An Introduction to the Theory of Superfluidity

KHALATNIKOV

An Introduction to the Theory of Superfluidity

I. M. KHALATNIKOV
L. D. Landau Institute of Theoretical Physics of the U.S.S.R. Academy of Sciences

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Pierre C. Hohenberg

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Vita

Isaac Markovich Khalatnikov

Isaac Markovich Khalatnikov is a Professor of Theoretical Physics at the Moscow Physiotechnical Institute. A graduate of Dniepropetrovsk University, Dr. Khalatnikov worked in the theoretical section of the Institute of Physical Problems at the U.S.S.R. Academy of Sciences under the direction of L.D. Landau. Khalatnikov and Landau together founded the theory of quantum liquids. Khalatnikov has worked as director of the L.D. Landau Institute of Sciences and has been appointed as an academician at the academy. Dr. Khalatnikov is a U.S.S.R. State Prize Laureate and is a past winner of the Landau Prize of the U.S.S.R. Academy of Sciences. His main research interests are in the fields of superconductivity and superfluidity theory, quantum field theory and cosmology.

Special Preface

Almost 25 years have elapsed since publication of the first version of this book. As the author of the book, I believe that the book, written as an introduction to the theory of the field of physics so advanced for this period, has not lost its importance. No essential changes have occurred in the fundamentals of the superfluidity theory based on Landau's ideas.

The book consists of 4 parts. Part 1 covers main properties of the excitation spectrum in superfluid ^{4}He and the thermodynamics determined by the spectrum. The material presented in Sections 1-6 of Part 1 needs no additional comments. Section 7 is devoted to the description of the interaction of elementary excitations. In reading this section one should bear in mind that our notions of the long wavelength phonon part of the spectrum have undergone certain alterations. Experiment has revealed that in the initial part of the phonon spectrum the sign of dispersion is such that conservation laws permit decay of one phonon into two. Therefore, although this does not affect qualitative results of the calculations of the kinetic effects, described in Part 3, quantitative results should be insignificantly modified with processes of phonon decay taken into account.

Part 2, "Hydrodynamics," has remained, as it should, invariant and no statement has lost its importance.

When working with Part 3, one should remember the remark concerning the phonon decay effect [1].

A brief Part 4 is devoted to solutions of impurities in superfluid ^{4}He. As an introduction to this field it retains its validity.

The book was intended for young researchers involved in theory and experiment of low temperature physics.

Professor I.M. Khalatnikov

[1] See I.M. Khalatnikov "Phenomenological Theory of Superfluid ^{4}He "in the book "The Physics of Liquid and Solid Helium," Part 1, ed. by K.H. Bennemann & J.B. Ketterson, John Wiley & Sons, New York

Editor's Foreword

The problem of communicating in a coherent fashion the recent developments in the most exciting and active fields of physics seems particularly pressing today. The enormous growth in the number of physicists has tended to make the familiar channels of communication considerably less effective. It has become increasingly difficult for experts in a given field to keep up with the current literature; the novice can only be confused. What is needed is both a consistent account of a field and the presentation of a definite "point of view" concerning it. Formal monographs cannot meet such a need in a rapidly developing field, and perhaps more important, the review article seems to have fallen into disfavor. Indeed, it would seem that the people most actively engaged in developing a given field are the people least likely to write at length about it.

"Frontiers in Physics" has been conceived in an effort to improve the situation in several ways. First, to take advantage of the fact that the leading physicists today frequently give a series of lectures, a graduate seminar, or a graduate course in their special fields of interest. Such lectures serve to summarize the present status of a rapidly developing field and may well constitute the only coherent account available at the time. Often, notes on lectures exist (prepared by the lecturer himself, by graduate students, or by postdoctoral fellows) and have been distributed in mimeographed form on a limited basis. One of the principal purposes of the "Frontiers in Physics" series is to make such notes available to a wider audience of physicists.

It should be emphasized that lecture notes are necessarily rough and informal, both in style and content, and those in the series will prove no exception. This is as it should be. The point of the series is to offer new, rapid, more informal, and it is hoped, more effective ways for physicists to teach one another. The point is lost if only elegant notes qualify.

A second way to improve communication in very active fields of physics is by the publication of collections of reprints of recent articles. Such collections are themselves useful to people working in the field. The value of the reprints would, however, seem much enhanced if the collection would be accompanied by an introduction of moderate length, which would serve to tie the collection together and necessarily, constitute a brief survey of the present status of the field. Again, it is appropriate that such an introduction be informal, in keeping with the active character of the field.

A third possibility for the series might be called an informal monograph, to connote the fact that it represents an intermediate step between lecture notes and formal monographs. It would offer the author an opportunity to present his views of a field that has developed to the point at which a summation might prove extraordinarily fruitful, but for which a formal monograph might not be feasible or desirable.

Fourth, there are the contemporary classics—papers of lectures which constitute a particularly valuable approach to the teaching and learning of physics today. Here one thinks of fields that lie at the heart of much of present-day research, but whose essentials are by now well understood, such as quantum electrodynamics or magnetic resonance. In uch fields some of the best pedagogical material is not readily available, either because consists of papers long out of print or lectures that have never been published.

"Frontiers in Physics" is designed to be flexible in editorial format. Authors are ncouraged to use as many of the foregoing approaches as seem desirable for the project hand. The publishing format for the series is in keeping with its intentions.

Finally, suggestions from interested readers as to format, contributors, and tributions will be most welcome.

David Pines
Urbana, Illinois
August 1964

Preface

The present book is an exposition of the modern theory of superfluidity, a phenomenon which occupies a distinctive place in contemporary physics. This is first of all due to the fact that superfluidity is a macroscopic manifestation of quantum laws. The phenomenon of superfluidity was at first thought to be rather exotic, and restricted to liquid helium. It is now known that, in one form or another, superfluidity is found in all macroscopic bodies, wherever quantum laws are applicable.

Intensive interest in this phenomenon has led to progress in solid state physics as a whole. The ideas and methods of the theory of superfluidity have turned out to bear fruit in many branches of physics, quite far removed from solid state physics, such as the theory of nuclear structure.

This book is intended for research workers and graduate students, and may serve as an introduction to this most interesting field of contemporary physics. It is assumed that the reader is familiar only with the fundamentals of quantum mechanics and statistical physics.

The author wishes to express his deep gratitude to Professor David Pines, who first suggested writing this book. The author is likewise grateful to Dr. Pierre Hohenberg for his considerable labor in translating and editing the manuscript.

I.M.K.
Moscow
April 1965

Translator's Preface

The task of translating this volume would have been much less rewarding for me had I not had the pleasure of meeting the author, and discussing certain questions with him in person. I would like to take this opportunity to thank Professor Khalatnikov for the unforgettable experience of working for a year within the theoretical group at the Institute for Physical Problems.

P.C.H.

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