulae used. These are only included where they aid understanding. It is hoped that this approach will be more suitable for those students who, as a result of their earlier experiences in the subject area, do not regard themselves as having any mathematical ability. The inclusion of worked examples in the text and exercises with answers in worked form (at the end of the book) are intended to help such students.

The material is selected so as to increase in difficulty as the book progresses. The introductory chapter on Linear Equations leads to the more general chapters on Elementary Matrix Algebra and Non-linear Equations. Chapter 4 on Series deals with applications of immediate relevance as well as providing the groundwork for the chapters on calculus. Chapters 5 and 6 are concerned with the relationship between two variables whilst more general relationships are covered in Chapter 7. This chapter ends with the problem of maximisation subject to constraint, and a variation on the same problem is presented in Chapter 8. The final chapters deal with dynamic relationships, in continuous terms in Chapter 9 and discrete terms in Chapter 10. Trigonometric functions are considered in detail in Appendix A.

The order in which the material is covered can be varied. For example some teachers may prefer to leave Matrix Algebra to the end of a course, or to follow Integral Calculus with Differential Equations.

While the text is intended primarily for students of economics it has proved to be extremely helpful in the teaching of mathematics to students of business studies. It should also be useful to managers who need to renew their acquaintance with the basic mathematical techniques relevant to operational research.

We are grateful to Roger Latham and David Peel of the University of Liverpool for reading through an earlier draft and making many useful suggestions, and also to Alan Whittle for his constructive comments on the final draft. They are, of course, in no way responsible for any errors. We are also grateful to Miss Pauline Conley who typed a difficult manuscript in record time.

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A.W.P.

Chapter 1
Linear Equations

1.1 Introduction

An equation can be a simple means of summarising the important features of a particular situation. It may be used for descriptive purposes or it may provide information which is very useful for decision making.

To illustrate this it is useful to look at a particular case. If we choose one from the production-management area we may have a situation in which the total cost of production is determined by the level at which a particular process is operated, ie by the level of output. This could be expressed as follows:

$$TC = f(q)$$

which is read 'TC is a function of q', where TC represents total cost

and q represents quantity or level of output.

The symbol f coupled with the brackets is a shorthand way of saying that the two quantities or *variables* are related in some way, which is as yet unspecified but is assumed to be single valued, ie for each value of q there is only one value of TC.

This may not appear to be very useful and indeed it is not unless we go further and attempt to identify the form which the relationship between the two variables will take. To do this we make two assumptions, which would have to be verified in practice, but which would not be considered unreasonable in many situations. These are as follows:

(a) There are parts of the total cost of production which will not be affected by changes in the level of output because they must necessarily be incurred if the process is adopted and they do not increase as production is increased. These are known as *fixed costs* and include such items as rent, rates, and wages of the labour force which within certain limits are not affected by changes in the level of output.

(b) There are other parts of the total cost of production which increase as the level of output increases. These are known as *variable costs* and include such items as raw materials, power, and parts of the labour force which can be employed on the process if and when required.

It is clear that in many practical situations the split between fixed and variable costs cannot be made very clearly and that in the long run all costs tend to be variable. However, a simple breakdown into these two categories can prove to be very useful in establishing such points as the breakeven level of production, as we shall see later. But first, let us consider an example in which the available information about the production process indicates that the fixed costs amount to £100 and that the variable costs associated with manufacturing 100 units of the product amount to £300. We will assume that the variable costs are directly proportional to the number of units of output and hence that the total cost varies linearly with the level of output.

The relationship between costs and output can then be written

$$\text{Total cost} = \text{fixed cost} + (\text{variable cost per unit}) \times (\text{level of output})$$

or

$$TC = a + b \times q$$

where a and b are two constants which represent fixed cost and variable cost per unit respectively.

1.2 Graphical representation

For a two-variable relationship a common method of presentation of information is by means of a *graph*. This is a two-dimensional diagram with two *axes*, one of which represents TC (in our example the total cost of production) and the other q (the level of output). These axes are generally drawn at right angles to each other and their point of intersection, the origin, O, is where both total costs and the level of output are zero. By choosing suitable scales to represent different values of the variables we can construct a graph from any given set of data.

In our example when the level of output is 0, the fixed costs and hence the total costs are equal to £100, ie when $q=0$, $TC=100$. Also, when the level of output is 100, the total costs are £400, ie when $q=100$, $TC=400$.

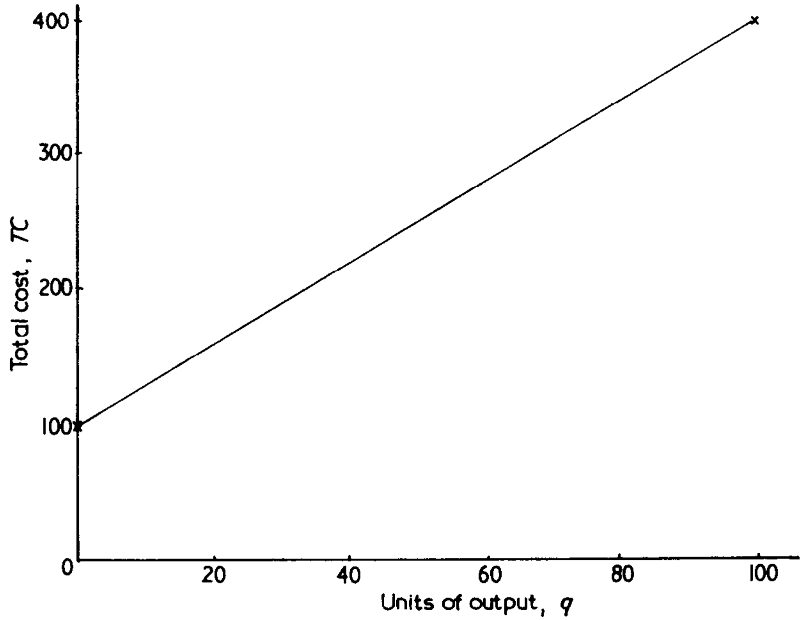

Fig 1.1

Since we are assuming that total cost increases uniformly and continuously with output we can join the two points by a straight line. Hence $TC=a+bq$ is an equation of a straight line, and in this case

$$a = \text{fixed costs} = £100 \text{ and}$$

$$b = \text{variable cost per unit of output} = £3$$

Effectively we are determining the values of a and b by using the information about the two points to solve the equation $TC=a+bq$.

This information enables us to locate two points on the graph with the *coordinates* (0, 100) and (100, 400), where the two numbers in brackets refer to the values of the variables measured along the horizontal and the vertical axes respectively. In general, any point on this graph (Fig. 1.1) is represented by the coordinates (q, TC).

For the first point: $q=0$, $TC=100$ and so

$$100=a+b\times 0$$

$$a=100$$

and for the second point $q=100$, $TC=400$ and so

$$400=a+100b$$

Substituting the value $a=100$ in this equation gives

$$400=100+100b$$

Subtracting 100 from each side of this equation leaves us with

$$300=100b \qquad \text{and so} \qquad b=3$$

Hence $TC=100+3q$ is the equation which summarises the available information about the production process and describes how costs vary with output.

This equation can now be used to determine the total cost of production at other levels of output. For example, when $q=20$, $TC=100+3\times 20=£160$.

If this point is included on the graph we find that it lies on the straight line joining the two points corresponding to the initial data. We would also find that all other points which satisfy the equation lie on the same straight line. For this reason $TC=100+3q$ is known as a *linear equation*.

In general the choice of symbols for the variables in an equation is made by the people concerned with the presentation and analysis of the information. Some letters tend to be used more frequently than others, and some have fairly agreed usage for particular variables. However, there are no exact rules to be followed, and it is important that the principle is understood, and that emphasis is not placed on the symbols themselves. It follows that $Y=a+bX$ could have been used in the example we have just considered, with Y replacing TC and