VOLUME II

THE COLLECTED SCIENTIFIC PAPERS OF
PAUL A. SAMUELSON

Edited by Joseph E. Stiglitz

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AUTHOR'S PREFACE

Over the years it has been suggested that I bring out some collected papers. As the supply of my reprints has become more and more exhausted, I have in principle become more and more impressed with the force of this argument. But I have always felt hesitant to interrupt current new research merely in order to reassemble old research. Now the matter has been taken out of my hands. The M.I.T. Press and kind colleagues have undertaken to do the job. Mr. Joseph Stiglitz, an able young graduate student, has agreed to do the minimum editing needed for such a collection.

My own preference was for as complete and unpretentious an assemblage as possible. I could not see why anyone else's research program should be interrupted by the arduous job of proofreading completely reset mathematical symbolism. As to selection of articles, the choice has been that of Mr. Stiglitz alone. A number of previously unpublished papers, such as the widely circulated RAND Memorandum on "Market Mechanisms and Maximization," have been included. My advice on selection was: When in doubt, include — of course excluding all nonscientific writing, such as periodic financial journalism for the *London Financial Times*, the *Nihon Keizai Shimbun*, and the *Washington Post*, and excluding most book reviews.

Personally, I should have preferred a simple chronological listing, so that any continuities of thought might reveal themselves. But wise friends, such as Professors Robert M. Solow and Charles P. Kindleberger, have prevailed upon me to agree to an arrangement by subject

matter. Mr. Stiglitz has alone taken the responsibility for the selection and arrangement of articles. And I think he would be the first to admit that alternative classifications might have been made with equal advantage, since topics like Welfare Economics and International Trade, and indeed all of the topics of modern economics, are so interdependent as to defy any arbitrary classification.

I must confess that I have been tempted to make an editorial change here or there. This temptation I have resisted. In several places where an outright error of substance occurred, I have added correcting paragraphs that are clearly marked to be of 1965 vintage. When tempted to omit a particular article, I was taken aback to have a colleague say, "Why I thought that one of your most interesting pieces." Only one of the items included seems to me to express some bad temper, but I decided that to exclude it would be invidious and would also deprive the reader of the opportunity to judge whether, as I have sometimes heard it said, I have mellowed over the years.

Finally, my thanks go to Joseph Stiglitz for a difficult and thankless job well done. Since I have continued to write articles at a steady pace and since he cannot expect to stay young forever, Mr. Stiglitz has wisely decided to call a halt at the arbitrary date of late 1964. And high time, says my wife and fairest critic.

Paul A. Samuelson

Cambridge, Massachusetts
September 1965

EDITOR'S PREFACE

These two volumes contain virtually all of Professor Paul A. Samuelson's contributions to economic theory through mid-1964. These articles have been collected from the economic journals, *Festschrifts*, and several books on current economic problems. A few of the articles were unpublished RAND Memoranda, and others were lectures.

The arrangement by topics has not been easy; some of the articles, or chapters, properly belong in several sections; some might be put most properly in sections of their own. A few of the final decisions had to be made somewhat arbitrarily. The parts are arranged into books of closely related subjects. Within the parts, the articles are arranged chronologically, except where several articles were very closely tied together. For instance, the 1963 article on the "Gains from International Trade Once Again" immediately follows the 1939 article "The Gains from International Trade." I hope that this arrangement will make the book more useful to the reader than a strictly chronological ordering. For those who prefer the latter, however, I have included in the acknowledgments a chronological list of the articles.

Although no major changes in text have been made, a number of minor corrections have been made. After several of the articles, a 1965 postscript has been inserted in which Professor Samuelson states his present position on these topics.

On some of the topics discussed, a more recent exposition will be

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found in *The Foundations of Economic Analysis* (Cambridge: Harvard University Press, 1948) by Professor Samuelson, and *Linear Programming and Economic Analysis* (New York: McGraw-Hill, 1958) by Robert Dorfman, Paul A. Samuelson, and Robert M. Solow.

Cambridge, Massachusetts
July 1965

JOSEPH E. STIGLITZ

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WELFARE ECONOMICS AND INTERNATIONAL TRADE

International trade theory was developed by practical men interested in normative, welfare problems. By making rigorously abstract assumptions, we may consider trade between two individuals instead of between countries. For each individual the technical conditions of production can be expressed in terms of a family of substitution curves. As between individuals three types of movements are distinguished: (1) both individuals get more of every commodity with less of every productive service; (2) each individual moves higher on his preference scale, even though less of particular commodities may be received; (3) one individual moves to a higher position as the other moves to a lower. The first two are clearly beneficial to both parties. About the third nothing can be said in the absence of special and complete welfare judgments. It is demonstrable that free trade (pure competition) leads to an equilibrium in which each country is better off than in the absence of trade, and that no movements of the first or second kinds are possible. Nevertheless, this does not prove that each country is better off than under any other kind of trade; indeed, if all others are free trading, it always pays a single country not to trade freely.

Historically the development of economic theory owes much to the theory of international trade. Precisely because the classical theory of international trade arose in the thought of "practical" men, interested as *citizens* in problems of public policy, the normative and welfare aspects of the subject have received considerable attention. This is clearly seen in the agitation for and against free trade.

Since welfare economics still constitutes a vexing problem in the pure theory of value and distribution, it would perhaps be useful to examine some normative aspects of the conventional theory of international trade in order to determine the extent to which and the senses in which the conclusions reached in that field are valid.

At the outset, it is understood of course that the very discussion of welfare economics implies certain ethical assumptions. I do not propose, however, to discuss the philosophical grounds for holding or rejecting different ethical precepts or assumptions. Rather will the discussion be confined to the implications of different ethical assumptions and the necessary and sufficient presuppositions for the truth of various theorems.[1]

Since the real world presents almost infinite complexity, it is always necessary in matters of this sort to resort to ideally strong cases to bring out the essential theoretical issues. For this purpose I shall consider not trade between national countries as actually existing in the modern world, but rather have recourse to an analogous situation of trade and barter between two or more individuals. This may be regarded as trade between countries with perfect social solidarity and consensus (e.g., totalitarian states); or between units each consisting of identical (representative) individuals; or better still, merely as trade between individuals which illuminates the

[1] To indicate the "objectivity" of our discussion, it may be remarked that the whole argument will be deductive, consisting essentially of *propositional functions* at the same level of thought as a maximum problem in the theory of calculus.

processes of international trade. In this way the problem of weighing and combining different individuals' advantages within each country is avoided. Our license for employing this convention is amply provided by innumerable examples in the classical theory of international trade.

From the consideration of the problem of bilateral monopoly it is hoped that much light will be thrown on the welfare problems of international trade, and that at least one important misconception in the conventional theory will be revealed as such.

I

Consider first a self-sufficient individual (country) engaging in no outside trade. We take as given all technological relations, i.e., production functions; tastes of the individual in an ordinal—as opposed to a cardinal—sense. For generality, we include in the individual's scale of preferences the amounts of the various kinds of productive services (inputs) rendered.

Under these assumptions it is possible to reduce our technological relations to the following implicit form:

$$\varphi\,(x,\ y,\ a,\ b) = 0$$

where x and y are the amounts of commodities produced per unit time, and a and b are amounts of productive services rendered per unit time. This relationship is to be interpreted as follows: for any pre-assigned amounts of y, a and b, there is a maximum amount of x which can be produced; x, a and b being held constant, a similar argument holds for y. Furthermore, for given amounts of x, y, and b, there is a minimum amount of a necessary; and likewise for b.

If we regard the amounts of a and b as fixed, the resulting relationship between x and y is the familiar *substitution* or *production indifference curve*. Contrary to the usual exposition, this curve is not a technological datum. Its derivation is essentially an economic problem and imposes certain equalities on the marginal physical productivities of non-specific factors. We shall take its derivation as having been performed and shall make the usual assumptions as to its shape, deducible from the law of variability of proportions (see Figure I). Be it noted that there is not one substitution curve, but one curve for any pre-assigned pair of values for a and b.

Taking account of this implicit relationship between our variables, the individual acting in isolation selects that combination of variables, consistent with the above relationship, which is most preferable to him, or which maximizes any index of his utility. This imposes as a condition of equilibrium the equivalence of the ratio of marginal utilities (rate of consumer substitution or indifference) to the slope of the production indifference curve, drawn up as of the optimum values of a and b. The optimum values of a and b are determined at the point where the derived utility of

them, i.e., utility of their marginal physical product, is equated to their marginal disutility.[2]

It will be seen from the above that the doctrine of opportunity cost, properly stated, in no way contradicts the so-called pain-cost theory of value. In fact, when stated with full qualifications, the doctrine of opportunity cost inevitably degenerates into the conditions of general equilibrium.
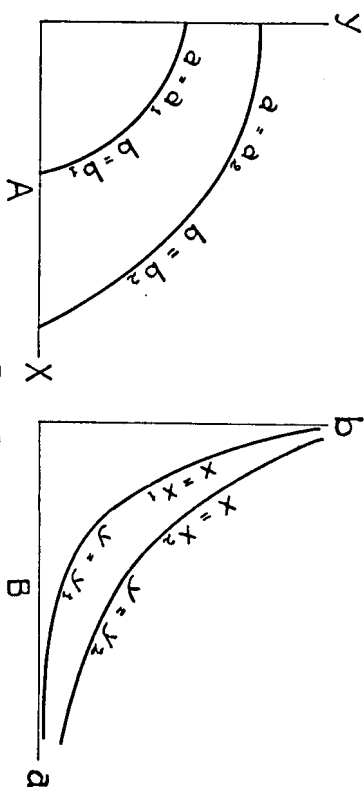
FIGURE I

It may be well to point out that the maximum conditions stated above could have been achieved by means of a system of pricing. As a matter of bookkeeping the individual might employ the fiction of setting provisional prices upon all commodities and factors of production. After a series of successive approximations, the equilibrium set of prices achieved would be such as to satisfy the various marginal conditions. Undoubtedly it is this "parametric" behavior of prices, which does yield a maximum position in a Robinson Crusoe economy, which accounts for the popularity of perfect competition with many orthodox economists, and the identification of this condition as an optimum one. That this involves the fallacy of composition will be demonstrated later.

One final point deserves emphasis. Since there exists only a scale of preference in an ordinal sense, it is impossible from the very nature of our original assumptions to evaluate numerically the magnitude of the amount of gain to an individual in moving from one position to another.[3]

[2] Although use is made of utility and disutility, only ratios of marginal utility and disutility are involved, and so all conditions could just as well have been expressed in terms of indifference directions, since the cardinal magnitude of utility is not in question.

[3] The attempt to measure gain by consumer's surplus is valid only under much more restrictive Marshallian assumptions of the existence of independent utilities with constant marginal utility of money. The use of index numbers can really only serve in this connection to determine under favorable circumstances the direction of the change—whether more or less preferred. This is not intended to be in contradiction to the use of an index number as a measure of the cost of living in different situations.

Thus, the attempt to compare the relative gains of different individuals is ruled out, entirely apart from the possibility of making inter-individual comparisons.

II

Hitherto, we have been dealing with an individual in isolation. Turning now to the consideration of two or more individuals engaged in some form of trade, the essential difficulties of the problem arise.

For simplicity let us consider only two individuals under the conditions elaborated above. It is necessary to distinguish sharply among at least three different kinds of movements which might take place as the result of the opening up of trade or of a change in the form which trade takes. (1) Both individuals may get more of every commodity while performing less of every productive service. Such a movement from almost any ethical standpoint must be regarded as an improvement and as a desirable one. (2) Each individual moves to a position higher up on his preference scale, although some of one commodity may be foregone in order to receive a more than compensating amount of another. Similarly there may be changes in the amounts of productive services rendered which are more than compensated for. Such a movement also would probably be considered desirable, although considerable knowledge of the scale of preference of each individual would be needed in order to identify such a movement. (3) One individual is moved to a lower position while the position of the other is improved. In the absence of additional ethical assumptions of a very complete kind, it would be impossible to determine whether or not such a movement would be desirable. To say that the marginal social utility of each good should be equal for every individual is to leave entirely unanswered the question as to how such a magnitude is defined. Here no attempt will be made to inquire into the various possibilities in this field.

In the case of the individual's acting in isolation, the maximum position achieved was such that a movement in any direction led to a less preferred situation, just as any movement from the top of a hill must be in a downward direction. It is precisely because of the usual assumptions of continuity that the mathematical characterization of a maximum position by certain equalities of differential coefficients (equivalences at the margin) is able to express certain inequalities for finite movements.

It would be tempting in the case of trade between two individuals to define as the equilibrium position any situation from which there is not possible any movement of the first two kinds. Clearly any tentative position reached from which both could move and be improved could not be regarded as an optimum position. However, it will be shown that there does not exist one point—or even a finite set of points—which forms the solution of these conditions. That is to say, there exists an infinite set

of conceivable situations such that no movements are possible which better both parties.

This may be illustrated by the extremely simple case of barter between two individuals, each endowed with initial amounts of the various commodities.

In this case it is well known that by mutual agreement each party will finally land somewhere on the Edgeworthian contract curve. This is the locus of points (note, not a single point!) at which the ratio of the marginal utilities of the various goods are equal for both individuals. From such a locus there is no possible movement which does not injure one party. Moreover, from any point not on the contract curve, there always exist possible movements of the first two kinds. The ruling out of such movements does not serve, therefore, to pick out an optimal point of equilibrium, but rather narrows down the possibilities to a locus of points, still infinite in number. A movement along the contract curve is necessarily of the third kind concerning the desirability of which the economist, as such, has nothing to say.

Consider then the equilibrium which will result when both individuals behave like competitors, i.e., each considers prices as given, but both together determine the prices at that level which will equate the amounts demanded and supplied of all commodities. This equilibrium is represented by the intersection of the familiar Marshallian offer curves.

Two things are obviously true in this case. First, in the final equilibrium established, each individual will be better off *than in the absence of any trade at all*. Second, the equilibrium point will lie somewhere on the contract curve, since the ratios of marginal utilities of all goods are equal to the ratios of the common prices, and hence equal to each other.

Thus, and this is the crux of the argument, under free trade both parties are better off than under no trade at all, but are not necessarily in *the* optimum position. There is absolutely no presumption whatsoever that this equilibrium point is superior in any sense to any other point on the contract curve; for the movement between any two such points is of the third kind, about which nothing can be said.

The very fact that any trade takes place is an indication that both individuals are better off, since each can at the very worst refuse to trade. Economists have proved this at great length in many ways under the mistaken impression that they were at the same time proving the desirability of free trade.

The free trade equilibrium point very obviously is not the most preferred point to any one country. Its maximum would occur when the other country consumes nothing, and it consumes all. Obviously the other country would not consent to this, since it need not trade at all. Under favorable circumstances of higgling, one country might be forced to a point on the contract

curve at which it received an infinitesimal gain from trade, and vice versa. There is absolutely no ground for saying, or no sense in stating, that the free trade point is the point of fair compromise, since only movements of the third kind are involved. In fact, one country behaving like a competitor, it can be shown that it is always to the advantage of the other not to so behave, but rather to take account monopolistically of its own effect on price.[4]

A very similar argument holds *a fortiori* in the more general case where output in each country is variable. It could be easily shown that there exists a production locus, analogous to the contract curve, along which certain ratios of equivalence of marginal productivities hold, and from which all movements are of a kind to decrease the total potential productivities of all goods in both countries. But, as before, this is a locus and not a point. The demonstration that under free trade this locus is attained, in no way establishes a presumption in favor of the point so reached.

III

Nothing said here is in fundamental contradiction to the orthodox theory of comparative costs in international trade. But if the thesis here presented be accepted as valid, it should serve as a warning against a possible misinterpretation of the classical theory.

Furthermore, this thesis must not be construed as being necessarily contrary to the political question of free (or freer) trade. It may well be argued that modern tariff and quota restrictions are of the sort that their abolition would in many cases result in the betterment of all parties concerned. But, as a matter of scientific integrity, it would seem desirable to clarify economic theory on these issues.

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[4] The monopolistic country will move the other along its offer curve up to a point of tangency of that locus with the monopolist's indifference curve.

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THE GAINS FROM INTERNATIONAL TRADE

[1] In a recent paper[1] the thesis was advanced that while it is not possible to demonstrate rigorously that *free* trade is better (in some sense) for a country than *all* other kinds of trade, it nevertheless can be shown conclusively that (in a sense to be defined later) free trade or some trade is to be preferred to *no* trade at all. I should like here to amplify these remarks with respect to the last point, that some trade is better than no trade.

This is by no means a novel proposition. Indeed, it can be traced back to the beginnings of the Classical theory of international trade. It has become associated, however, quite unnecessarily in my opinion, with a labour theory of value, or a "real cost" theory of value, or more recently, with an opportunity cost theory of value. All of these have come in for considerable criticism in recent years as restrictive special cases of the so-called theory of general equilibrium. Those writers who have insisted on the need for a modern theory of value for a positive description of behaviour in international trade have in general ignored the normative aspects of international trade, presumably in the belief that as soon as one gives up the inadmissible special theories indicated above, nothing can be said concerning this problem.[2] It will be argued here that this is a mistake, that from the most general theories of equilibrium all valid normative propositions can be derived.

[2] It is well to indicate clearly the assumptions under which our analysis is to take place. We shall consider a single economy consisting of one or more individuals enjoying a certain unchanging amount of technological knowledge, so that we may take as data the production functions relating the output of each commodity to the amounts of inputs devoted to its production. Any number of commodities is assumed; there may also be any number of inputs or productive services. These are not necessarily fixed in amount, but may have supply functions in terms of various economic prices. Moreover, for our purposes the differentiation of the factors of production can proceed to any degree; thus, labour services of the same man in different occupations are not regarded as the same factor of production unless the provider of these services is indifferent as between these two uses. Similarly, in order that the productive services rendered by different individuals may be consid-

[1] P. A. Samuelson, "Welfare Economics and International Trade" (*American Economic Review*, June, 1938).
[2] A recent exception is provided by P. T. Ellsworth's *International Economics* (New York, 1938). However, the problem is posed, not settled. Professor Haberler in his *The Theory of International Trade* (London, 1936) does not employ a full general equilibrium approach.

ered the same service, it is necessary that in every use they be infinitely substitutable.

In order to ensure that perfect competition is possible, we rule out increasing returns, and assume that all production functions show constant returns with respect to proportional changes of *all* factors. Each individual acts as if he were a small part of the markets which he faces and takes prices as given parameters which he cannot influence by changes in his own supplies or demands. It is assumed that for each individual there exists an *ordinal* preference scale in which enter all commodities and productive services, and that subject to the restraints of fixed prices he always selects optimal amounts of each and every commodity and every productive service (some zero in amount). Each individual is better off if he receives more of every commodity while rendering less of every productive service. No attempt is made to render the "utilities" and "disutilities" of different persons comparable.

[3] Under these conditions, for any assumed set of prices there will correspond definite demand and supply reactions on the part of every individual. Moreover, the total outputs of each commodity will be determined, and the total amounts of productive factors necessary to produce these outputs will be determined. If the economy is isolated, it will be necessary as conditions of equilibrium that prices of commodities and factors of production be such as to equalize the amounts produced and consumed of each and every commodity, and to equalize the amounts supplied and demanded of every productive factor.

Under assumed conditions of ownership of the factors of production and assumed scales of preference for commodities and productive services on the part of every individual, there will result in general (waiving possible multiplicities of equilibrium raising problems not peculiar to international trade) unique equilibrium quantities of consumption goods and productive services for each and every individual. It is unnecessary to write down mathematically these equations to deduce the familiar fact that not enough has been assumed to be able to deduce the absolute level of commodity and factor prices, but that these are determined except for a factor of proportionality; i.e., relative commodity and factor prices are determined. Let us write as follows the equilibrium set of prices, determined to within a factor of proportionality, which will be established for our economy when isolated,

$$p_1{}^0, p_2{}^0, \ldots, p_n{}^0, w_1{}^0, w_2{}^0, \ldots, w_s{}^0,$$

with corresponding equilibrium total quantities of the respective commodities and productive services,

$$x_1{}^0, x_2{}^0, \ldots, x_n{}^0, a_1{}^0, a_2{}^0, \ldots, a_s{}^0.$$

The total amounts produced of the respective commodities will be indicated by the barred letters,

$$\bar{x}_1{}^0, \bar{x}_2{}^0, \ldots, \bar{x}_n{}^0,$$

equal respectively in the isolated state to the quantities (unbarred) consumed.

[4] Before introducing possibilities for trade into our system, it will be useful in view of the later discussion first to develop certain relations which must hold in the field of *production*. Confronted with given factor prices, firms will combine factors of production in such proportions as to produce any selected quantity of consumers' goods at the lowest total money cost. In consequence of this, certain marginal conditions of equality will be attained (or at least certain inequalities with respect to finite movements). Although the proof is not given here,[3] it can be shown that this places restrictions on the possible combinations of factors of production and commodities which can occur. Indeed, it will be found that the totals of commodities produced and the totals of pro- ductive services must obey an implicit equation of the following form:

$$\phi[\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n, a_1, a_2, \ldots, a_s] = 0. \qquad [1]$$

This is capable of the following interpretation: for preassigned values of all productive services and all but one commodity, this equation gives the *maximum* amount of the remaining commodity which can be pro- duced with the given state of technology. Moreover, with preassigned amounts of all commodities and all but one productive service, this shows the *minimum* amount of this one productive service which is necessary.

Utilizing the well-known law of variable proportions, the following remarkable theorem can be established. Consider any set of commodity and factor prices,

$$p_1', p_2', \ldots, p_n', w_1', w_2', \ldots, w_s'.$$

Since each entrepreneur is trying to maximize his profits, there will result an optimal set (not unique) of commodities produced and productive services used, indicated by

$$\bar{x}_1', \bar{x}_2', \ldots, \bar{x}_n', a_1', a_2', \ldots, a_s',$$

satisfying, of course, equation [1]. Our theorem says that *for such preassigned prices the resulting optimal quantities of commodities and productive services maximise for the economy as a whole the algebraic differ- ence between total value of output and total factor cost, as compared to any other commodity and factor combinations satisfying equation* [1]. This is equivalent to the following inequality:

[3] In a forthcoming paper on the conditions of equilibrium in international trade I have gone more fully into these and other matters.

$$[p_1' \bar{x}_1' + p_2' \bar{x}_2' + \ldots + p_n' \bar{x}_n'] - [w_1' a_1' + w_2' a_2' + \ldots + w_s' a_s'] \geq$$
$$[p_1 \bar{x}_1 + p_2 \bar{x}_2 + \ldots + p_n \bar{x}_n] - [w_1' a_1 + w_2' a_2 + \ldots + w_s' a_s], \quad [2]$$

where the unprimed x's and a's represent *any* point satisfying equation [1]. This inequality merely places certain curvature restrictions on the surface represented by equation [1], for the various ratios between respective prices correspond in a well-known manner to the respective slopes (when prices exist) of this surface.[4] In figure I are presented typical shapes for
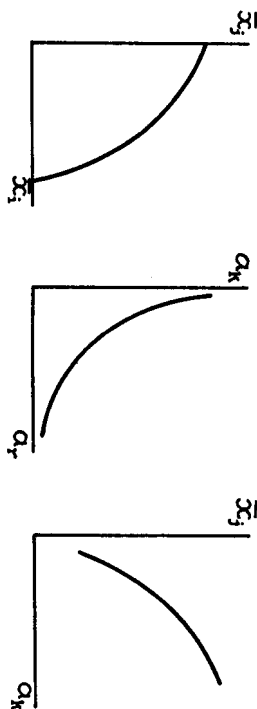

FIGURE I

various cross-sections of this surface. In the first diagram is shown the amount that must be given up of one commodity, x_i, in order to get more of another, x_j, with all other variables held constant. This substitution curve must be concave to the origin. The next diagram shows the amount of one input, a_r, which must be added to compensate for the withdrawals of a_r, all other variables being held constant. The last diagram shows the amount of commodity, x_j, that can be secured with additional amounts of a_k, with constant levels for the remaining outputs and inputs.

The above inequality can be written symbolically

$$\Sigma p' \bar{x}' - \Sigma w' a' \geq \Sigma p' \bar{x} - \Sigma w' a, \quad [2]$$

where it is always understood that the summations are over the respective n commodities and s productive services. Of course, a similar inequality holds for any other preassigned set of prices.

[5] Trade can be introduced very simply into our system without explicitly dealing with any new country or countries. This is done by the useful device of supposing that there exists an outside market in which there prevail certain arbitrarily established (relative) prices at

[4]The equality sign can hold if all the x's and a's are respectively proportional (or equal) to the primed x's and a's. In the singular (and rare) case where the preassigned factor prices are such that all factors of production are used in equal proportions by all commodities, it is possible for the equality sign to hold. This constant cost case does not essentially modify the analysis.

which this country can buy or sell various commodities in unlimited amounts without changing those quoted prices. It does not matter for the present purposes how, in fact, such prices would be established in this outside market or source, but rather we are interested in the effects upon this country of the existence of such quoted prices.

The fact that this outside market will both buy and sell at the new quoted prices will compel the prices of respective goods in the domestic economy to assume equivalent ratios, or else corrective arbitrage movements would take place. Obviously, therefore, we have introduced new forces to determine some of the prices. It is necessary, then, to drop some of our previous conditions. In particular, we must dispense with the condition that the amounts of commodities produced domestically and consumed domestically must be equal. Instead we have the single condition that the total value of imports must equal the total value of exports, or

$$\Sigma p x = \Sigma p \bar{x}. \quad [3]$$

It is clear that for any preassigned prices of internationally tradable goods there will result certain equilibrium values for all the other variables, quantities produced and consumed, productive services supplied, and prices of non-tradable commodities and services.

For one set of prices, namely those proportional to the set $[p_1^0, p_2^0, \ldots, p_n^0]$ which would prevail if the economy were isolated, no trade will result.[5] For these particular prices are such as to equalize the domestic production and consumption of each and every good. For any other set of prices, some trade will result, and there will emerge new equilibrium values for all of our unknowns. By assigning appropriate values to our outside prices, we can obviously reproduce all possible conditions of trade which could conceivably arise. This is the justification for introducing a simplifying device which enables us to ignore the existence of outside economies. Of course, if we were trying to explain the actual prices with which our economy will be confronted, it would be necessary to consider outside conditions.

[6] I first apply our analysis of the effect of introducing relative prices, different from those which would be established if our system were isolated, to a simplified case in which all members of our economy are identical in every respect. That is, the same ordinal preference schedule relating commodities and productive services is assumed for every individual, and also the same ownership in the means of production.

[5]A trivial exception is provided by the constant cost case mentioned in the previous foot-note. Here at the isolated state prices there might be an unimportant possibility of neutral equilibrium as recognized in the Classical theory of international trade. I adopt the convention of defining trade to exclude this possibility.

This does not mean that the utilities of different individuals are comparable. Indeed, since all individuals are identical, if one is bettered (in an ordinal sense) by the introduction of trade, then all will be bettered, and there will be no necessity for making any welfare comparisons between individuals.

In these circumstances, the following theorem can be established: *the introduction of outside (relative) prices differing from those which would be established in our economy in isolation will result in some trade, and as a result every individual will be better off than he would be at the prices which prevailed in the isolated state.* The truth of this has been intuitively apprehended by a great many economists, but I do not believe that there exists anywhere in the literature a rigorous proof of this proposition.

To illustrate the difficulties which must be encountered in establishing this theorem I present a table showing some possible results of the introduction of trade. In the first two columns are respectively the prices and quantities consumed of three commodities; in the next two columns, the prices and quantities produced of the same three goods; in the last two columns, the prices and quantities of two factors of production.

Case I gives a hypothetical set of prices which would prevail in the isolated state with equal production and consumption of all commodities. The amounts corresponding to each individual would be some constant fraction of the total quantities. Although actual prices are given to avoid the asymmetry of using any one good or service as *numéraire*, only relative prices are of importance.

If a new set of relative prices are imposed from without, new equilibrium values will be appropriate. Cases II, III, and IV indicate possible sets of equilibrium values which might emerge, depending on the particular make-up of tastes of the individuals in question.[6] In Case II after trade is established, it will be noted that more of every commodity is consumed, while less of every productive service is provided. Obviously, Case II is an instance of our theorem. But what can be said of Case III? Here, the same amounts of all productive services are provided, but not more of every commodity is consumed. More of commodities x_2 and x_3 will be consumed, but less of commodity x_1. In Case IV things appear to be still worse. Not only does the quantity of some commodity decrease, but also more of the productive service a_2 is provided. Is it possible to say in the general case that the new situation is better than the old, or is our theorem false?

It is obvious that a labour theory of value cannot be of any aid in the

[6]Cases II, III, and IV are alternative and mutually exclusive possibilities. Hence, although each is consistent with Case I, they are not necessarily consistent with each other.

analysis of this problem, since two factors of production have been assumed. The opportunity cost doctrine as presented by Professor Haberler could be applied only to Case III, where the total amounts of the various factors of production remain unchanged after trade has taken place. Contemplation of the behaviour of the terms of trade would suggest that an improvement has taken place, but it would be easy to construct examples for which this test would give a spurious result. None of the usual methods throws any light on the question as to whether Case IV represents an improvement over the condition which would

TABLE I

	p	x	p	x	w	a
Case I—no trade.......	1	10	1	10	4	5
	2	15	2	15	2	20
	1	20	1	20		
Case II.........	3	11	3	20	9	4
	2	17	2	15	3	18
	1	23	1	0		
Case III.........	3	8	3	20	6	5
	2	17	2	15	3	20
	1	32	1	0		
Case IV.........	3	8	3	20	6	4
	2	17	2	15	3	22
	1	32	1	0		

prevail in the absence of trade. And yet there can be no doubt that the situation represented in Case IV is the typical case when trade occurs. If we assume that in the real world there are innumerable commodities and productive services, it is scarcely conceivable that after trade takes place more of each and every commodity and less of each and every productive service will result. The introduction of trade would be expected to result in less of one or more commodities and more of one or more productive services.

Still, if the theorem given above is valid, it must follow that we can very definitely show that all the given cases in which trade takes place are better than the original situation illustrated by Case I. It remains only, therefore, to prove our theorem, after which all the illustrative examples will emerge as special instances. It will be noted that the proof to be given depends only on the elementary operations of arithmetic: addition, subtraction, equality, inequality, etc.

To ensure generality, consider any initial set of prices prevailing in the isolated state,

$$p_1^0, p_2^0, \ldots, p_n^0,$$

and the corresponding equilibrium values of the remaining variables,

$$x_1^0, x_2^0, \ldots, x_n^0, \bar{x}_1^0, \bar{x}_2^0, \ldots, \bar{x}_n^0, a_1^0, a_2^0, \ldots, a_s^0, w_1^0, w_2^0, \ldots, w_s^0.$$

Now consider any new set of prices leading to trade,

$$p_1', p_2', \ldots, p_n',$$

and the corresponding new equilibrium values

$$x_1', x_2', \ldots, x_n', \bar{x}_1', \bar{x}_2', \ldots, \bar{x}_n', a_1', a_2', \ldots, a_s', w_1', w_2', \ldots, w_s'.$$

From the production inequality of equation [2] we know that

$$\Sigma p' \bar{x}' - \Sigma w' a' \geq \Sigma p' \bar{x}^0 - \Sigma w' a^0. \quad [4]$$

But from the condition that the total value of imports must equal exports, or that the total value of goods produced must equal the total value of goods consumed, a similar inequality will hold if we leave the bars off the x's and consider goods consumed instead of goods produced. This gives

$$\Sigma p' x' - \Sigma w' a' \geq \Sigma p' x^0 - \Sigma w' a^0. \quad [5]$$

I now assert that this condition (barring the unimportant case of equality sign mentioned in foot-note 4) assures us that each of our identical individuals is better off in the second case than in the first.

Imagine an individual confronted with commodity and productive service prices $[p_1', p_2', \ldots, p_n', w_1', w_2', \ldots, w_s']$. Subject to these prices, his most preferred position with respect to consumption and the providing of services is shown by his behaviour to be $[x_1', x_2', \ldots, x_n', a_1', a_2', \ldots, a_s']$. By considerations similar to the economic theory of index numbers as developed by Pigou, Haberler, Konüs, Staehle, Leontief, Frisch, et al., it can be shown that this combination is preferred in an ordinal sense to $[x_1^0, x_2^0, \ldots, x_n^0, a_1^0, a_2^0, \ldots, a_s^0]$.[7] If at the primed set of prices the individual would have bought the original combination of goods $[X^0]$, and provided the original amounts of productive services $[A^0]$, the total algebraic cost would have been less than that of what he actually bought and sold $[X', A']$. In addition, therefore, something more could have been bought of every commodity, and a little less of every productive service supplied. This proves that $[X', A']$ is better than $[X^0, A^0]$, for if this were not so, why did not the individual actually choose $[X^0, A^0]$, and perhaps a little more of every good and a little less of every service, in preference to $[X', A']$? If the individual was in a true maximum position at the primed prices, it must necessarily follow from

7For many reasons I regard the index number approach as a clumsy device for solving the problem at hand. A more convenient test as to the ordinal desirability of two situations is presented in my "Note on the Pure Theory of Consumers' Behavior" (*Economica*, March, 1938).

our inequality that $[X', A']$ is better than $[X^0, A^0]$. Thus, our theorem is proved.

To appreciate the true meaning of this theorem and its proof, the reader may make the experiment of dropping one or more of our premises to show how the proof will break down. Such an exercise is provided by the well-known Graham's Paradox.

No modification in the proof is required by the assumption that there exist domestic consumers' goods which cannot be traded under any circumstances. With slight modification transportation costs could be introduced into the analysis without affecting appreciably the results. It will be noted that the proof is still valid in the case where there exist no resources transferable between different production uses. Indeed, if the commodities are not produced at all, but fall from heaven in fixed amounts per unit time, the theorem still applies. Moreover, the introduction of discontinuities requiring modifications of the usual marginal analysis is already covered in our theorem.

[7] If, as I have shown, the introduction of outside prices different from those which would prevail in the isolated state betters all of our identical individuals, a possible generalization suggests itself. Is it possible to state that the more prices "deviate" (according to some convention) from those of the isolated state, the better off all individuals will be? The answer is in the affirmative. In order not to complicate the present exposition, I withhold the rigorous proof of this proposition until a future occasion.

[8] Before going on to consider more realistic cases where individuals are not all alike, I should like to point out two interesting special cases covered by the previous theorem. The limiting case of an economy in which all individuals are exactly alike is that of a single household or Robinson Crusoe economy consisting of but one unit. Moreover, from a formal point of view a completely unified economy under perfect control of some central authorities interested in maximizing some ordinal preference scale is like a one individual economy. For such single individual economies, pretending to play the game of perfect competition is one possible way of arriving at optimal equilibrium values. If self-sufficiency is not an end in itself, it follows from our previous theorem (and even under less stringent assumptions) that for an individual or unified economy trade is always preferable to no trade, although it is not necessarily true that free trade is the best trading policy.

[9] I now drop the assumption that all individuals are alike with respect to tastes, abilities, ownership of the means of production, etc. The introduction of changed prices leading to trade cannot, of course, be expected always to better each and every individual. After trade, the

prices of items chiefly consumed by a particular individual may have risen making him worse off than before. (It is not possible, however, for every individual to be made worse off.)

In order to evaluate the resulting situation, it would be necessary to have some scale which would take into account comparisons as between different individuals. For some type of weighting of the fortunes of different individuals, the result might be judged an improvement. For some other, such as an egocentric evaluation on the part of those rendered worse off, the resulting situation might be judged to be worse than that which prevailed in the isolated state. If nothing more than this could be said, the problem of the benefits from trade would be of limited theoretical and practical importance.

Fortunately, definite results which do *not* depend upon the comparisons of the real incomes of different people can be derived. Although it cannot be shown that every individual *is* made better off by the introduction of trade, it can be shown that through trade every individual *could* be made better off (or in the limiting case, no worse off). In other words, if a unanimous decision were required in order for trade to be permitted, it would always be possible for those who desired trade to buy off those opposed to trade, with the result that all could be made better off.[8] This can be deduced from the fact that as a result of trade larger (or in the limiting case, equal) amounts of every commodity can be secured with smaller (or equal) amounts of every productive service. Without trade the range of possible commodities which are available with preassigned amounts of all productive factors is given by the implicit equation [1]. If outside prices are introduced, it will always be desirable for production policy to be aimed at maximizing the total value of output at the outside prices, with any preassigned amounts of each and every productive factor. For this will yield a larger sum of money than any other production policy, and with a larger sum of money more can be bought of every commodity than with a smaller one. As a result, each of the following three statements is true: [1] more can be had of every commodity as of the same totals of all productive services; [2] of the same preassigned quantities of all consumers' goods, less of every productive service need be rendered;[3] after trade, more of every commodity can be secured with less of every productive service. This ensures us that by Utopian co-operation everyone can be made better off as a result of trade.[9]

[8]See Professor Viner's interesting remarks in his *Studies in the Theory of International Trade* (New York, 1937), pp. 533-4.

[9]Mathematically, subject to preassigned outside prices and with preassigned quantities of all productive services, there will result optimal production quantities

I shall make no attempt to construct a numerical index of the gains of trade. In the simplest case of a single individual, only an ordinal preference scale is assumed so that only better or worse comparisons can be made. Such constructs as consumers' surplus are in general inadmissible. Even in the singular cases where they are able to be employed, they are perfectly arbitrary and conventional, adding nothing to the analysis.

[10] In conclusion, I should like to point out that in the above exposition an attempt has been made to demonstrate rigorously with little reliance on intuition the truth of the theorems advanced. Whether or not this should be done is, of course, a matter of taste. Much more important than the carrying through of the formal steps of the argument is the realization that the theorems are true consequences of the premises, and do not rest on *presumption* or *probability*. For in pointing out the consequences of a set of abstract assumptions, one need not be committed unduly as to the relation between reality and these assumptions. On the other hand, in advancing a presumption in favour of an undeducible proposition, the suggestion is conveyed that the difficult task of interpreting reality has already been performed.

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which are functions of the preassigned variables and satisfy the production limitation of equation [1]. These optimal production quantities will sell for the largest possible total in the outside market, and hence the expression $\Sigma p'x$ is maximized subject to equation [1] and fixed amounts of productive factors. The resulting money sum will be sufficient to permit consumption of goods obeying the condition that all imports must be paid for by exports, or $\Sigma p'x = \text{maximum } \Sigma p'\bar{x}$. Because production is optimal, the result is more (or equal) consumption of every good. Moreover, for sufficiently small reductions of all production services, it will still be possible to have more of every commodity, and hence the truth of the third statement follows.

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THE GAINS FROM INTERNATIONAL TRADE ONCE AGAIN[1]

I. INTRODUCTION

In 1939 I wrote a paper that showed how some international trade makes a society potentially better off than it would be if restricted to autarky.[2] Although this paper has received a flattering amount of notice, I had always regarded it as somewhat incomplete and had long planned to follow it with a more definite companion piece. For it was written with two purposes in mind other than to say all that can be said about the gains from international trade.

First, it was an attempt to show how the new theories of revealed preference could be used to demonstrate important theorems in welfare economics. And second, it was intended to mediate the dispute between two of my famous teachers, Jacob Viner (then of Chicago) and Gottfried Haberler (Harvard), over the doctrine of opportunity cost in international trade and value theory: my 1939 article was shaped to show how the eclectic doctrine of general equilibrium could take changes in factor supplies in its stride and by the index-number methods of revealed preference illustrate how the Haberlerian transformation curve could be generalised.

Even after the passage of twenty years, the final chapter seemed still to be lacking in the literature. And an interesting 1958 Danish criticism of my earlier paper's treatment of income distribution by Mr. Erling Olsen[3] led me to defend the argument and at long last take up the thorough completion. This time there was no need to worry about the obsolete doctrine of opportunity cost; nor to use index numbers of revealed preference, since for better or worse this approach had already won its place in the literature of economic theory. Good fortune, however, brought Dr. Murray Kemp to M.I.T. as a visiting professor in 1959–61 on his way from Canada to a chair at the University of New South Wales. For, in discussing the present paper, Professor Kemp showed that my alternative approach of 1939 could indeed be carried through all the way to achieve the same final goals.[4] In a real sense, therefore, our two papers are complementary and benefit from simultaneous publication.

II. THE SMALL COUNTRY CASE

On the special assumption that our country under consideration is too small to affect its terms of trade, and on the assumption that the price ratios abroad differ from those that would prevail at home under autarky, Fig. 1's heavy line *EUF* represents our "consumption possibility frontier" with
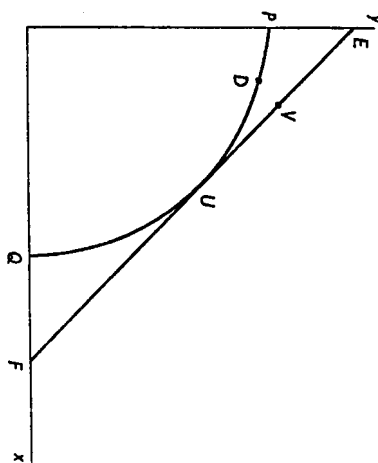

FIG. 1.—With no trade, we end up at *D*. With free trade, production ends up at *U*, consumption at *V*, with *UV* the vector of algebraic imports.

some trade. With autarky the consumption possibility frontier is given by the production locus *PDUQ*. Since the trade frontier lies everywhere[1] north-east of the autarky frontier, our society can have more of all goods (and less of all irksome inputs) with some trade. It is in this sense that trade makes us potentially better off.

III. AN IMPORTANT ENVELOPE

I wish to increase the generality of my 1939 argument by now dropping the assumption that our country is small. Let us be large enough to affect our terms of trade as we move along Fig. 2's Marshallian offer curve of the rest of the world for our two-goods.

[1] At *U* itself the frontiers coincide. Thus, if there were some distribution of income which brought us under autarky to *U* rather than *D*, opening up trade would in fact be followed by no international transactions taking place, and hence would (2) represent the limiting case where trade neither helps nor hurts us. (If individuals' tastes and endowments happen to be much alike at home there might be no redistribution of income that would, under autarky, get us to *U*. In such a case we would know that the *cum*-trade utility frontier of Fig. 4 does lie uniformly outside the autarky utility frontier. On the other hand, if *U* is a possible autarky point the *cum*-trade frontier will touch the autarky utility frontier at one or more points; but it must always lie north-east of the autarky point corresponding to *D*—as we shall see.)