

STATIC AND DYNAMIC ELECTRICITY

WILLIAM R. SMYTHE

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STATIC AND DYNAMIC ELECTRICITY

Third Edition, Revised Printing

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STATIC AND DYNAMIC ELECTRICITY:

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The late F. K. Richtmyer was Consulting Editor of the series from its inception in 1929 to his death in 1939. Lee A. DuBridge was Consulting Editor from 1939 to 1946; and G. P. Harnwell from 1947 to 1954.

TABLE OF SYMBOLS

Note: In this table bold-face symbols ($\mathbf{v}, \mathbf{u}, \boldsymbol{\phi}, \dots$) are space vectors. Phasors ($\tilde{I}, \tilde{\mathcal{E}}, \dots$), phasor space vectors ($\tilde{\mathbf{E}}, \tilde{\mathbf{B}}, \tilde{\mathbf{H}}, \dots$), conjugate phasors ($\hat{I}, \hat{\mathcal{E}}, \dots$) and conjugate phasor space vectors ($\hat{\mathbf{E}}, \hat{\mathbf{B}}, \hat{\mathbf{H}}, \dots$) are shown by an erect (\sim) or an inverted (\frown) flat vee above the symbol. Magnitudes of vectors and scalars, whether time-dependent or not, are written without designation.

$A, A_\phi, A_x, \text{etc.}$	Vector potential.
A^0	Normalized vector potential.
$A, A_x, \text{etc.}$	Quasi-vector potential
$B, B_\phi, B_x, \text{etc.}$	Magnetic induction
B	Susceptance.
B^0	Normalized or relative susceptance, $B\check{Z}_k$.
C	Capacitance. A constant.
C^0	Normalized or relative capacitance, $C\check{Z}_k$.
c	Velocity of light. A length.
c_{nn}	Self-capacitance.
c_{mn}	Mutual capacitance.
$D, D_\phi, D_x, \text{etc.}$	Electric displacement.
Dw	Dwight integral tables.
ds	Differential element along s .
dr	Differential change in r .
$E, E, \tilde{E}, \check{E}, \text{etc.}$	Electric field intensity.
$E(k)$	Complete elliptic integral.
e	Electronic charge. 2.71828.
$\mathcal{E}, \mathcal{E}, \tilde{\mathcal{E}}, \check{\mathcal{E}}, \text{etc.}$	Electromotance.
\mathcal{E}_e	Effective or rms electromotance.
F, F_x	Force
G	Conductance, $\check{Y} = G + jB$.
g	Acceleration of gravity.
$H, H, \tilde{H}, \check{H}, \text{etc.}$	Magnetic field intensity.
$H_n^{(1)}, H_n^{(1)}(v), H_n^{(2)}, H_n^{(2)}(v)$	Hankel functions.
h	Planck's constant.
h_1, h_2, h_3	In orthogonal curvilinear coordinates.
	Length elements are $h_1 du_1, h_2 du_2, h_3 du_3$.
$h_n^{(1)}, h_n^{(1)}(v), h_n^{(2)}, h_n^{(2)}(v)$	Spherical Hankel functions.

TABLE OF SYMBOLS

$I, I, \hat{I}, \hat{I}, \text{etc.}$	Electric current.
I_e	Effective or rms current.
i_e	Effective or rms current density.
$i, \hat{i}, i, i_x, \text{etc.}$	Current density. Current.
i, j, k	Unit x, y, z vectors.
$J_n, J_n(v)$	Bessel functions.
j	$(-1)^{\frac{1}{2}}$.
$j_n, j_n(v)$	Spherical Bessel functions.
K	Relative capacitivity, ϵ/ϵ_v .
$K(k)$	Complete elliptic integral.
K_m	Relative permeability, μ/μ_v .
$K_n, K_n(v)$	Modified Bessel functions.
$k_n, k_n(v)$	Modified spherical Bessel functions.
k	Boltzman constant.
L, L_{nn}, L_n	Inductance.
L_{mn}	Mutual inductance.
L^0	Normalized or relative inductance, L/\tilde{Z}_k .
l, m, n	Direction cosines with x, y, z axis.
M, M	Magnetization.
M	Mutual inductance.
$M_x, M, M, \tilde{M}, \tilde{M}, \text{etc.}$	Dipole or loop moment.
M', M'	Classical magnetic dipole moment (IX).
m	Mass. A number (usually integer).
N	Electric or magnetic flux.
n	Unit normal vector.
n	Index of refraction. A number.
$n_n, n_n(v)$	Spherical Bessel function.
$2n!!$	$2 \cdot 4 \cdot 6 \cdot \cdot \cdot 2n$
$(2n + 1)!!$	$1 \cdot 3 \cdot 5 \cdot \cdot \cdot (2n + 1)$
P	Polarization
P, \dot{P}	Power.
\bar{P}	Average power.
$P_n^m, P_n^m(\mu)$	Associated Legendre function.
Pc	Peirce integral tables.
p, p	Momentum.
p	A number. $\omega\mu\gamma$. ω .
Q	Electric charge. Quality of cavity.
Q	Quadrupole moment.
$Q_n^m, Q_n^m(\mu)$	Associated Legendre function.
q	Point or variable charge.
R, R_n, R_{nn}, R_{mn}	Resistance.
$R, R(r)$	Function of r only.
$R_n, R_n(v)$	Solution of Bessel's equation.

$R_n^0, R_n^0(v)$	Solution of Bessel's modified equation.
R, R	Distance between two points.
r, r	Distance from origin.
S	Area or surface.
S_c, S_o	Cavity cross-section areas.
S, S_n, S_n^m	Surface harmonic.
S, S_n, S_{mn}, S_{nn}	Elastance.
s_{nn}	Self-elastance.
s_{mn}	Mutual elastance.
s	Distance along curve. An integer.
T, T	Torque.
T	Absolute temperature. Period.
t	Time.
TE	Transverse electric.
TM	Transverse magnetic.
te	Subscript for TE wave quantities.
tm	Subscript for TM wave quantities.
U	Stream or potential function.
$[U]$	$\oint dU$ around constant V curve.
u, u	Velocity.
u	$\cos \theta$.
u_1, u_2, u_3	Orthogonal curvilinear coordinates.
V	Potential or steam function.
$[V]$	$\oint dV$ around constant U curve.
v	Volume.
v, v	Velocity.
$W, W, \tilde{W}, \tilde{W}$	Solutions of scalar wave equation.
W	Energy. $U + jV$.
W_{te}	Solutions yielding TE waves.
W_{tm}	Solutions yielding TM waves.
X	Reactance.
x, y, z	Rectangular coordinates.
$Y_n, Y_n(v)$	Bessel functions.
Y, \tilde{Y}	Admittance, $G + jB$.
\tilde{Y}^0	Normalized or relative admittance, \tilde{Y}/\tilde{Z}_k .
Z, \tilde{Z}	Hertz vector.
$\tilde{Z}, Z, Z, Z_{nn}, Z_n$	Impedance.
\tilde{Z}_k	Characteristic impedance.
\tilde{Z}^0	Normalized or relative impedance, \tilde{Z}/\tilde{Z}_k .
$Z, Z(z)$	Function of z only.
z	Complex variable, $x + jy$.
$\alpha, \beta, \gamma, \delta, \epsilon, \theta, \phi, \chi, \psi$	Often used for angles.

TABLE OF SYMBOLS

β	Ratio v/c . Ratio $(\mu - \mu_v)/(\mu + \mu_v)$.
β	Free space wave number, $\omega(\mu\epsilon)^{\frac{1}{2}}$.
β'_{mn}	Wave-guide wave number, $(\beta^2 - \beta_{mn}^2)^{\frac{1}{2}}$.
β_{mn}	Cutoff wave number.
β_{mnp}	Cavity resonance wave number.
$\Gamma, \tilde{\Gamma}$	Phasor propagation constant.
γ	Electrical conductivity.
Δ, Δ_{rs}	Determinant. A small part of.
δ	Skin depth. Phase difference.
δ	A small quantity. A small part of.
δ_n^m	Kronecker delta, zero if $m \neq n$, one if $m = n$.
ϵ	Capacitivity. A small quantity.
ϵ_v	Free space capacitvity.
ϵ, ϵ_n	Phase angle.
η	Intrinsic impedance.
$\Theta, \Theta(\theta)$	Function of θ only.
θ	Colatitude angle.
$\hat{\theta}$	Unit vector in θ direction.
$\theta, \theta', \theta''$	Angles of incidence, reflection, and refraction.
κ	Magnetic susceptibility. $(1 - \beta^2)^{-\frac{1}{2}}$.
λ	Wave length.
λ_{mn}	Cutoff wave length.
λ_g	Wave-guide wave length.
λ_{mnp}	Cavity resonance wave length.
μ	Permeability. $\cos \theta$.
μ_v	Free space permeability, $4\pi \times 10^{-7}$.
ν	Frequency in cycles per second.
ν_{mn}	Cutoff frequency.
ν_{mnp}	Cavity resonance frequency.
$\Xi, \Xi(\xi)$	Function of ξ only.
ξ, ζ, ϕ	Oblate spheroidal coordinates.
ξ, η, ϕ	Prolate spheroidal coordinates.
$\Pi, \tilde{\Pi}, \Pi$	Poynting vector.
$\bar{\Pi}$	Rms Poynting vector.
ρ	Distance from z - or θ -axis. Charge density.
ρ_1	Unit vector in ρ -direction.
σ	Surface electric charge density.
s	Area or surface resistivity.
τ	Volume resistivity. Density. Time.
Φ	Function of ϕ only.
$\hat{\phi}$	Unit vector in ϕ -direction.
ϕ	Longitude angle. Phase angle.
Ψ	Scalar potential.

TABLE OF SYMBOLS

xv

Ω	Magnetomotance. Solid angle.
ω	Frequency in radians per second.
∇	Vector operator, $i\partial/\partial x + j\partial/\partial y + k\partial/\partial z$.
∇_2	Two-dimensional vector operator.
$\mathbf{a} \times \mathbf{b}$	Vector product of \mathbf{a} and \mathbf{b} .
$\mathbf{a} \cdot \mathbf{b}$	Scalar product of \mathbf{a} and \mathbf{b} .
∇^2	Laplace operator.
$[v]$	Retarded v .

PREFACE TO THE THIRD EDITION, REVISED PRINTING

It is a source of great satisfaction to my father on his 95th birthday that a revision of this book is being published. In the past 50 years it has been used as a textbook by countless students who have, sometimes painfully, learned the value of a rigorous problem course. Others have found it valuable as a reference that contains solutions to hundreds of difficult electromagnetic problems with widespread applicability.

WILLIAM RODMAN SMYTHE

PREFACE TO THE THIRD EDITION

The arrival of the digital computer since the appearance of the second edition has considerably altered the electromagnetic computation problem and made methods feasible that were probably discarded years ago as impractical. These devices iterate so efficiently that the summation of a simple, slowly convergent series is often faster than closed-form evaluations requiring interpolation in an inadequate function table. Solution of linear simultaneous algebraic equations is now easy and very useful for obtaining coefficients in the series solutions of mixed boundary and mixed coordinate problems such as those added to the text and problems of Chap. V. Many of these appear here for the first time.

About 25 useful elliptic function conformal transformations are added to text and problems of Chap. IV as well as minor alterations and additions to Chaps. VI, VII, and XI (former Chap. XIII). Old Chaps. IX and X have been deleted because for about 10 years all physics and electrical engineering graduate students at the California Institute of Technology, and probably elsewhere, have seemed to be already familiar with linear circuits. The coverage of eddy currents in Chap. X (former Chap. XI) is greatly extended. Little or none of this added material appears elsewhere. Chapter XII (former Chap. XIV) on wave guides and cavities is entirely rewritten, with the addition of new matter and use of the variational method. The treatment of radiation from accelerated particles in Chap. XIII (former Chap. XV) has been expanded.

The new material herein, unlike that in former editions, is unchecked by students. As much as possible of the almost error-free last edition has been left undisturbed. The author has checked the new material as well as he can but knows from past experience that some errors may survive. It is hoped that those finding them will inform him so that they may be corrected in future printings.

WILLIAM R. SMYTHE

PREFACE TO THE SECOND EDITION

The wide use of rationalized mks units and the increased importance of microwaves made this radical revision of the first edition imperative. The units are changed throughout. The resultant extensive resetting of the text permits a modernization of nomenclature through such changes as "capacitor" for "condenser" and "electromotance" for "electromotive force." The original wording has been preserved only in the Cambridge problems. In static-field chapters, 40 problems of above-average difficulty have been added, usually covering boundary conditions omitted in the first edition. The expanded treatment of electromagnetic waves made necessary the rewriting of the parts of Chap. V dealing with Bessel functions and led to the introduction of vector surface harmonics, which greatly simplify some calculations. Much of Chap. XI on eddy currents has been rewritten, and two of the three electromagnetic-wave chapters are entirely new. Both the text and the 150 problems include methods and results not found in the literature. Two groups of advanced Ph.D. students worked over this material to get practice in attacking every type of wave-field problem. Many are too difficult for first-year graduate students, but every problem was solved by at least one of the advanced students. They can be worked either directly from the text or by fairly obvious extensions of it. Some useful results appear in the problems and are listed in the Index, which should be consulted by engineers with boundary value problems to solve. Chapter XV of the first edition is omitted because none of the remaining theory is based on it and because to bring it up to date would require an excessive amount of space.

None of the new topics appears to lie outside the scope of the mathematical preparation assumed for readers of the first edition. That the successful solution of electrical problems depends on physical rather than mathematical insight is borne out by the author's experience with the first edition, which shows that graduate students in electrical engineering and physics greatly excel those in mathematics.

It is believed that very few of the errors and obscure or ambiguous statements in the first edition escaped the scrutiny of the 375 students at the California Institute of Technology who worked it through. No infallible system for locating errors caused by the transposition of units has been found, and the author will appreciate letters from readers pointing them out.

WILLIAM R. SMYTHE

July, 1950

PREFACE TO THE FIRST EDITION

It has been found that, in most cases, the average graduate student, even though he seems to be thoroughly familiar with advanced electrical theory, is unable to solve the electrical problems he encounters in research if they fall outside the routine types and so must be worked out from first principles. The present book is the result of having taught, for the last twelve years, a course designed to train first-year graduate students in physics, electrical engineering, geophysics, and mathematics to apply the principles of electricity and magnetism. Students in these fields must show a proficiency equivalent to an average grade in this course to be admitted to candidacy for the Ph.D. degree at this institute. It is hoped that this book will also provide a reference where workers in these fields may find the methods of attack on those problems, common in research, for which the formulas in the handbooks are inadequate. It is assumed that the reader has the mathematical preparation usually acquired in a good undergraduate course in introduction to mathematical physics. This implies a reasonable familiarity with vector analysis, the calculus, and elementary differential equations. All the mathematical development beyond this point is done herein by methods that a reader with such training can follow. The author has succeeded, with some difficulty, in avoiding the use of contour integration but believes that anyone going further into the subject should acquire a working knowledge of this powerful mathematical tool.

As has already been implied, this book is written for the experimental research physicist and engineer rather than for the theoretical man. For this reason, only that theory which has applications is included, and this is developed by the most concise method compatible with the assumed preparation of the reader. No subject is included for its historical interest alone. More than the usual number of problems have been worked out in the text, and these have been selected for one or more of the following reasons: the result has important applications, it clarifies the theory, it illustrates some useful mathematical device, it proves the utility of some concept in the theory. At the end of each chapter is an extensive collection of problems involving nearly all the theory in the text. Many of these are taken from the Cambridge University examination questions as printed in Jeans's "The Mathematical Theory of Electricity and Magnetism," Cambridge, 1925, and are included with the permission of the Cambridge University Press to whom we express our thanks. Although the best students can work all the problems, the average student cannot. They provide an oppor-

tunity for the reader who is working up the subject by himself to test his proficiency. In many cases important results, which lack of space prevented working out in the text, are included as problems and are listed in the index.

The treatment and arrangement of the subject matter depart from the most common practice in several respects. In the first place, it will be noticed that all developments are based directly on the macroscopic experimental facts rather than on the hypothetical microscopic structure of conductors and dielectrics. There are two reasons for this. The first is that, although the microscopic theory meets the crucial test, namely, gives the observed macroscopic laws within the precision of observation, this does not imply that the theory is unique or that other deductions from it are correct. The second is that the development of the most satisfactory theories, those based on wave mechanics, require mathematical technique with which we have assumed the reader to be unfamiliar when starting this book, but with which he will be quite familiar after finishing two-thirds of it. The development of these theories, necessarily brief, is therefore postponed until the last chapter. The second departure from the most common practice consists in the complete omission of the concept of the isolated magnetic pole. All magnetic theory is therefore based on the interactions of electric currents or moving charges. This logically leads to the use of the magnetic vector potential rather than the scalar potential, so that the former is used extensively, although not exclusively, in all magnetic and electromagnetic theory. It will surprise many to find that this leads to considerable simplification in certain places, notably in the calculation of inductance and the treatment of eddy currents and electromagnetic radiation. Other minor departures from the usual practice include the more extensive use of Bessel functions and conformal transformations and the treatment of forces between moving charges exclusively by the methods of special relativity. The latter procedure, besides having a very firm experimental basis, makes hypotheses as to the shape or size of electric charges unnecessary and gives a clearer idea of the limits within which the standard formulas can be applied without using such hypotheses.

Several subjects usually included in books on electricity and magnetism are omitted here. The theories of electrolytic conduction, thermionic emission, photoelectric phenomena, thermoelectric effects, etc., of which the reader is assumed to have an elementary knowledge, are entirely omitted because it is believed that a treatment of them, on the same level as the remainder of the book, would require a background of physical chemistry, thermodynamics, and quantum theory not possessed by the average reader. The theory of electrical machines and instruments, including vacuum tubes, is also omitted because it is

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believed that such subjects are best treated in connection with laboratory courses. Certain alternative methods such, for example, as the Heaviside operational method and the dynamic method of circuit analysis have been omitted for lack of space.

Before starting this book the average reader will have become familiar with all the common systems of electrical units and will have his own preference. The actual system used will make little difference provided it is clearly indicated. For each section of the subject, the author has used that system which seemed simplest to work with. Thus the c.g.s. electrostatic units are used in Chaps. I to V, the c.g.s. electromagnetic units in Chaps. VII to XII, and the Gaussian system in Chaps. XIII, XIV, and XV. To avoid confusion, the units used are noted at the bottom of each page and a very complete system of conversion tables is given in the Appendix, enabling the results of any calculation to be expressed in any units. To see whether the use of rational units would have simplified calculations, all the numbered formulas occurring in the preliminary lithoprinted edition were scrutinized. It was found that the complexity of 1196 formulas would be unchanged, that of 169 would be decreased, and that of 123 would be increased. Thus in the theory there is little to choose between rationalized and unrationalized units. The results in problems were not investigated.

It has been the practice throughout, whenever an integration is performed or a mathematical transformation made, to refer, by number, to the appropriate formula in both Peirce's "A Short Table of Integrals," Ginn, 1929, and Dwight's "Tables of Integrals and Other Mathematical Data," Macmillan, 1934 [4th ed., 1961]. It is therefore advisable for the prospective reader to procure a copy of one of these inexpensive tables. The bibliographies at the ends of the chapters are not by any means complete but include merely those books that have come to the author's attention which contain useful additional material or instructive alternative treatments.

The author has used every device he can think of to eliminate mistakes but is perfectly certain that they still persist and will be grateful to anyone pointing them out.

The author wishes to express his gratitude to the hundred graduate students, especially Dr. Pasternack, who worked through the preliminary lithoprinted edition, zealously detecting errors and obscure spots, and checking the answers to problems. He also wishes to thank Professors Ira S. Bowen and William V. Houston for reading sections of the manuscript and for valuable discussions. In particular, he wishes to thank Dr. Charles H. Townes for checking all the derivations in the final manuscript and the answers to those problems not appearing in the preliminary edition.

WILLIAM R. SMYTHE

August, 1939

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