NONLINEAR
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Anjan Biswas
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Mathematical Theory of Dispersion-Managed Optical Solitons

色散管理光孤子的数学理论

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With 23 figures

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NONLINEAR PHYSICAL SCIENCE
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NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences

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Preface

The concept of dispersion-managed (DM) optical solitons was introduced in the early 1990s. The advent of such DM solitons has changed the world of optical solitons. In fact, they are governed by the dispersion-managed nonlinear Schrödinger's equations (DM-NLSE), unlike in the case of classical solitons which are governed by the pure nonlinear Schrödinger's equations (NLSE). It is to be noted that the pure NLSE is integrable by the classical method of Inverse Scattering Transform, while the case of DM-NLSE is not integrable. This leads to a lot of challenges and hindrances in studying the DM-NLSE. Many methods have been introduced in order to study the DM-NLSE. They are the variational principle, soliton perturbation theory, moment method as well as the asymptotic analysis. These methods of studying the DM solitons have introduced a wider picture in this area.

This book introduces and exposes the concept of DM solitons from scratch. Later the soliton perturbation theory and the variational principle are introduced to study the dynamics of pulses that propagate through optical fibers. The types of optical fibers that are studied in this book are the polarization preserving fibers, birefringent fibers and finally the case of multiple channels is also taken into consideration. The asymptotic analysis is used to study the quasi-linear pulses in optical fibers where along with the dispersion, the nonlinearity is also managed. Later, the Gabitov-Turitsyn equations are derived for these three types of optical fibers using the asymptotic analysis. Subsequently, higher order asymptotic analysis is carried out to derive the higher order Gabitov-Turitsyn equations for these types of optical fibers. Finally, the issue of optical crosstalk is touched upon to complete the discussion.

It needs to be noted that there are quite a few technical aspects that are skipped in this text. Those issues are the collision induced frequency and timing jitter along with the amplitude jitter. The other important issue that has been deliberately skipped is the issue of four-wave mixing. Finally, one other important issue of DM solitons that has not been touched upon is the aspect of soliton radiation. These issues are not yet exhaustively studied in the context of DM solitons and therefore requires further development before being incorporated in this text. Although there are a few papers that have been published in this context, a substantial amount of work is yet to be done to complete these chapters.

This book is organized as follows: Chapter 1 introduces the necessity and importance of studying the dispersion-managed optical solitons as opposed to the classical or conventional optical solitons. Chapter 2 introduces the technicalities of dispersion-managed optical solitons, the conserved quantities as well as the soliton perturbation theory. Finally this chapter ends with a brief introduction to the variational principle. Chapter 3 focuses on the polarization preserving fibers. Three types of pulses are studied in this chapter. They are Gaussian, super-Gaussian and super-sech pulses. The soliton parameter dynamics are derived. Finally the stochastic perturbation of optical solitons is studied with the aid of soliton perturbation theory. Chapters 4 and 5 deal with the birefringent fibers and multiple channels. Chapter 6 details out the aspect of optical crosstalk in both the linear as well as the nonlinear regime. Chapter 7 derives the Gabitov-Turitsyn equation for polarization-preserving fibers, birefringent fibers as well as in the case of multiple channels by using the technique of multiple-scale perturbation expansion. In Chapter 8, the issue of quasi-linear pulses are studied which another form of optical pulses that are studied where the nonlinearity is also managed in addition to the group velocity dispersion. Finally, in Chapter 9, the higher order asymptotic analysis is carried out to derive the higher-order Gabitov-Turitsyn equations that serves as an opening to the future research in this direction.

This book is primarily intended for graduate students at the Masters and Doctoral levels in Applied Mathematics, Applied Physics and Engineering. Also undergraduate students, with senior standing, in Physics and Engineering will benefit out of this book. The pre-requisite of this book is a knowledge of Partial and Stochastic Differential Equations, Perturbation Theory and Quantum Mechanics.

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Daniela Milovic
Matthew Edwards

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Contents

Chapter 1
Introduction

Dispersion-Managed optical solitons or DM solitons was first introduced in the early 1990s and since then it became a very attractive topic for optical communications. Dispersion-Management is very important in dense wavelength-division multiplexed (DWDM) systems. DM solitons allow the formation of ultra-long-haul Tera bit level optical networks working in all optical mode and maintaining optical transparency over vast geographical regions. The more wavelengths in use, the greater the need for dispersion-management. In high channel count systems, channel spacing is very close and requires continuous broadband management of dispersion and dispersion slope [1–10].

The discovery of dispersion-managed optical soliton has introduced plenty of new methods for high rate data transmission. By proper choice of parameters both Gordon-Haus effect and four-wave-mixing can be significantly suppressed thus providing nearly error-free transmission. It was shown in 1997 that bright solitons can exist in DM fibers and they can exist at normal path average dispersion. Dark solitons can also exist in such fibers, even at anomalous path averaged dispersion. This implies that the range of existence of bright and dark solitons overlap and therefore it becomes possible, for the first time, to analyze interaction between bright and dark solitons.

The propagation of DM solitons is governed by the dispersion-managed nonlinear Schrödinger's equation (DM-NLSE). Dispersion management forces each soliton to propagate in the normal dispersion regime of a fiber during every map period. When the map period is a fraction of the nonlinear length, the nonlinear effects become insignificant, leading to linear pulse evolution over the map period. If the self-phase modulation (SPM) effects are balanced by the average dispersion, solitons can survive in an average sense even on a longer length scale.

The NLSE that governs the classical or conventional soliton propagation is integrable by the classical method of Inverse Scattering Transform (IST), unlike DM-NLSE which is not integrable by IST, since the Painleve test of integrability [2] will fail. So, several methods have been introduced in order to study DM-NLSE that includes variational approach, collective variables approach, soliton perturbation theory, moment method as well as the asymptotic analysis. These methods really brought wider picture in the area of DM solitons.

The main feature of DM soliton is that it does not maintain its shape, width or peak power, unlike a fundamental soliton. However, DM soliton parameters repeat through dispersion map from period to period. This makes DM solitons applicable in communications in spite of changes in shape, width or peak power. From a systems standpoint, these DM solitons perform better.

By a proper choice of initial pulse energy, width and chirp will periodically propagate in the same dispersion map. The pulse energies much smaller than critical energy should be avoided in designing DM soliton system, whereas if the pulse energy is the same as the critical energy, it is the most suitable situation. An inappropriate choice of initial pulse energy may cause pulse interaction and thus lead to detrimental pulse distortion. The required map period becomes shorter as the bit rate increases.

The main difference between the average group velocity dispersion (GVD) solitons and DM solitons lies in its higher peak power requirements for sustaining DM solitons. The larger energy of DM solitons benefits a soliton system by improving the signal-to-noise ratio (SNR) and decreasing the timing jitter. The use of periodic dispersion map enables ultra high data transmission over large distances without using any in-line optical filters since the periodic use of dispersion-compensating fibers (DCF) reduces timing jitter by a large factor.

An important application of the dispersion-management is in upgrading the existing terrestrial networks employing standard fibers. Recent experiments show that the use of DM solitons has the potential of realizing transoceanic light-wave systems capable of operating with a capacity of 1 Tb/s or more.

Optical amplifiers compensate fiber losses but on the other hand induce timing jitter. This phenomena is mainly caused by the change of soliton frequency which affects the group velocity or the speed at which the pulse propagates through the fiber. The timing jitter can become an appreciable fraction of a bit slot for long-haul systems as bit slots become less than 100 ps. If left uncontrolled, such jitter can cause large power penalties. For DM solitons timing jitter is considerably smaller than that for fundamental solitons and the physical reason for jitter reduction is related to the enhanced energy of the DM solitons. From a practical point of view, reduced timing jitter of DM solitons permits much longer transmission distances.

As the bit rate increases, soliton-soliton interaction becomes a critical issue. Collision length depends on the details of dispersion map. The system performance can be optimized by an appropriate choice of map strength.

Dispersion-management can be efficiently used in several situations as follows:

1. For optimum pulse generation in a mode-locked laser operating at a wavelength around 1 μm or shorter. The normal chromatic dispersion has to be over compensated in order to utilize the anomalous dispersion regime, where soliton effects can help to obtain shorter pulses. It is usually also necessary to compensate carefully the higher-order dispersion, i.e., to control the group delay dispersion over a significant optical bandwidth.
2. In a mode-locked fiber laser, dispersive and nonlinear effects can become so strong that the pulse parameters (including the pulse duration and chirp) vary significantly during each resonator round-trip. With a suitable combination of fibers exhibiting normal and anomalous dispersion, a stretched-pulse fiber laser can be realized, which can generate pulses (DM solitons) with significantly higher pulse energy than with e.g. soliton mode locking.
3. Similar effect can be used in optical fiber communications: a fiber-optic link consisting of a periodic arrangement of fibers with normal and anomalous dispersion can help to suppress nonlinear effects such as channel crosstalk via four-wave mixing. It is possible to suppress the Gordon-Haus timing jitter at the same time, if the average chromatic dispersion is zero.

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Chapter 2
Nonlinear Schrödinger's Equation

In this chapter the nonlinear Schrodinger's equation (NLSE) will be derived from the basic principles of Electromagnetic Theory. This equation will be then modified in presence of dispersion-management. The conserved quantities of this dispersion-managed NLSE (DM-NLSE) will be derived. The variational principle used for solving the DM-NLSE will be introduced. Finally, this chapter will end with a brief introduction to the soliton perturbation theory.

2.1 Derivation of NLSE

Pulse propagation through nonlinear and dispersive medium is governed by the NLSE that is derived from Maxwell's equation. Maxwell's equations arise in Electromagnetic Waves and are described in a medium that is assumed to be isotropic with no free charges (i.e., no plasma). NLSE can be modified for the case of plasma generation by adding terms to the NLSE that account for multi-photon absorption and plasma index change.

The starting point is the Maxwell's equations which takes the following forms, assuming no free charges:

$$\nabla \cdot \boldsymbol{D} = 0 \tag{2.1}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{2.2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{2.3}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{2.4}$$

where $\boldsymbol{E}$, $\boldsymbol{H}$ are electrical and magnetic fields while $\boldsymbol{D}$, $\boldsymbol{B}$ are the respective flux densities and all vectors depend on three spatial coordinates and time