

M<sup>3</sup>D:

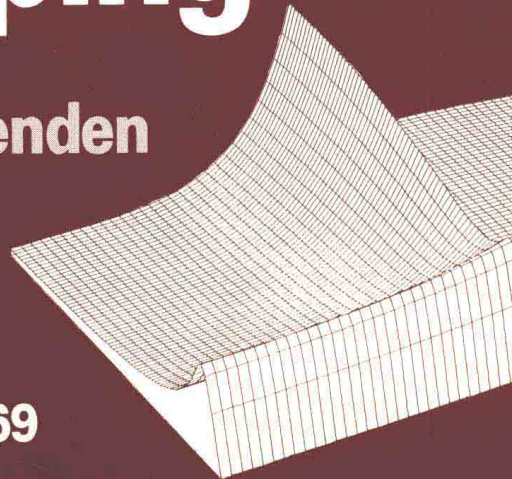
**Mechanics *and***  
**Mechanisms *of***  
**Material**  
**Damping**

**Kinra/Wolfenden**

*editors*



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# *M<sup>3</sup>D: Mechanics and Mechanisms of Material Damping*

*Vikram K. Kinra and Alan Wolfenden, Eds.*

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**IN MEMORIUM  
HERBERT KOLSKY  
1917–1992**



*Professor Herbert Kolsky (“Harry” to all who knew him personally), was a pioneer and leader in the field of experimental investigation of the behavior of materials under sudden loads. He was also a splendid teacher, whose graduate students will remember him affectionately for his painstaking guidance and a warm sense of humor. One of us, V. K. Kinra, had the privilege of being his graduate student at Brown University. He taught courses at all levels with memorable zeal and gusto as a faculty member since 1960 at Brown University, first in the Division of Applied Mathematics and later with a dual appointment in the Division of Engineering. Kolsky was a lively and original lecturer, and was welcomed as speaker at numerous seminars and conferences; most recently he delivered an invited keynote lecture at this Symposium. He was in demand as a visiting scholar, and lectured in China, France, India, and Israel. He accepted invitations, among others, for sabbatical visits at Imperial College in London, Middle East Technical University in Ankara, Turkey, Oxford University, the Swiss Federal Institute of Technology (E.T.H.) in Zurich, and the University of California at Berkeley.*

*Born in London, England, and graduating in 1937 from Imperial College with honors in physics, Kolsky earned Ph.D. and D.Sc. degrees from the University of London in 1940 and 1957, respectively. He was awarded an honorary D.Sc. by the E.T.H. in Switzerland in 1984. Following wartime work in a research group at Imperial College, London, he served for nine years as head of the physics department at Butterwick Research Laboratories of Imperial*

*Chemical Industries in Welwyn, England. Supported by a Fulbright Fellowship he then spent two years as visiting professor at Brown University, returning to England in 1958 to accept a "Special Merit" appointment as Senior Principle Scientific Officer at the Royal Armament Research and Development Establishment at Fort Halstead, Seven Oaks, Kent. He was persuaded to return to America in 1960, and enjoyed more than thirty fruitful years of research until his death; his last paper appears in this volume. He was a Fellow of the Institute of Physics, the American Physical Society, the Acoustical Society of America, and the American Academy of Mechanics.*

*Kolsky published about 100 papers on a variety of subjects, including the properties of metals and polymers under dynamic conditions, the propagation and interaction of cracks in brittle materials, the penetration of solid bodies by projectiles, and the performance of fibre-reinforced composites under impact conditions, to name only a few interests. But two accomplishments in particular should be singled out. Kolsky had an extraordinary influence on the field of research which is concerned with the characterization and quantitative measurement of properties of materials as they respond to loads that are suddenly applied with high intensities, so that damage in the form of permanent deformation or rupture occurs. This field of investigation is of obvious practical importance, for example, in the design of aircraft and other high speed vehicles, and in the analysis of penetration of armor by projectiles. It involves the intrinsic difficulty that the basic properties can only be determined if the deformation of the specimen being tested can be analyzed mathematically, whereas such an analysis can only be made if the properties of the material are already known. Kolsky's achievement was to devise an ingenious but simply constructed apparatus which enables this dilemma to be largely circumvented. In the literature this apparatus is generally referred to as the "split-Hopkinson bar," because it derives from old ideas of a pressure bar due to B. Hopkinson. However, Kolsky's method involved ideas far beyond the Hopkinson bar, and the apparatus that more properly should be called the Kolsky bar has become a standard, very widely used device, with hundreds in many variations being employed in laboratories around the world.*

*Kolsky's other signal contribution was a landmark book "Stress Waves in Solids" published in Oxford in 1953, and since reprinted by Dover. It has been translated into Russian and Chinese. It is a concise treatment, but so authoritative on the basic theory and its experimental underpinning that it still serves as a kind of "Bible" for students and researchers, as it has done for nearly 40 years. This exemplary contribution was honored in the citation for the Worcester Reed Warner gold medal of the American Society of Mechanical Engineers, awarded to Kolsky in 1982.*

*Kolsky was a man of bright spirit who will be greatly missed by his friends and colleagues around the world. They will remember among other things his marvelous ability to come up with a verse of poetry or an anecdote precisely fitting the occasion. As a homage to his many contributions to the field of experimental mechanics, we dedicate this book to Professor Harry Kolsky.*

*Vikram K. Kinra  
Alan Wolfenden  
College Station, Texas*

# Foreword

The symposium on M<sup>3</sup>D: Mechanics and Mechanisms of Material Damping was presented in Baltimore, Maryland, 13–15 March 1991. The symposium was sponsored by ASTM Committee E28 on Mechanical Testing in cooperation with The Office of Naval Research. Vikram K. Kinra and Alan Wolfenden, Texas A&M University, presided as chairmen of the symposium and are editors of this publication.

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# Overview

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In Baltimore on 13–15 March 1991 the International Symposium on “M<sup>3</sup>D: Mechanics and Mechanisms of Material Damping” took place. The Symposium was sponsored by the ASTM Committee E28 on Mechanical Testing in cooperation with the Office of Naval Research. More than 69 colleagues representing several countries participated. The main objective of the Symposium was to bring about a synergistic interaction among researchers in the field of mechanics of solids and materials science, with the aim of promoting increased collaboration between the two disciplines. In retrospect, and judging from the comments made by many of the participants during and after the meeting, the organizers of the Symposium feel that the objective was well met.

Within the two and a half days' duration of the Symposium there were five technical sessions. Each session was under the control of two specially invited Session Chairmen and started with a keynote address given by an internationally recognized authority in a particular aspect of damping. The keynote address was then followed by several contributed talks. The Session Chairmen ensured that ample time was available for questions and discussion as an integral part of each talk. The discussions were judged to be particularly valuable. In addition to these five technical sessions, and due to the overwhelming response from the call for papers, the Organizers arranged a Poster Session at which around twenty colleagues set up displays detailing the latest results of their research on many aspects of damping. The Poster Session allowed ample time for researchers to get acquainted, or, in many cases, re-acquainted. This session also allowed the participants the opportunity to grasp a better understanding of the varied techniques used for damping measurements and to develop ideas for future research.

As a record of the International Symposium on M<sup>3</sup>D we have produced this Special Technical Publication (STP). It is comprised of 35 papers which cover a wide range of aspects of damping, from fundamental work to technological applications. There were five invited keynote speakers who led off the sessions of the Symposium and whose efforts were focused on highlighting the complementary roles of mechanics and materials in our understanding of damping phenomena. The first five papers in the STP are their keynote papers. Most of the authors contributing to the STP are affiliated with universities or national laboratories, but two representatives are from industry. The international interest in damping is reflected *in the fact that authors from eight countries contributed to the STP*. Although most of the papers had a natural bias to either mechanics or materials, many of the papers point up the necessity of merging these disciplines for advances in understanding damping.

That damping is a complicated phenomenon can be judged by the breadth of the mechanics and materials topics that are covered in this STP: thin-layer materials, high damping materials, metal matrix composites, ceramic matrix composites, polymer matrix composites, phase changes (including martensitic phase changes and solid-liquid phase changes), non-linear effects of boundaries, coupled modes of vibration, magnetomechanical damping, thermoelastic damping, finite element modeling, specific materials (tungsten carbide-cobalt, aluminum, spinels, niobium, tantalum, zircons, rubber, zirconium-niobium, carbon-carbon, superalloys, silicon carbide-vitroceraamics, steels, and aluminum-silicon), grain boundary effects, and longitudinal and flexural vibrations. (One wonders what does NOT affect damping!).

The keynote paper by Berry emphasizes the power of damping measurements in understanding and controlling the mechanisms of damping at the atomic level in a variety of

materials that will be of great technological impact, including the new superconductors with high critical temperatures. Professor de Batist's keynote paper is on the subject of the role of martensitic phase transformations in promoting high damping in alloy systems. The keynote paper by Kolsky covers the history of the development of techniques for measuring damping over many orders of magnitude of frequency. Some innovative nontraditional applications of damping measurements were the subject of the keynote paper by Professor Gibson. Finally, the fifth keynote paper by Wong and Holcomb deals with some aspects of the mechanics of damping in ceramic reinforced metals.

Readers who are interested in the history of the development of techniques for measuring damping will appreciate the keynote paper by Kolsky and the invited paper by Plunkett. In particular, the Kolsky paper has an ample supply of references. Other papers, such as that by Lesieutre, start with the traditional tools of mechanics (for example, finite element modeling) and lead to suggestions for critical measurements for determining mechanisms of damping. A few papers attack the problems of getting exact solutions for specific aspects of flexural vibrations or coupled modes of vibration.

For the future, much research is left to be done. For all the techniques of measuring damping, the availability of standard reference materials with known values of damping (for specific vibrational mode and frequency) is going to be essential for assessing accuracy of damping values.

The organizers of the Symposium have had lots of help from many sources. Truly the team spirit is alive in the damping field! We express our appreciation to: the ASTM staff for their efforts in promoting the Symposium, in having the arrangements at the location run so smoothly, and in providing the infrastructure for getting this STP available in print; the Session Chairmen for being firm but diplomatic in running their sessions punctually and for sparking lively discussions; the invited keynote speakers for permitting us to peek into their vast reservoir of expertise on damping gained over many years; the participants from overseas for providing the international flavor; our network of anonymous reviewers for their time and most helpful suggestions for improvements in many of the manuscripts (each manuscript was reviewed by three reviewers); our colleagues in Committee E28 for their enthusiasm in backing the Symposium; the Office of Naval Research, represented by Dr. Don Polk, and Ms. Cathy Wong, for its interest and sponsorship of the Symposium.

As a final note, as organizers of the Symposium and editors of the STP, we admit reluctantly that we have enjoyed all the hustle and bustle associated with this activity. Many hours of our time went into seeing that things were done correctly and on time. We thank our families for putting up with us!

*Vikram K. Kinra*

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# Relationship Amongst Various Measures of Damping

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(valid for small values of damping:  $\tan \phi < 0.1$ )

$$Q^{-1} = \frac{\Psi}{2\pi} = \eta = \frac{\delta}{\pi} = \tan \phi = \phi = \frac{E''}{E'} = 2\zeta = \frac{\Delta W}{2\pi W} = \frac{\lambda\alpha}{\pi}$$

$Q$  = Quality Factor

$\Psi$  = Specific Damping Capacity

$\eta$  = Loss Factor

$\delta$  = Logarithmic Decrement

$\phi$  = Phase Angle by which Stress Leads Strain

$E''$  = Loss Modulus

$E'$  = Storage Modulus

$\zeta$  = Damping Ratio or Damping Factor

$\Delta W$  = Energy Loss Per Cycle

$W$  = Maximum Elastic Stored Energy

$\lambda$  = Wavelength of Elastic Wave

$\alpha$  = Attenuation

Herbert Kolsky<sup>1</sup>

# The Measurement of the Material Damping of High Polymers Over Ten Decades of Frequency and Its Interpretation

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**REFERENCE:** Kolsky, H., "The Measurement of the Material Damping of High Polymers Over Ten Decades of Frequency and Its Interpretation," *M<sup>3</sup>D: Mechanics and Mechanisms of Material Damping*, ASTM STP 1169, V. K. Kinra and A. Wolfenden, Eds., American Society for Testing and Materials, Philadelphia, 1992, pp. 4–27.

**ABSTRACT:** This paper describes a number of experimental methods that have been used to measure the internal energy loss that occurs in many solids, especially high polymers, when they are taken around deformation cycles. This loss can be measured directly, but when its magnitude is small and when the response is linear, it is found to be proportional to four other quantities: namely, the logarithmic decrement of free oscillations of systems where the material is used as a restoring elastic element, the sharpness of resonance of forced oscillations of such systems, the angle by which the strain lags behind the stress in sinusoidal oscillations, and the attenuation coefficient of sinusoidal waves propagated through the material. The magnitude of this mechanical loss is found to vary with the frequency of the oscillations, and these variations can be correlated to microscopic processes taking place in the solid. A variety of experimental techniques have to be used to cover the large ranges of frequency that are needed in order to study such correlations and it is highly desirable that all the measurements be for the same type of mechanical deformation (for example, shear). The results of measurements extending over about ten decades of frequency are reviewed, and the interpretation of these in terms of the various microscopic processes in the material are discussed.

**KEY WORDS:** material damping, mechanical properties, internal friction, internal stress, high polymers, mechanical deformation

When a sample of a material is taken around a deformation cycle, some mechanical energy is always dissipated, and this energy generally appears in the form of heat. Now, if the maximum amount of elastic energy that is stored in the sample during the cycle is  $W$  and the energy dissipated is  $\Delta W$ , the ratio,  $\Delta W/W$ , is called the "specific damping capacity" or the "specific loss" of the material. This ratio may be regarded as the ratio of the area of the hysteresis loop in the (stress, strain) curve to the maximum energy stored in the sample during the cycle; it is generally found that the value of this ratio depends on the amplitude of the cycle, as well as on the rate at which the sample is deformed. For sufficiently small deformation amplitudes, however, the ratio becomes independent of the amplitude of the cycle, and if the material behaves in a linear manner, the specific loss can be related to other measures of internal friction, such as the logarithmic decrement,  $\Delta'$ , of an oscillating mechanical system in which the sample is the restoring elastic element, and the "half breadth" of the resonant peak, when such a system undergoes forced oscillations. It is also roughly proportional to the phase angle,  $\delta$ , which is the angle by which the strain lags behind the stress in forced sinusoidal oscillations, and to the attenuation coefficient,  $\alpha$ , of a sinusoidal stress wave propagated through the material.

<sup>1</sup>Professor, Brown University, Providence, RI 02912.

Two fundamentally different types of deformation can occur and, in general, the magnitude of the mechanical loss is very different for them. These two types are shear deformation, where the shape of the material is changed without any change in volume, and dilatational deformation, where there are volume changes but no change in shape. When a specimen of cylindrical cross section is deformed in torsion, only shear deformations are produced. It is very much more difficult to produce pure dilatation, without any shear, in dynamic tests. Other types of deformation, for example, simple extension, involve a combination of dilatation and shear.

Now, although it is often possible to make direct measurements of the specific loss, such measurements are generally very difficult and are not found to be very accurate, so that other methods of measuring mechanical loss are normally employed. For deformation cycles that take place very slowly, that is, are of very low frequency, it is most convenient to measure the phase lag,  $\delta$ , between the driving force and the deformation, in sinusoidal deformations. The relationship is

$$\Delta W/W = 2\pi \tan \delta \quad (1)$$

At somewhat higher frequencies, the sample is often tested by employing it as the elastic restoring element in an oscillating mechanical system and measuring the ratio of successive oscillations. The natural logarithm of this ratio is called the logarithmic decrement, generally it is denoted by  $\Delta'$ . The approximate relationship to the specific loss is then

$$\Delta W/W = 2\Delta' \quad (2)$$

The sharpness of the resonance peak for forced oscillations provides an alternative method of determining internal friction. The half breadth of the resonance peak is defined as being equal to  $\Delta N/N$ , where  $N$  is the resonant frequency, and  $\Delta N$  is equal to  $N_1 - N_2$ , where  $N_1$  and  $N_2$  are the frequencies on the two sides of the resonance peak where the amplitude is one half of its value at  $N$ . The relationship here is

$$\Delta N/N = \{(\sqrt{3})/2\pi\}\Delta W/W \quad (3)$$

(It should perhaps be mentioned here, that some workers define  $N_1$  and  $N_2$  as the frequencies at which the "energy" is one half its value at  $N$ , and this leads to the relationship,  $\Delta N/N = (\Delta W/W)/2\pi$ .)

By analogy with electrical resonance, the sharpness of resonance is sometimes defined in terms of the quantity,  $Q$ , the quality factor.  $Q$  is defined as being equal to  $(\Delta N/N)\sqrt{3}$ , so that we have

$$Q = (\Delta N/N)/\sqrt{3} = (\Delta W/W)/2\pi \quad (4)$$

At higher frequencies, wave propagation methods are generally employed; thus, when a train of sinusoidal waves of angular frequency,  $\omega$ , travels through a liner material, it is found that the amplitude of the wave decreases exponentially with the distance of travel. If the oscillating strain at the origin,  $x = 0$ , is given by

$$\epsilon(0,t) = A \cos \omega t \quad (5)$$

then, after the wave has traveled a distance,  $x$ , the expression is

$$\epsilon(x,t) = A \exp(-\alpha x) \cos[\omega(t - x/c)] \quad (6)$$

where  $\alpha$  is called the attenuation coefficient. The relationship between  $\alpha$  and the phase lag,  $\delta$ , is

$$\alpha = (\omega/c)[\tan (\delta/2)] \quad (7)$$

where  $c$  is the phase velocity of the wave. For low damping, Eq 7 approximates to

$$\alpha = (\omega/2c) \tan \delta = (\omega/4\pi c)\Delta W/W \quad (8)$$

The velocity,  $c$ , depends on the type of wave being propagated through the material. For extensional waves, it is given by

$$c = \{\sqrt{(E^*/\rho)}\} \sec (\delta/2) \quad (9)$$

where  $E^*$  is Young's modulus for the material at angular frequency  $\omega$ , and  $\rho$  is the density. Experimental measurements of damping by the wave propagation method can be carried out either by propagating a continuous train of sinusoidal acoustic waves along a filament of the material, and observing the phase and the amplitude of the wave at different points along it or, at higher frequencies, by propagating an ultrasonic wave packet through a specimen and measuring both the time of transit of the packet and the attenuation of its amplitude, as it travels back and forth along the specimen.

Both from the point of view of engineering applications, and in order to study relationships between material damping and the microscopic or molecular processes that produce it, it is desirable to make measurements over extremely wide ranges of frequency. The next section of this paper outlines some of the experimental methods that the author and his coworkers, as well as other investigators, have employed during the last few decades to make such measurements. Subsequent sections discuss the conclusions to which the results of these studies have led.

### Experimental Measurements

As mentioned earlier, it is highly desirable to be able to measure internal friction of materials over a very wide range of frequency; the methods described here cover the range from one cycle in several hours to several tens of megahertz. It is also desirable to make all these measurements for one type of deformation, for example, shear. A considerable amount of experimental work has been carried out on the measurement of the mechanical loss in high polymers. Measurements using torsional deformations appear to have first been carried out by Lethersich [1] and Markovitz et al. [2]. An extensive investigation, over a wide frequency range, was carried out in the present author's laboratory by Benbow [3] both on high polymers and on low molecular weight resins. Benbow used a number of different experimental techniques, and some of the apparatus designed for this purpose is described later.

In order to measure the phase lag,  $\delta$ , by which the strain lags behind the applied torque, Benbow employed the apparatus illustrated in Fig. 1. This apparatus is designed to make measurements at very low frequencies; rod specimens of circular cross section are employed, the bottom of the specimen being fixed to the base of the apparatus and the top end attached to an oscillating torsion head through a metal wire of known torsional stiffness. The head is driven through a twelve-speed gear box, and a harmonic transformer converts the rotation into an oscillatory sinusoidal motion.

This setup covers the frequency range of about one or two cycles per second up to one

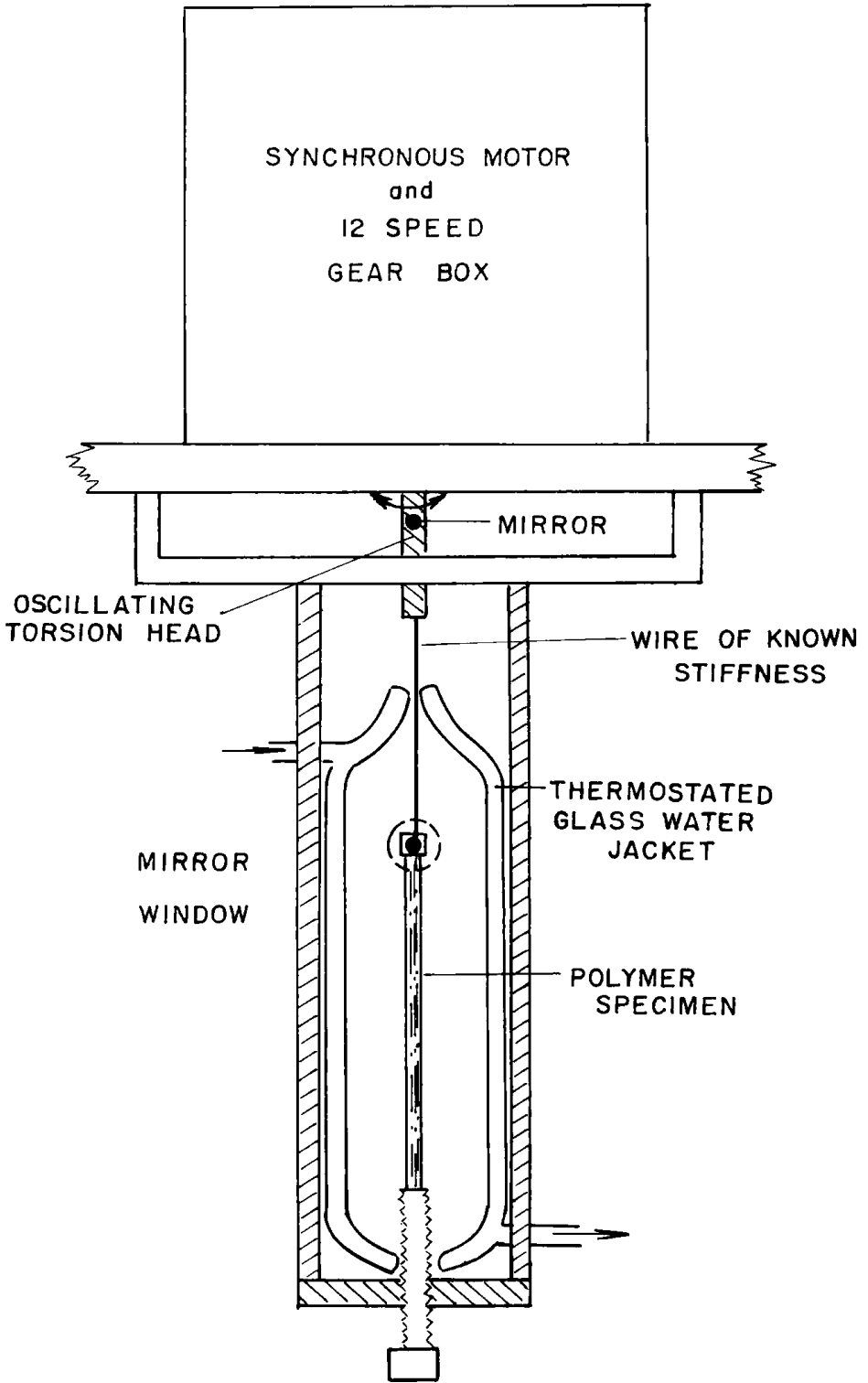


FIG. 1—Apparatus for measuring linear viscoelastic properties at low frequencies.



cycle in about 8 h. The top frequency is still low enough for the effect of the inertia of the setup not to affect the observations appreciably. The measurements were made by reflecting collimated light beams from two mirrors; one mirror was attached to the torsion head, the other to the top of the specimen. The angles of rotation of the two mirrors were measured by the use of conventional galvanometer and scale arrangements. From the angle of rotation of the mirror at the top of the specimen, the torsional strain in the specimen could be calculated, and from the difference between the angles of rotation of the two mirrors, the torque applied to the specimen could be found. The specimen is surrounded by a thermostatic jacket so that tests could be carried out over a range of temperatures.

At somewhat higher frequencies, the method described here becomes difficult because the rotary inertia of the moving parts has to be allowed for, and the recording of  $\delta$  can no longer be done optically. It is then preferable to make measurements by setting up oscillations of a mechanical system with the sample under investigation as its elastic element. When such a system is set into free oscillation, the logarithmic decrement,  $\Delta'$ , can be measured. Then, if forced oscillations are applied over a range of frequencies, the half breadth of the resonance peak can be determined. The results of the free oscillation method are generally simpler to interpret than the resonance measurements, since in the latter method, magneto-mechanical coupling between the driving force and the specimen affects the observations, and has to be allowed for. Resonance methods, however, are often used when the damping is so large that free oscillations cannot be set up.

An apparatus for carrying out tests in torsion on disk-shaped specimens, by measuring both the logarithmic decrement for free oscillations and the sharpness of resonance, is illustrated in Fig. 2. As can be seen from the figure, the bottom face of the specimen is cemented to a rigid block, while the top face is cemented to a circular metallic disk. This disk is supported by a thin vertical piano wire that is strong enough to support the weight of the disk, but thin enough to produce negligible restoring torque when the disk is rotated. A solenoid is wound at a point on the edge of the disk. This is in the field of a permanent magnet, so that when an oscillatory electric current passes through the solenoid, the disk is set into oscillatory motion. The amplitude of the motion is measured by means of a capacitor detector that is attached to a point at the opposite edge of the disk. The capacity of this detector changes as the disk moves, and the sinusoidal changes in capacity that are produced by the oscillation of the disk are used to frequency-modulate the output of a 15 Mhz oscillator. This modulated output is then demodulated by standard frequency-modulation techniques, to give a dc voltage proportional to the distance that the disk moved during the oscillation. This type of detector is very sensitive and has the added advantage that it does not load the system appreciably.

The internal friction of the specimen can be measured by this apparatus in one of the two following ways. One is to set the system into oscillation at its natural frequency, with the driving current switched off, the system goes into free oscillation; from observations of the amplitudes of these oscillations, the logarithmic decrement can be obtained. Alternatively, by measuring the amplitudes of the oscillations for a series of driving frequencies around the natural frequency, the half breadth of the resonance peak can be determined. The resonance method has two disadvantages. The first is that since the measurements are made over a range of frequencies and the mechanical loss is frequency dependent, the measured half breadth is an average of the values around the resonant frequency. In practice this is not of much concern, since the value of the mechanical loss changes rather little over the comparatively narrow frequency ranges covered by such measurements. The second is more serious, it is the magneto-mechanical coupling between the driving coil and the magnet. This effect tends to broaden the resonance peak, and this, in turn, leads to too high values being obtained for the half breadth of the response curve. The difficulty can be overcome